

Problem 9.13, A. F. Grandt, Jr., *Fundamentals of Structural Integrity – Damage Tolerant Design and nondestructive Evaluation*, John Wiley & Sons, 2004.

9.13 Frustrated by their unsuccessful efforts to tear down the goal posts following several recent football victories, a group of students decide to *cut notches* in the posts to hasten the nucleation of fatigue cracks as they plan to bend the goal posts back and forth following their next victory. During a full-scale fatigue test of one of the notched posts, it is discovered that *ten students* can exert a nominal cyclic stress amplitude  $\pm S$  that causes a crack to form in 10,000,000 reversals. Since the students don't expect the stadium guards to allow them to remain on the field for the time required to apply 10,000,000 reversals (and since the students are anxious to return to their studies following the game), it is decided to recruit more students in order apply a larger cyclic stress, and, thus, hasten the fatigue failure of the goal posts. How many students are required to develop fatigue cracks in the goal posts in *100 reversals*? You may assume:

- The goal posts are made from a material with the following properties: Elastic modulus  $E = 10,000$  ksi, fatigue strength coefficient  $\sigma'_f = 100$  ksi, fatigue strength exponent  $b = -0.07$ , fatigue ductility coefficient  $\epsilon'_f = 0.6$ , fatigue ductility exponent  $c = -0.6$
- The completely reversed nominal stress amplitude is directly proportional to the number of students who rock the posts back and forth (i.e., 10 students causes  $\pm S$ , 20 students cause  $\pm 2S$ , etc.)
- Uniform notches are cut into the posts (same depth, radius of curvature, etc.)

*This is a notch problem that employs Neubers Rule (Eq. 9.24).*

$$K_f^2 \frac{\Delta s}{2} \frac{\Delta e}{2} = \frac{\Delta \epsilon}{2} \frac{\Delta \sigma}{2}$$

*If one assumes nominal elastic behavior,  $\frac{\Delta e}{2} = \frac{\Delta S}{2E}$  and we obtain Eq. 9.25*

$$\Rightarrow \frac{K_f^2}{E} \left( \frac{\Delta S}{2} \right)^2 = \frac{\Delta \sigma}{2} \frac{\Delta \epsilon}{2}$$

*For failure to occur at  $2N_f = 10^7$  reversals as observed in the original experiment with the goal posts*

$$\frac{\Delta S}{2} = \pm S = \text{nominal stress applied by 10 students}$$

*The goal post fails at the notch root in  $2N_f = 10^7$  reversals when the cyclic stress is obtained from Basquin's rule (Eq. 9.1).*

$$\frac{\Delta \sigma}{2} = \sigma_{f'} (2N_f)^b = 100 (10^7)^{-0.07} = 32.36 \text{ ksi}$$

Also the strain amplitude that causes failure at this life is given by the strain life-relation (Eq. 9.12)

$$\begin{aligned}\frac{\Delta \epsilon}{2} &= \frac{\sigma_{f'}}{E} (2N_f)^b + \epsilon_{f'} (2N_f)^c \\ &= \frac{100}{10,000} (10^7)^{-0.07} + 0.6 (10^7)^{-6} = 0.0033 = \frac{\Delta \epsilon}{2}\end{aligned}$$

Thus, by Neuber's rule

$$\begin{aligned}K_f^2 &= \frac{\frac{\Delta \sigma}{2} \frac{\Delta \epsilon}{2} E}{\left(\frac{\Delta s}{2}\right)^2} = \frac{(32.36)(.0033)(10,000)}{(s)^2} \\ \Rightarrow K_f &= 32.5/s = \text{fatigue notch factor}\end{aligned}$$

Now determine the cyclic load (i.e. number of students needed to vibrate the posts) to fail the posts at  $2N_f = 100$  reversals. In this case, failure occurs when

$$\begin{aligned}\frac{\Delta \sigma}{2} &= 100 (100)^{-0.07} = \sigma_{f'} (2N_f)^b = 72.4 \text{ ksi} = \frac{\Delta \sigma}{2} \\ \frac{\Delta \epsilon}{2} &= \frac{\Delta \sigma}{2E} + \epsilon_{f'} (2N_f)^c = \frac{72.4}{10,000} + .6 (100)^{-6} = .0451 = \frac{\Delta \epsilon}{2}\end{aligned}$$

Again, by Neuber's Rule

$$\begin{aligned}K_f^2 \left(\frac{\Delta s}{2}\right)^2 &= \frac{\Delta \epsilon}{2} \frac{\Delta \sigma}{2} E \\ \Rightarrow \frac{\Delta s}{2} &= \frac{\sqrt{\frac{\Delta \epsilon}{2} \frac{\Delta \sigma}{2} E}}{K_f} = \frac{\sqrt{(.0451)(72.4)(10,000)}}{\frac{32.5}{S}} \\ \Rightarrow \frac{\Delta s}{2} &= 5.55 S\end{aligned}$$

Since  $S \equiv 10$  students, we need ~56 students to fail the goal posts in 100 reversals. Assuming that this number of reversals can be applied relatively quickly, the students are now free to return to their dormitory to study for their next exam.