Durability of Automobile Gears

PART ONE

Spiral-Bevel Gears

by

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PART TWO

Transmission Gears

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Durability of Spiral-Bevel Gears for Automobiles

Part One

By J. O. Almen

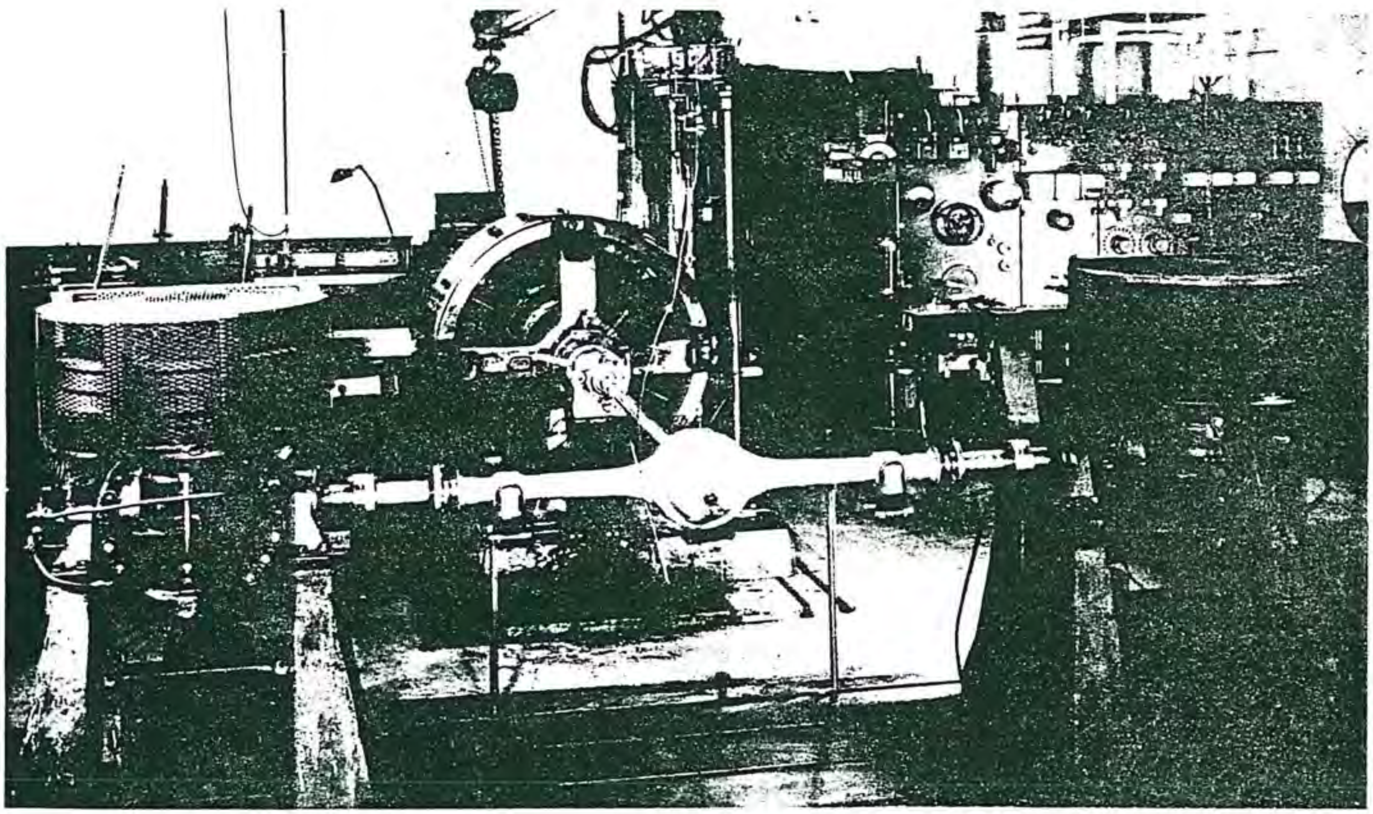


Fig. 1—Special type of rear axle equipment used by G-M Research Division in durability tests.

THE cause of failure of gears in service varies with the type of gears and with the service they perform but, in general, failure is due to tooth breakage, to the destruction of the tooth surfaces by wear of several kinds, and to pitting. For the past eight years, General Motors Research Division has made an organized effort to determine the causes of failure in automobile gears and to find practical remedies. In the course of this investigation, laboratory breakdown tests have been run on some four hundred automobile and truck rear axles, and thousands of service records have been examined for the purpose of correlating the laboratory tests and the performance of the gears in normal service. This study has led to some definite conclusions, particularly for spiral-bevel rear-axle gears, but it is probable that many of the factors that influence the service life of such gears will apply also to gear sets in general.

It has been found that gear-tooth breakage in service is due to fatigue, and that this fact provides data for more accurate tooth stress calculation than has been possible heretofore. Destruction of rear-axle gear-tooth surfaces by scoring is due to welding of small areas of the mating teeth under the influence of high pressure and high temperature. Pitting results from fatigue of the tooth surface due to repeated high compressive stresses, and is of the same nature as is encountered in ball and roller bearings.

Of the alloy steels and heat treatments that have been used for carburized automobile rear-axle gears, one cannot be shown to be superior to another, except for warping tendencies and consequent variation in stress concentration.

The potential load-carrying capacity of automobile gear teeth is not realized, due to stress concentrations resulting from deflection of the gears and their supports, unequal tooth spacing, eccentric and wabble mounting, etc.

Axle-Testing Equipment

The specialized type of rear axle-testing equipment used by General Motors Research Division is shown in Fig. 1. The propeller shaft of the rear axle undergoing test is coupled to a large direct-current, cradled dynamometer capable of 1570 lb.-ft. torque. The axle shafts of the rear axle are connected to two cradle-mounted alternating-current power-absorption units through a 13.66 to 1 speed increasing gear. This speed-increasing gear is made from a commercial worm-gear truck axle in which the worm wheel is used as the driving member and the worm as the driven member. A ten-pole alternator is mounted directly on each of the worm shafts. During tests, the alternators are usually run at 720 r.p.m., which results in synchronism with the alternating-current supply to which they are connected. This not only provides simple speed control means, but has the further advantage of reducing the current consumption through regeneration.

The load applied to the axle gear during the test is usually equal to maximum torque of the engine multiplied by the transmission low-gear ratio, that is, approximately three times maximum engine torque. The test at this load is continued until failure occurs, which, in practically all cases, is by breakage of one or more pinion teeth. Records are kept of the propeller-shaft load, axle-shaft loads, oil temperature and the number of revolutions of the propeller shaft during the test. It is customary to run at least three axles of each design. The average duration of these runs is considered a measure of the relative merit of the axle for direct comparison purposes. As a fatigue value, it is, of course, always better to use an average of several tests than to rely upon a single specimen. In the accompanying charts, each plotted point is, therefore, in most cases, an average of results from three or more axles.

It is not generally realized that no form of test short of operation in actual service will produce reliable data. When all the test conditions are made to faithfully reproducce actual service conditions for the rear axle, it will be found that not only must complete vehicles be used, but they must include driving the vehicle under all conditions of roads and loads, including the type of driving that is usually considered abusive, such as driving through deep mud or sand, carrying large overloads, etc. It will not suffice that the test be conducted by a test driver, since the proper proportions in various types of service will only be found through records of owner-driven cars. These data must be accumulated in the same manner as mortality tables are accumulated for the calculation of insurance rates, with the difference that the automobile designer does not have the privilege of eliminating the poor risks, that is, the abusive driver.

Efforts to draw conclusions from arbitrary laboratory tests all too frequently lead to misconceptions of design and material requirements that persist for long periods. An example is found in the long-held belief that axle-gear failure in service was due to shock loading, with the consequence that tests and material specifications were drawn to meet conditions that did not exist in service, as will be discussed.

Same years ago, Buckingham† suggested the general theory that gears in some types of high-speed service are subjected to high impact loads due to oscillation of the gears, and that gear-fatigue failure is often due to the overstress resulting from such shock loads, rather than to the normal driving loads.

Elastic Characteristics

To determine whether such impact loads were contributing to automobile rear-axle failures, measurements were made of the elastic characteristics of the rear-axle structure, from the front end of the propeller shaft to the road tires. It was found that the elasticity in this driving train, relative to the moment of inertia of the gears, is such as to make impact loading of the type described by Buckingham practically impossible in automobile rear-axle gears. Furthermore, the bulk of rear-axle gear failures in service were found to originate in a few isolated sections of the country during the spring and fall seasons. Their geographical distribution and seasonal character indicated hard pulling rather than fast driving as the reason for failure. However, as a further check for impact loading, road tests were conducted with telephone receivers coupled across the gear teeth, which showed continuous closed circuit under all driving conditions, except when the torque was reversed.

In the beginning, the tests herein reported were conducted for the General Motors car divisions merely as routine durability (or breakdown) tests, without any detailed consideration as to the manner of gear-tooth failure or to the factors influencing this failure. The main purpose of the test was to establish the durability of a new design relative to a past design on which service experience data had been compiled. Initially, then, the procedure was simply a matter of orderly recording and study of test data. As the tests progressed, however, it became apparent that many popular conceptions of materials and design did not agree with test results. The formula in general use for calculating bending stress in gear teeth was found unreliable. No consistent difference could be found among the various steels and heat treatments used in production. As later analyzed, variations in stress concentration due to deflections, tooth forms, machining scratches, etc., had so great an effect on fatigue resistance as to obscure the effects of various alloys and heat treatments. Laboratory tests on standard specimens were not in quantitative, and few in qualitative, agreement with these tests on gears.

It must be emphasized that, at the time this type of breakdown-test equipment was first put into operation, it was not generally recognized that normal pinion-tooth failures in service were due to fatigue. Therefore, before the results obtained by this laboratory fatigue test would be admissible, it was necessary to establish that this test rated gear assemblies in the same order as these gear assemblies were rated in actual owner service. Furthermore, even if normal service failure was found to be due to fatigue, as was indicated by examples of failures showing typical fatigue fracture, it would still be necessary to find out how a laboratory fatigue test should be conducted to rate the gears in their proper order. That is, should the gears be run at relatively low loads and high speeds, or should they be tested at high loads and low speeds? This question could be answered only by searching the service records for examples of production axles that were representative of several degrees of durability. The type of information sought for was found after reviewing thousands of service reports; that is, records were found showing relatively larger and smaller numbers of service failures for several production designs. With this information as a guide, it was found that axles tested under maximum low-gear torque on the axle-testing equipment, showed the same relative resistance to failure that was shown by the service records.

From these checks, it was believed reasonable to conclude that, when a

* Presented at the eighteenth annual meeting of the American Gear Manufacturers Association, Niagara Falls, Canada, Oct. 14-15 1935.

† Buckingham, "Dynamic Loads on Gear Teeth," Research Publication, Am. Soc. Mechanical Engrs. (1931).

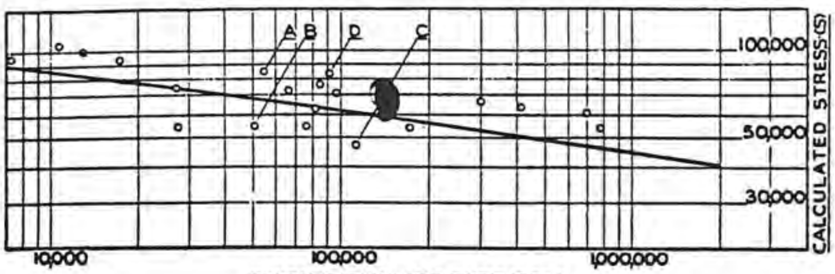


Fig. 2—Spiral bevel gear fatigue curve calculated by "Lewis" formula.

rear-axle gear fails in service, the failure is due to fatigue of the gear tooth as a result of intermittent loading; and that the fatigue life of a gear tooth is used up in the accumulative effects of short periods of operation under maximum low-gear torque.

From the comparison of the laboratory tests with service records, it was found that operation for 100,000 pinion cycles under maximum low-gear torque on the test equipment was equivalent to a lifetime of service in the automobile under the most severe operating conditions. The requirements for truck and bus axles are somewhat higher, owing to greater average severity of service.

In commercial gears and many other machine elements, such as connecting rods, valve springs, crankshafts, etc., the number of maximum stress cycles occurring during the life of the machine is such a large proportion of total cycles that the permissible stress at the fatigue limit is the important value to be used in design. Many automobile parts, such as rear axle gears, axle shafts, transmission gears, etc., differ from ordinary machine elements in that the number of maximum stress cycles is such a small proportion of the total cycles that they may be designed to operate under maximum loading at stresses far above the permissible stress at the endurance limit. As he been stated, the minimum required life of a rear-axle pinion at maximum stress is only 100,000 cycles, equivalent to about 30 miles of road travel. The tooth stress in normal car operation, that is, in high gear, is so low as to be negligible from a fatigue standpoint.

Several cases are known of service failures of rear axle gears in automobiles in actual owner service, the entire history of which were known. On the average, the total number of pinion cycles of operation in the car was 1000 times the number of cycles run in the laboratory test. In other words, the severity of service in the hands of the hard driver is approximately one one-thousandth as great as the service in our laboratory test. With due qualification, it might be roughly stated that, in the car, there is only one maximum stress cycle per thousand total cycles.

Need for Modified Stress Formula

From the beginning, the values obtained from these rear-axle-breakdown tests were plotted in the usual manner for fatigue specimens, as shown in Fig. 2. In this log-log plot, the calculated stress of the specimen is shown on the vertical scale and the number of stress cycles before failure occurs is shown on the horizontal scale. Data points plotted in this manner should have lain reasonably close to a straight line, if gear-tooth failure were really due to fatigue. However, the plotting of the first 20 points, corresponding to 62 individual axles broken in test, resulted in wide scattering of the points, as shown in the figure. This scattering was most disconcerting, since it indicated that gear failures were too erratic to permit their study as fatigue specimens, unless some rational explanation for the scattering could be found. Among the reasons that might be accountable for the unusual scattering was possible errors in the method by which gear tooth stresses were calculated. The method that had been used was the gear-tooth-stress formula originally introduced by Dr. Wilfred Lewis some forty years ago.

A modification of the Lewis formula had been suggested by McMullen and Durkan in "Machinery," June, 1922, but the new formula did not come into general use. When the tooth stress of the same 62 axle gears was recalculated by McMullen and Durkan modification, they plotted to the curve shown in Fig. 3. Here we find little evidence of the disorder that characterized the original plot. The points lie as close to a straight line as could be expected from highly accurate laboratory fatigue specimens, notwithstanding the fact that these test points represent ordinary production axles of many sizes and designs. It is a striking proof of the greater accuracy of the modified gear tooth-stress formula. Note that the gears represented by points A and B failed after approximately the same number of stress cycles, indicating that they were actually stressed alike. In Fig. 2, the gears represented by point A were calculated by the Lewis formula to be stressed 85,000 lb. per sq. in., whereas the gears represented by point B were stressed, according to the Lewis formula, to 55,000 lb. per sq. in. When calculated by the modified formula, these gears were found to be stressed alike, that is, approximately 47,000 lb. per sq. in. Or, comparing points D and C, Fig. 2, calculated by the Lewis formula, point D was stressed to 85,000 lb. per sq. in. and point C stressed to 47,000 lb. By the modified formula and as plotted in Fig. 3, the stress was calculated at 43,000 lb. per sq. in. for point D and 41,000 lb. per sq. in. for point C. Thus, the Lewis method of calculating stress may introduce inaccuracies on the order of 80 per cent, whereas the McMullen and Durkan method reduces these inaccuracies to negligible amounts.

In comparing Figs. 2 and 3, it will be noted that the stress scales in the two plots are quite different. No means are yet available for determining the actual stresses, and the stress scales used are, therefore, purely relative. Either stress scale may be multiplied by a constant without altering the real value of the plot.

For commercial gears of a type and material similar to rear-axle gears, it may be found necessary to design for continuous operation at maximum

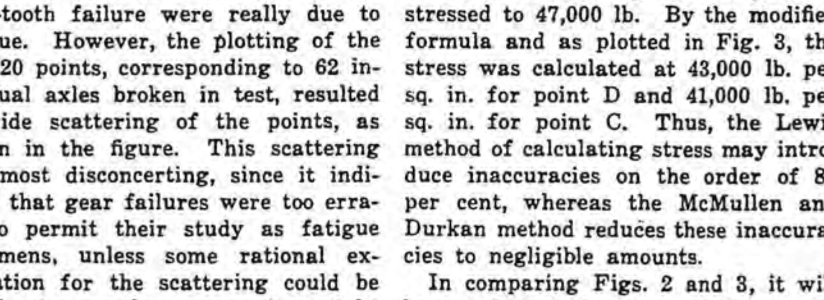


Fig. 3—Spiral bevel gear fatigue curve stress calculated by modified formula.

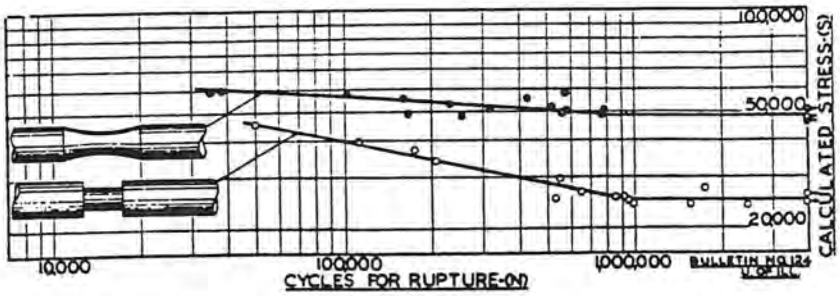


Fig. 4—Effect of shape of specimen on SN diagram from rotating beam tests.

stress. The permissible stress for such gears may be found by extending the fatigue curve shown in Fig. 3 to 10,000,000 cycles, which may be taken as the fatigue limit. This will show that gears designed for 20,000 lb. per sq. in. should be capable of operating continuously at this stress without tooth breakage.

Effect of Stress Concentration on Fatigue Life

Fig. 4 shows two fatigue curves taken from the University of Illinois Bulletin No. 124, by H. F. Moore and J. B. Kommers. The rotating beam specimens from which these curves were made were identical as to material and heat treatment, but different in form. The specimen for the upper curve, having an endurance limit at 49,000 lb. per sq. in. stress, is of the form shown on the chart; the lower curve, showing an endurance limit of 24,000 lb. per sq. in., is for the shouldered specimen. This stress difference, however, is not real, but is simply the result of calculating the stress in the shouldered specimen without allowance for the stress concentration resulting from the sudden change in section. The fatigue tests, which respond accurately to the real stresses, provide means for correcting the stress formula for the shouldered specimen in terms of the uniform stress specimen, since the real stress is obviously the same for both forms.

The difference between the real stress and the calculated stress will not be the same for different forms of fatigue specimens; that is, the errors in stress calculation will vary, depending upon local conditions producing stress concentrations. The true stress in any form of laboratory specimen having stress concentration can only be found, so far as the author is aware, by a sufficient number of fatigue tests on the specimen to construct a fatigue curve from which the stress concentration factor may be determined by comparison with similar data on a simple specimen. This method of determining true stress is, unfortunately, not yet applicable to machine elements, since it only admits of stress determination in terms of a uniformly-stressed, or other form of standardized specimen. General Motors Research Division has attempted to correlate tests of rotating beam-fatigue specimens and rear-axle gear tests, with the hope that the true stress could be determined and materials evaluated, but the results have been disappointing.

The Carburized Gear Tooth—Specifications for Case and Core

The shock theory of tooth failure in axle gears has led to erroneous conclusions with regard to relative strength requirements of case and core. In consequence, laboratory tests designed to aid in evaluating materials have led the investigator astray, since these tests were responsible for the widely-held belief that any carburized gear tooth should have a hard case and a tough core. The fatigue theory of failure alters the conception of case and core requirements.

For many years it was customary to test automobile gears in various impact machines. The designer of this type of test assumed that gear teeth in service were subjected to hammer-like blows, and he, therefore, attempted to duplicate in the laboratory this type of load. Under this test the best material was that which resisted the greatest number of blows, notwithstanding the fact that the gear was usually ruined after the first impact. Hence, the specification that carburized gears must be of such materials and heat treatment as would produce a hard surface to resist wear, and a tough core to resist breakage by impact.

When it is realized that rear-axle gear teeth are not subject to hammer-like blows, the tough core requirement vanishes. As an intermittently loaded beam, the tooth surface must not only resist wear, but, since the bending stress varies from a maximum at the surface to zero near the tooth center, it becomes important to provide a surface highly resistant to fatigue in bending; less fatigue resistance is required of the core, depending upon the depth of carburization.

Fig. 5 shows a qualitative distribution of stress between the case and the core. The vertical ordinate represents the half thickness of a gear tooth, the horizontal scale represents stress. The actual surface stress is usually far greater than the calculated value, due to stress concentration caused by surface irregularities, as illustrated by the dotted line. As calculated, the maximum stress in the core is less than the stress in the case, by a relatively small amount if the case is thin. Unless the fatigue resistance of the core is proportionally as great as the fatigue resistance of the case, fracture will start in the core. Stress concentration factors, such as rough or scratched surfaces, change of section, etc., have the effect of increasing the stress difference between the case and core, thereby reducing the fatigue-strength requirement of the case below the value indicated by direct calculation.

Fig. 6 shows the results of three groups of rear-axle gears carburized to different depths. Point E is the aver-

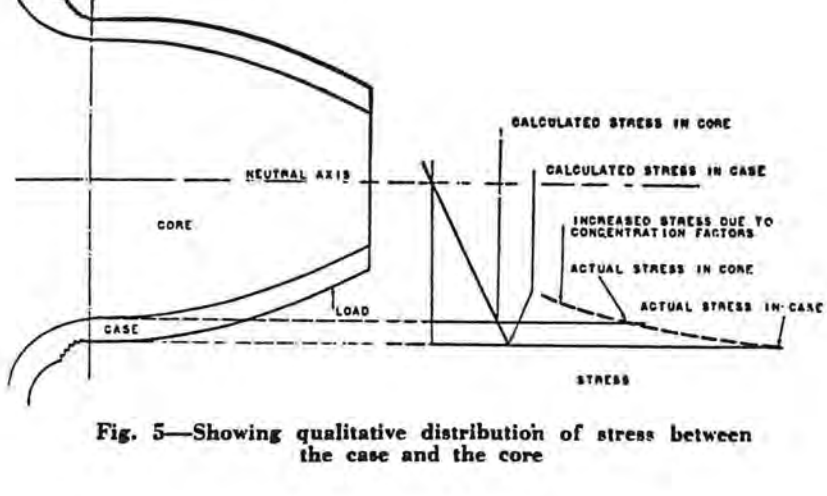
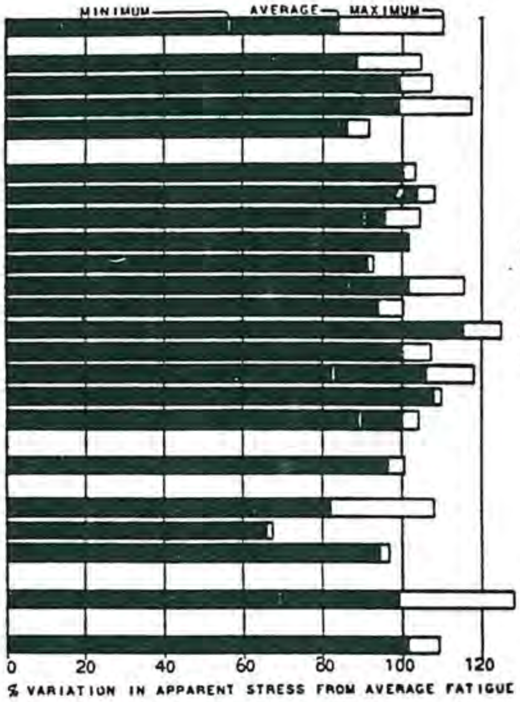


Fig. 5—Showing qualitative distribution of stress between the case and the core

Table I



STEEL SPEC.	CARB. TEMP. °F.	CASE DEPTH INS.	QUENCH	REHEAT TEMP. °F.	QUENCH	DRAW TEMP. °F.	DRAW MINS.	NO. OF AXLES TESTED
2315-A	1660	.050-.055	IN OIL	1325-35	IN OIL	300	30	18
2315-A	1600	.045	IN OIL	1325-35	IN OIL	275	30	89
2315-A	1675	.050-.055	IN OIL	-	-	300	30	14
2315-A	1660	.040-.050	IN OIL	-	-	-	-	7
2515-A	1650	-	COOL IN BOX	1475	IN OIL	-	-	3
2515-A	1700	.050-.055	COOL IN BOX	1400	IN OIL	300	-	9
2515-A	1675	.045	COOL IN BOX	1380 CYN.	IN OIL	250	30	11
◆2515-A	1675	.045	IN OIL	-	-	250	30	3
◆2515-A	1675	.045	COOL IN BOX	1380 CYN.	IN OIL	250	30	2
2515-A	1675	.045	IN OIL	-	-	250	-	10
◆2515-A	1675	.045	IN OIL	-	-	250	30	3
*2515-A	1675	.045	IN OIL	1380	CYN. FLASH & QUENCH IN OIL			2
◆2515-A	1675	.045	TRANS. CYN. & REDUCE TEMP. TO 1350 & OIL QUENCH					12
2515-A	1700	.050-.055	COOL IN BOX	1400	IN OIL	300	-	12
2515-A	1660	.040-.050	IN OIL	-	-	-	-	3
2515-A	1675	.045	COOL IN BOX	1380	IN OIL	300	COOL IN AIR	5
3115-A	1675	.050-.055	IN OIL	-	-	300	30	5
4615-A	1675	.050-.055	IN OIL	-	-	300	30	14
4615-A	1650-80	.050	IN OIL	1425	IN OIL	300	60	2
4615-A	1675	.040-.050	IN OIL	-	-	300	60	3
4620-A	1675	.040-.050	IN OIL	-	COOL IN AIR	300	60	14
4815-A	1680	.050-.055	IN OIL	1420-30	IN OIL	300	30	7

*COARSE GRAINED STEEL. ◆FINE GRAINED STEEL.

age of three gears carburized to approximately 0.037 in. depth, point F is the average of three gears carburized to the normal depth of approximately 0.045 in. depth and point G is the average of three gears carburized to a depth of approximately 0.082 in. These data are too meager to be conclusive, but they indicate that resistance to fatigue is improved with reasonable increase in depth of carburization. It should be noted that the normally carburized gears, point F, failed earlier than normal expectancy, and that the deeply carburized gears, point G, about matched normal expectancy.

The Effect of Material and Heat Treatment

Notwithstanding the many metallurgical reasons and tests that are advanced to show this or that alloy steel is best for rear axle-gear purposes, there is no evidence from the present study to show that, among the alloys usually used for this purpose, one is superior to another. Fig. 7 is the same average fatigue curve as Fig. 3, except that it records tests of 250 axles which were made from 22 combinations of alloys and heat treatments as noted in Table 1. It will be seen that, regardless of material, heat treatment or grain size, the test points for any given material lie about equally divided above and below the average curve. Large deviations from the average are due to other reasons than metallurgical ones, as will be shown.

It is not intended to deny that there are real differences in the various alloys or in the effects of the various heat treatments. These differences, however, have been determined from rather ideal laboratory tests, under which conditions such differences are large enough to become appreciable. In highly-finished, uniform-section structures, such as ball and roller bearings, wrist pins, ground shafts, and the like, the superior properties of expensive alloy steels are usually realized. In structures having high stress concentration, such as production rear-axle gears and many other machine elements, the properties of alloy steels, as determined by the usual laboratory tests, are not realized. The selection of steel for rear-axle gears should be governed by warping tendencies, machining characteristics and cost.

In Table 1 are listed the 22 combinations of alloys and heat treatments represented by the test points shown in Fig. 8. The bar chart at the left of the table compares these combinations on a stress basis as indicated by the tests. Note that, on the basis of this comparison, coarse grained 2515-A steel makes the best showing and that 4615-A steel is the poorest. This comparison, however, is not a true measure of the material or heat treatment. The differences are due to other factors, as discussed below.

Fig. 8 gives fatigue data for a series of tests that was designed to evaluate materials and heat treatments other than those used in production at that time. The series consisted of three axles each of nine combinations of materials and heat treatments. The early failure of these gears demonstrates the difficulty of producing good gears in small quantities, due to the effects of unfamiliar warping characteristics of the steel. In preparing for tests of new

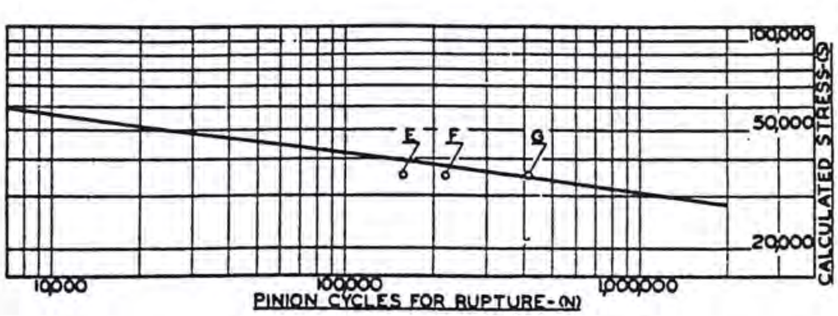


Fig. 6—Spiral bevel gear fatigue curve. Case depth $E = 0.032$ in., $F = 0.045$ in., $G = 0.082$ in.

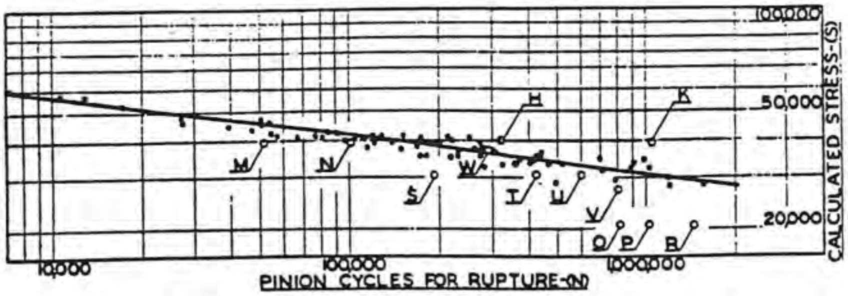


Fig. 7—Spiral bevel gear fatigue curve representing 250 rear axle gears.

production axle gears, it is customary to make a large number of gears and select from them the few that are good enough for use. The cutting is then adjusted to compensate for distortion.

The Effect of Stress Concentration Factors on Axle Life

Properties of the alloys are obscured by the much greater effects of stress concentrations, which, in gears, result from the tooth shape, machining scratches, deflection of the gears, shafts and bearings, eccentric assemblies, warping during heat treatment, etc. When considerable gain or loss is shown in the performance of a machine element, it is often ascribed to the particular alloy used, when, in fact, it is probable that the gain or loss resulted from a change of one or more of the stress-concentration factors.

Point H, Fig. 7, lies well above the average curve. This point is the average of three experimental rear axles which differed from production axles in that they were more rigidly supported, so that the stress concentration due to deflection was reduced.

The gears represented by points K, M and N, Fig. 7, were of identical design, material and heat treatment, notwithstanding which their lives varied as much as twenty to one. Point K is far above normal expectancy. The advantage in this case was reduced stress concentration, due, largely, to smoother finish of the roots of the teeth. Point M, lying below the average, was the result of bad machining scratches, together with bad tooth contact, which increased the stress concentration. The gears representing the intermediate point N had good tooth contact as for K but severe machining scratches as for M.

The photographs, Fig. 9, show the fractured pinion teeth from the axles represented by points K and N, Fig. 7. The one at the left was cut with slow cutter feed, the other with fast cutter feed. The difference in test durability was in the ratio of ten to one; that is, the coarse feed reduced the life of the gear from 1,000,000 cycles to 100,000 cycles. This test was not a deliberate test on the effect of cutter notches. The fine cutting feed was used while the gear cutting machine was being adjusted; the coarse feed was the normal cutting rate in production, aggravated by the fact that the cutting blades were not of uniform height. The use of cutter blades of equal height and with corners rounded to the maximum permissible radius, together with slower feed, greatly reduced the stress concentration due to cutter scratches. Note, however, that the fractures in these gear teeth follow the cutter scratches in both cases, demonstrating that further increased durability may be expected if practical means for producing smoother finish can be found.

The photographs show that these teeth were more highly stressed at one end than at the other, since the characteristic fatigue fracture does not extend the entire length of the tooth. Failure started at the root of the heel (large end) of the tooth and moved radially outward at that point as the fracture progressed toward the toe (small end) of the tooth. This is typical of all gear failures in our tests, and is mainly the result of elastic deformations, principally in the pinion antifriction bearings, causing load concentration at the heel of the tooth, as discussed later.

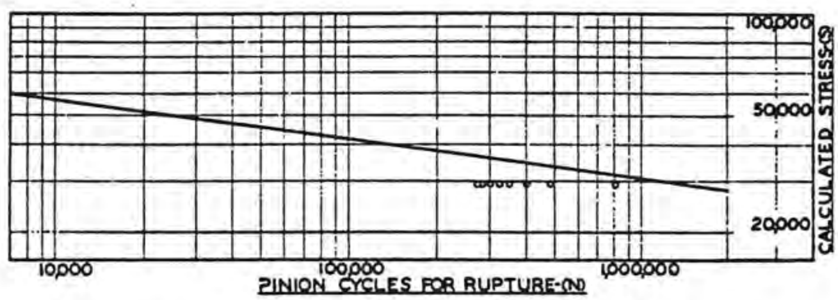


Fig. 8—Spiral bevel gear fatigue curve representing 27 special series rear axle gears.

Table II gives data on axles represented by points O, P, R, S, T, U and W, Fig. 7.

Points O, P and R represent axle gears that were unusually large for the car in which they were used, having, by extrapolation, a normal expectancy of 20,000,000 cycles, or two hundred times the minimum requirement. Because of their size, these gears were safe, notwithstanding extremely high stress concentration factors, and there was, therefore, no great incentive to improve their performance. The gears

Toe Heel Toe Heel

Fig. 9—Views of broken pinion teeth from the axles represented by points *K* and *N* in Fig. 7.

represented by points S, T, and U were for the same make of car but of somewhat smaller relative size. Although still over-size, an improvement was made in the rigidity of the pinion bearing, which is reflected in the position of the test points relative to the average curve. Subsequent increase of engine size required further improvement in design and shop practice, with the result that these smaller gears now have somewhat more than normal expectancy, as given by point W. The two gears represented by point O are responsible for the poor showing of 4615-A steel in Table I, which, as has been explained, was not the fault of either the material or heat treatment.

Point V, Fig. 7, represents the average of seven axle gears of identical design, material and heat treatment, with gears that formerly had given normal expectancy. This test point was low because the manufacturing had been transferred to a new plant, and new personnel and the shop technique had not yet been mastered. Later gears of this design and material returned to the normal curve.

Table II

Plot Point	No. of Tests	Steel Spec.	Minimum Life	Maximum Life	Average Life	Average Life in % of Expected Life
O	2	4615-A	656,700	969,000	813,000	4.8
P	5	4615-A	556,000	2,072,000	1,178,000	7.
R	4	2315-A	1,120,000	1,700,000	1,422,000	8.4
S	2	2315-A	141,000	242,000	191,600	19.
T	3	4615-A	233,000	537,000	427,600	43.
U	3	2315-A	218,500	1,275,000	604,000	60.
W	3	4615-A	195,500	450,800	293,000	122.

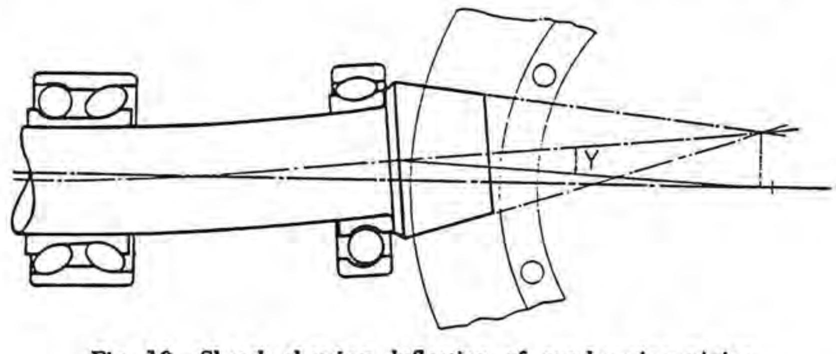


Fig. 10—Sketch showing deflection of overhanging pinion (exaggerated)

Fig. 10 shows an exaggerated sketch of deflection of an overhung mounted pinion under load, and the stress concentrating effect of such deflection on the contacting teeth. Although the sketch shows a ball bearing mounting, the results are the same for other forms of anti-friction bearings. Theoretically, the apex of the pinion cone coincides with the cone apex of the gear. This ideal condition does not prevail in practice, inasmuch as all parts of the gear assembly are deformed when load is applied. The pinion-bearing deformations are such as to shift the pinion axis through an angle which tends to localize the tooth loads at the heel of the teeth. A measure of this concentration is the angle Y, Fig. 10. Deflections in other directions are less serious in effect, since they do not result in as serious shifting of the load lengthwise of the teeth. Where space permits a better pinion support can be obtained by the use of straddle mounting. With this arrangement of bearings, it is theoretically possible to completely compensate for angular deflections, as shown in Fig. 11. If the rigidities of the two bearings are inversely proportional to their distances from the pinion apex, deflections of the pinion bearings under load will merely result in the pinion rotating about its apex. In practice, it would be necessary to overcompensate to allow for the deflection of the ring gear assembly and its supports. This means that the forward bearing of a straddle mount should have high radial elasticity and that the rear bearing should be highly rigid.

Fig. 12 shows the results of a series of tests made to determine the relative deflections of several pinion bearing combinations used in production automobiles. The heights of the bars are the measure of angular deflections for forward and for reverse drive, for each of the bearing types illustrated. The load applied was the resultant force corresponding to maximum low-gear torque, and was the same for all bearings. The tests were made on new bearings having equal capacity ratings and fitted in accordance with the tolerances specified by the manufacturers. The solid bars show the angular deflection readings obtained with bearings fitted to maximum shaft and housing tightness; the open bars show the difference between tight and loose fits according to the manufacturers' tolerance limits. The data shown are corrected for pinion-shank deflections.

The angular deflections found for the straddle-mounted pinions shown in Fig. 12 are small when compared to the deflections of overhung pinions, as would be expected from the deformations shown in Figs. 10 and 11. For complete compensation, however, the angular deflection of the pinion should be negative, that is, the pinion apex should fall below its original position (Fig. 11) as the pinion rises under load since the ring gear apex moves downward due to deflection of its supports. This can be accomplished by increasing the radial deflection of the forward pinion supports.

Economic considerations do not always justify the adoption of straddle-mounted pinions. Reduction of stress concentration in overhung pinions through the use of preloaded bearings, smooth fillets, reduced warpage, etc., permit the use of relatively small gears. The additional potential saving in size and weight that accompanies controlled elasticity can often not be realized because of design limitations, such as the available space between the pinion and the differential case.

In spiral bevel gears, the effect of deflection in the direction of the pinion axis is to partially compensate for angular deflection. Deflection in the direction of the ring-gear axis results in contact errors of the same kind as

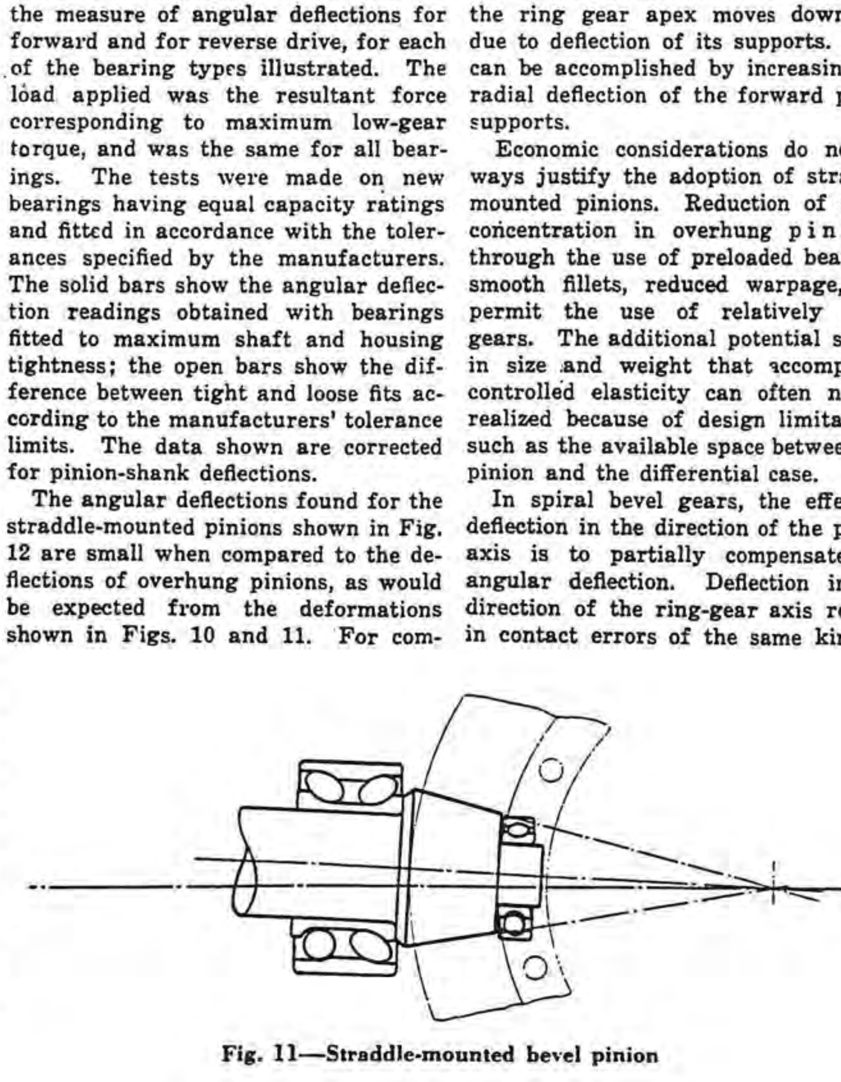


Fig. 11—Straddle-mounted bevel pinion

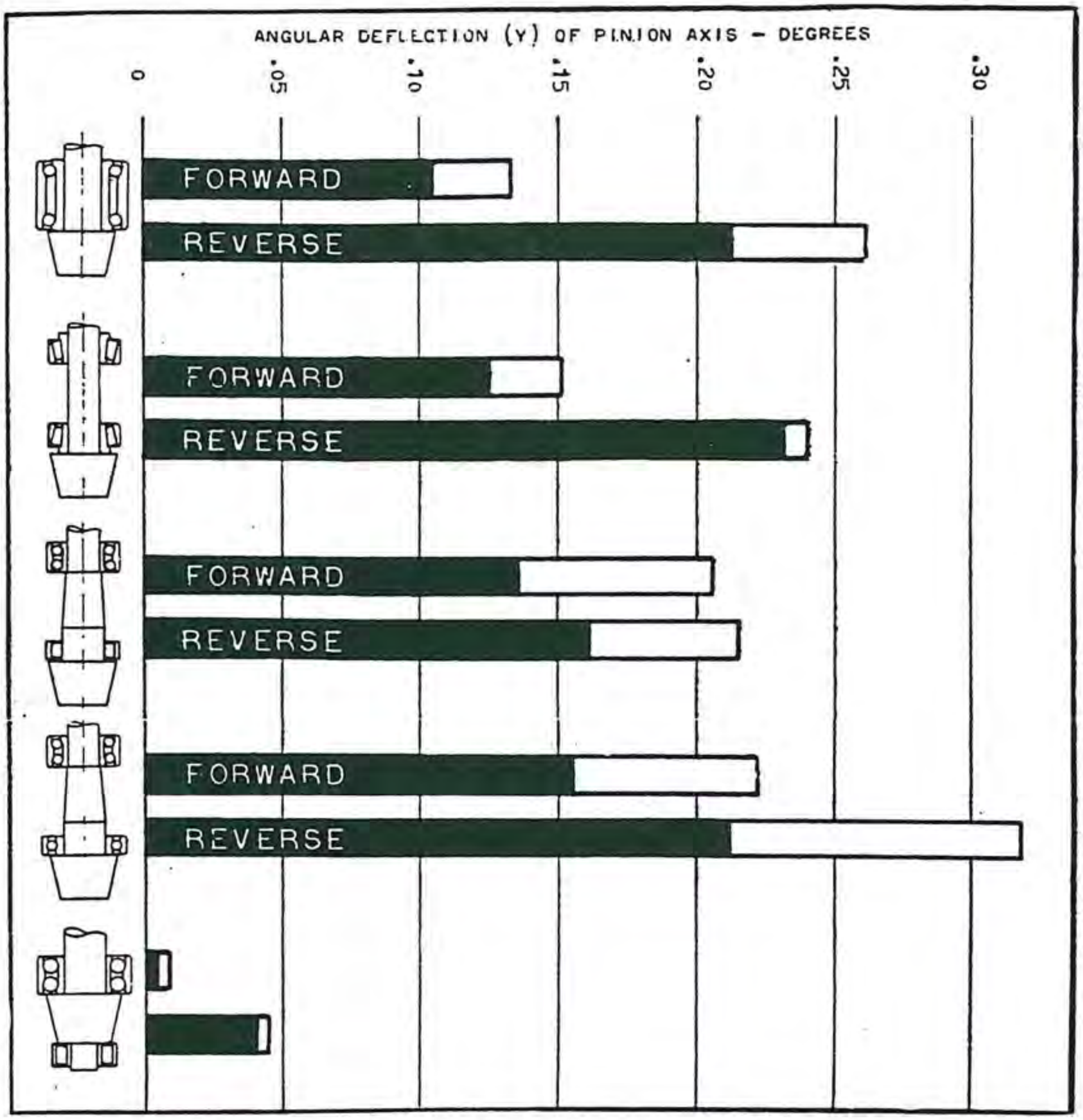


Fig. 12—Results of a series of tests made to determine the relative deflections of several pinion-bearing combinations

result from angular deflection, but of very much smaller amount.

The method used by General Motors Research Division for calculating gear-tooth stress assumes that the entire gear load is applied to one tooth only, notwithstanding the fact that several teeth may theoretically be in contact. Load concentration at the large ends of the teeth resulting from deflection of the kind illustrated in Fig. 10 reduces the overlap of the teeth, which, together with spacing errors, is sufficient to completely destroy the theoretical load distribution.

Photographs Taken While Gears are Running at Constant Speed

A part of normal routine in our gear testing is the photographing of the tooth-contact patterns of all gears tested. Photographs are taken while gears are running at constant speed under constant load, by means of a neon-lamp stroboscope. The rear axle is installed as shown in Fig. 1, with the back cover plate removed. The axle is brought up to speed and load. A neon lamp of special form is flashed in time with the gear teeth, by means of a cam on the input dynamometer which has as many lobes as the pinion has teeth, with the result that the gears appear to be stationary. The teeth are then sprayed with a specially prepared, quick-drying paint, which dries before reaching the point where the gears are in mesh. This paint must not only cover the teeth and dry quickly, but must also possess lubricating qualities, since the gears must be free from oil during the photographing process. As the ring-gear teeth come into contact with the teeth of the mating pinion, the paint is wiped off in the contact region, giving sufficient color contrast to enable pictures to be taken with an ordinary camera. As shown in Fig. 13, photographs are made under four loads, corresponding to a light driving load, full engine torque in direct drive, full engine torque in second gear, and full engine torque through low gear. Note that at light load the gears made contacts well toward the small ends of the teeth, and that as the load was increased, the contacts shifted toward the large ends of the teeth and away from the small ends. This is what would be expected from deflections of the kind shown in Fig. 10 and from the fractures shown in Fig. 9. These photographs measure the combined effect of deflections in all directions, as well as runout of all kinds.

To obtain this information by the

use of indicators is difficult, if not impossible. It has been shown that the most important measure of deflection in rear-axle gears is the relative position of the gear apexes. Separation of the apexes in the vertical plane is shown to be bad. Separation of the apexes fore and aft has a partial compensation effect on vertical apex separation. Apex separation in line with the ring-gear axes has relatively small effect. It is difficult to determine the movement of the apexes by indicator measurements. Furthermore, tooth contact conditions as affected by runout of the gear teeth due to warpage in heat treatment, or to wabble or eccentricity of the tooth cutting relative to other machined surfaces, can not be found by ordinary indicator measurements.

Fig. 13 (b) shows a set of tooth-contact photographs made from a gear identical in every respect with the gear shown in Fig. 13 (a), except that the gear-cutting machine was adjusted to give greater toe contact. Note that the shifting of contact with increasing load is less severe in Fig. 13 (b) than in Fig. 13 (a). This is most apparent when comparing the light-load photographs and the highest-load photographs. The reduction in stress concentration following the altered machine setting increased the life of the gear from 104,000 to 264,000 cycles.

Reduction of load concentration by setting the machine to cut toe contact is limited by the tendency of the gear to be noisy. Partial compensation for the inevitable shifting of contact with load is accomplished by cutting the teeth of the pinion to a spiral curve that differs from the spiral curve of the mating gear, as is shown in Fig. 14. The smaller curvature of the pinion-tooth curve permits this tooth to rock on the greater curvature of the gear tooth as the angular deformation varies. If these radii were equal, it is evident that the slightest angular deformation would shift the load from one end of the tooth to the other, with consequent high stress concentration. The greater the angular deflection, the greater must be the difference in radii of the teeth of the two mating gears, but this also leads to greater load concentration, since it limits the useful length of the teeth. Obviously, improvement would follow reduction in angular deflection permitting less difference in the radii of the teeth and thus producing more uniform stress distribution.

Photographs of the type shown in Fig 13 are satisfactory for routine estimates of deflections and resulting load concentrations. They are quickly made and easily interpreted, and in the General Motors Research Laboratory they have supplanted the old method of deflection measurements by indicators. This method, however, does not permit quantitative measures of load distribution. For this purpose, in one case, a series of etching tests was made by applying a static load equal to second-gear torque on an assembled axle and etching the surfaces of the gear teeth by passing sulfur dioxide and water vapor through the axle housing for a period of 30 hr. The areas of the gear teeth in actual contact were not etched, and it was, therefore, possible to measure the extent of the contacting areas. From these measurements and the known curvature of the teeth, it was possible to calculate the actual compressive stresses and the distribution of load, which were checked against the applied load and found to agree within about 10 per cent. The tests were repeated a sufficient number of times with slight changes in the phase relationship of the gears to construct a plot of load distribution over the face of the teeth.

Fig. 15 shows, as a composite of the loads on the three teeth in simultaneous contact, the qualitative distribution of load along the pitch line of the pinion tooth. The load concentration at the heel (large end) of the tooth is clearly indicated; likewise, the fallacy of rating gear-tooth loads in terms of the length of the tooth face. The biased distribution of load would be even more pronounced under maximum low-gear torque.

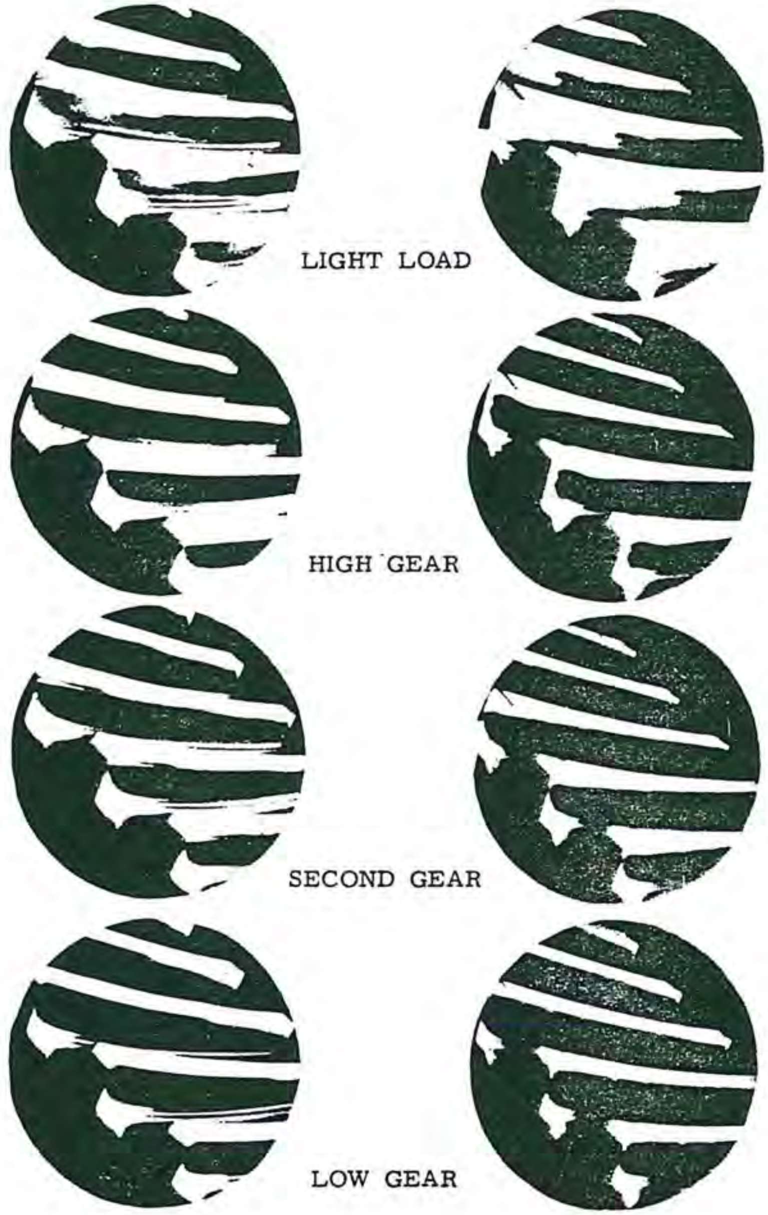


Fig. 13—Photographs are made under four loads

Stress Distribution Between Pinion and Ring Gear

An important consideration in the design of spiral bevel gears is the proper proportioning of stress between the two mating gears. As stated previously, the permissible calculated stress in the pinion teeth has been found to be approximately 42,000 lb. per sq. in., equivalent to 100,000 stress cycles. The permissible calculated stress in the ring gear has, by these same tests, been found to be 62,000 lb. per sq. in. With this stress distribution, either the pinion or ring gear would be liable to failure by fatigue. The higher permissible ring gear stress is due in part to the fact that automobile rear-axle gear ratios are on the order of four to one and, therefore, each ring-gear tooth is stressed only one-fourth as often as the pinion tooth. The required minimum life of the ring gear, therefore, need be only 25,000 stress cycles, which corresponds to approximately 51,000 lb. per sq. in. (see Fig. 3). The balance of the permissible stress discrepancy, that is, the difference between 51,000 lb. per sq. in. and 62,000 lb. per sq. in., appears to be due to lower stress concentration in the ring-gear teeth, probably as a result of less severe cutter scratches.

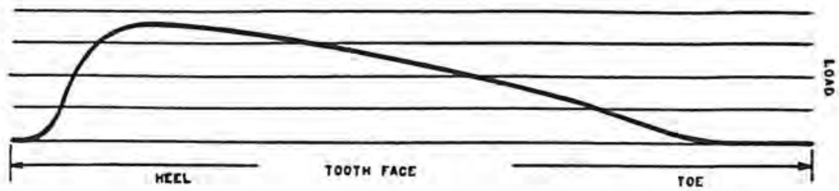


Fig. 15—The composite of the loads of three teeth in simultaneous contact, the qualitative distribution of load along the pitch line of the pinion tooth

Pitting

Surface pitting occurs in regions subjected to repeated high compressive stresses, and is a common form of gear failure. In automobile spiral bevel gears, pitting starts just below the theoretical pitch line of the pinion near the large ends of the teeth, and progresses toward the small ends of the teeth. In tests run at maximum low-gear torque, minute pits begin to appear after approximately 150,000 cycles of the propeller shaft and increase in size and number with continued running. However, pitting is not serious even after 2,000,000 cycles, which is far beyond the life requirements of the gears. In spiral-bevel truck gears, pitting may sometimes cause trouble due to the greater life requirements. Final failure in such cases is usually tooth breakage, but pitting aggravates stress concentration in the region where stress concentration is already serious.

The etching tests described above supplied data from which it was possible to calculate the unit load between the gear teeth by the Hertz method. These tests showed that pressures of the order of 300,000 lb. per sq. in. are reached under second-gear torque. The maximum pressure occurs some distance below the theoretical pitch line of the pinion and corresponds with the location of initial pitting.

If pitting is due to compressive fatigue of the tooth surface, it follows that the pitting tendency is at a minimum when the load is uniformly distributed over the teeth. Stress concentrations, due to deflections, warpage, runout, etc., promote pitting, as well as reduce the resistance to breakage. There is, however, considerable evidence indicating that gear-tooth pitting in these and many other cases is not caused by compression fatigue. The fact that pitting starts at a point below the theoretical pitch line may simply mean that the actual pitch line, under the conditions of operation, does not coincide with the theoretical pitch line. It is also noted that as the pits spread toward the small ends of the teeth, they follow a line parallel to the theoretical pitch line and do not follow the line of maximum unit pressure. Furthermore, when the pits first appear, they are of very small size, requiring considerable magnification for satisfactory observation. Once started, these small pits rapidly increase in size through breakdown of the side walls of the original pit. Pits due to compression fatigue should be of relatively large size on first appearance.

In dynamometer durability runs on automobile transmissions, it sometimes happens that pitting develops in gears that carry no load, such as the small reverse idler.

It is possible that a form of pitting may result from corrosion. It is well known that ball and roller bearings are subject to "corrosion brinell"; that is, the contacting surfaces become indented when the bearings are given slight motion while under load, even though the load is far below that required for true pressure indentation. When automobiles are shipped long distances in freight cars, it is usually found that the wheel bearings are slightly indented due to the small wheel motion resulting from the vibration of the freight car. "Corrosion brinell" is also common in automobile kingpin bearings, valve-rocker-arm bearings, and the like. Gear action at and near the pitch line appears to offer all the conditions necessary for "corrosion brinell."

We do not have sufficient data on the effect of materials on pitting in carburized gears to be conclusive. Tests have shown a tendency toward increased pitting with increased depth of case, but because many variables introduce load concentration, these data must also be accumulated from a larger number of tests before a definite trend can be established.

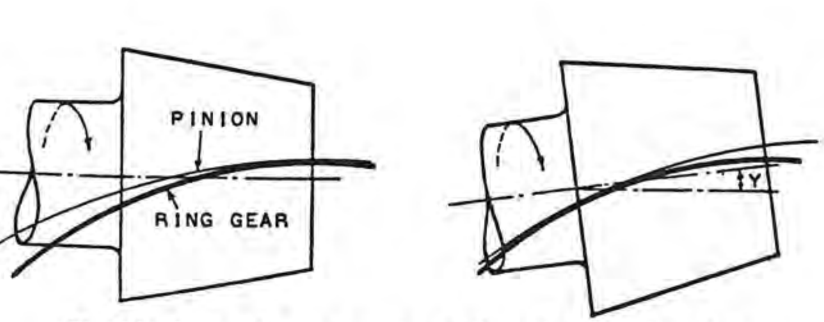


Fig. 14—Shows the compromises made to accommodate the teeth to a wide range of loads

Scoring

The type of gear-tooth wear variously referred to as scoring, roping, spalling, etc., occurs in highly-loaded spiral-bevel axle gears on the road when running at high speed, or on the application of overloads at moderate speed, such as dropping in the clutch while coasting down hill. Scoring is characterized by scratches in the direction of sliding between the mating teeth and appears to be caused by the welding of small areas of the contacting surfaces under the influence of high heat of friction and high unit pressure. Scoring does not occur in our laboratory tests, because the tests are designed to produce failure by breakage, which demands high stress. In the laboratory tests, the unit pressures are high, but the rubbing velocity is too low to generate sufficient heat to produce welding.

The same procedure that was used to obtain basic data on the resistance to breakage of gear teeth was followed to obtain basic data on gear scoring; that is, service records were examined to find the gear designs that scored in normal owner service and to determine the conditions under which this type of failure occurred. The designs that scored in service were then compared with designs that were free from this type of trouble, with the object of finding a practical measure for predetermining scoring tendency. The etching tests previously discussed supplied data for finding the unit pressure over the tooth surface. On the assumption that the instantaneous temperature is proportional to the product of unit pressure and the sliding velocity for poorly lubricated surfaces, a series of calculations were made to determine the pressure-velocity (PV) values over the tooth surface under various operating conditions. These calculations gave the greatest PV value at the top of the pinion tooth under a load corresponding to direct drive at road speeds somewhat below the maximum speed of the car.

In service scoring had been found to occur at high driving speeds and on severe use of the clutch on coasting. The latter condition did not permit calculation, since the applied loads were unknown, but the PV values calculated from the etching tests for forward drive agreed well with service experiences. A number of production cars and buses, some of which were subject to scoring and others in which no scoring had occurred, were calculated for PV factors under the conditions producing maximum PV values. The results are shown in Table III. It will be seen that no scoring occurred in the gears having PV values of less than 1,500,000, in which P equals the Hertz pressure in lb. per sq. in. and V the rubbing velocity in ft. per sec. All gears having PV values above 1,800,000 were subject to occasional scoring in service when lubricated with ordinary mineral oil. For purpose of design, a high PV limit of 1,500,000 is used for gears lubricated with ordinary mineral oil. In designs having higher values of PV, an E.P. lubricant must be used. For production spiral bevel gears, a mild type of E.P. lubricant is satisfactory. It should be remembered that these data have been taken from production, carburized, spiral-bevel gears, and that they do not necessarily apply to other forms of gears such, for example, as hypoids, in which the sliding velocity is somewhat greater. The formula used for all PV calculations was derived from the one series of etching tests. It is to be expected that there is some variation in the load distribution over the surfaces of the teeth in different spiral-bevel gear set designs, but the formula used gives values sufficiently accurate for practical purposes.

Stress concentration promotes scoring, just as it promotes breakage and pitting, particularly on the coasting side of the teeth. Since scoring in forward drive occurs at relatively low torque, when deflections are small (see Fig. 13), the load on the teeth is more uniformly spread over the surfaces of the teeth. However, the compromises that are made to accommodate the teeth to a wide range of loads, as illustrated in Fig. 14, prevent attainment of the best conditions. Warpage, runout, spacing errors, etc., also increase the scoring tendency. Reducing the rubbing velocity is effective in reducing scoring. This may be accomplished by reducing stress concentration factors to secure adequate resistance to breakage with a finer pitch and shorter teeth.

The most practical remedy for scoring is the use of an E.P. lubricant. Economy of material and weight demands the smallest gears that will carry the load, and by using E.P. lubricants, the gears for automobile rear axles may be designed from this standpoint, with, of course, due consideration to noise.

It has been found that gears are more likely to score when new than after they have been run for some time. For moderate PV values, it is sufficient that an E.P. lubricant be used for the original filling of the axle. Ordinary mineral oil may be used for subsequent fillings. This is due to the greater smoothness and in some instances to the work hardening of the bearing surfaces with use.

It is important to produce the maximum hardness of the surfaces of the teeth. In the heat-treating process, it sometimes happens that a thin surface layer is soft. This layer is so thin that it cannot be detected by the penetrating type of hardness testers, and recourse is had to the file test as the most practical and reliable method for determining surface hardness, especially in the hands of a skilled operator. Skin softness may be caused by decarburization where the gear is too long exposed to air while at a high temperature, or it may be the result of the sequence of operations followed in carburizing and hardening the gear. High drawing temperature, either in the furnace or through running too hot in service, may soften the gears and aggravate scoring.

Table III

Car	Scoring	Pinion R.P.M.	Pinion Torque Lb. Ft.	Compressive Stress Lb./Sq. In. "P"	Sliding Velocity Ft./Sec. "V"	"PV"
1	None	2400	328	71,900	16.70	1,200,000
2	None	3450	112	73,400	18.42	1,355,000
3	None	3710	100	84,700	16.40	1,392,000
4	None	3720	96	94,700	14.75	1,402,000
5	None	3820	90	87,200	16.05	1,403,000
6	None	3850	87	87,600	16.20	1,420,000
7	None	2970	91	80,000	17.85	1,430,000
8	None	4060	92	88,200	17.05	1,505,000
9	Occasional	3600	118	80,000	19.35	1,548,000
10	Occasional	4000	95	86,600	17.80	1,551,000
11	Occasional	3800	107	92,500	17.10	1,595,000
12	Serious	3600	97	109,200	15.40	1,676,000
13	Serious	3800	109	108,000	16.60	1,800,000
14	Bad	4266	88	79,300	23.25	1,848,000
15	Bad	3810	90	101,200	18.32	1,852,000
16	Bad	3800	109	95,800	20.05	1,932,000
17	Bad	2400	328	75,000	26.75	2,010,000

Other Types of Wear

Wear may be distinguished from scoring as a process in which the rubbing surfaces are wasted away. Wear may be slow lapping as a result of abrasives. When the lapping particles are large, such as sand or metal particles, scratched surfaces that resemble scoring may be observed. Fine abrasives, however, leave smooth surfaces. Abrasive materials may be introduced through insufficient cleaning of the gears, carrier or housing; they may be present in the lubricant, or they may be composed of metal particles from the surfaces of the teeth, as from initial roughness or incipient scoring.

Wear may also be a process of corrosion through chemical action. Materials such as free sulfur or chlorine, in the presence of water, will form acids that will attack the metal surfaces, unless there is also present in the lubricant a material that acts as an inhibitor to this action. For example, lead soap, in the case of a "lead soap plus free sulfur" E.P. lubricant, is an inhibitor for this type of corrosion. This corrosive action may pass unnoticed because the surfaces of the teeth retain their polish, since the products of corrosion are constantly rubbed off. E.P. lubricants depend on chemical activity for their action. Such lubricants are normal mineral oils to which have been added one or more of several chemicals that will combine chemically with the surfaces of the teeth to form a thin protective coating that prevents welding and, therefore, scoring. If this coating is rubbed off, a new coating is formed. Each time the coating is removed, some of the tooth material is lost. The difference in action of E.P. lubricants and corrosive agents is that the chemical bond is stronger and, therefore, can better resist being rubbed off. The quantity of metal removed by corrosive agents is large, whereas the quantity removed by E.P. lubricants is extremely small.

Ball bearings do not wear when lubricated with clean inert oil. When wear is observed in ball bearings, it may be taken as evidence that the oil used contains corrosive acids or abrasives.

Oil

Besides supplying lubrication in the lower load range, oil is a cooling agent. More often than not, the oil used in gear sets is too viscous to function as a good coolant, and frequently in high-speed gears the quantity used is so great as to add heat through churning. The use of low viscosity oil not only improves cooling but reduces churning losses. The viscosity of the oil that is doing the actual lubricating is the viscosity corresponding to the temperature of the metal to be lubricated. When the metal temperature is high, the oil viscosity is low, notwithstanding the apparent high viscosity of the oil in the housing. The use of low-viscosity oil does not mean correspondingly low viscosity as a lubricant, but does mean reduced metal temperature and increased efficiency. These statements in regard to the effect of viscosity are true only when the oils that are being considered are equal in their E.P. or their "oiliness" properties. There may be cases where small differences in the inherent E.P. or "oiliness" properties of the oils make appreciable differences in these results.

Durability of Automobile Transmission Gears

Part Two

By J. O. ALMEN and J. C. STRAUB

ACCURATE means of determining tooth stress is more important to the automobile gear designer than to the designer of industrial gears. Considerations of cost, weight and space demand that automobile gears be reduced to the smallest possible size consistent with satisfactory service. This paper deals with an investigation conducted by the General Motors Research Laboratory for the purpose of finding which of several methods of calculating the bending stresses in helical automobile transmission gears is the most reliable.

It is impossible to compute by rigorous mathematics the actual bending stresses in gear teeth, due to the many indeterminate variables that are involved. The usual gear formulae assume that the gears are accurately cut and mounted and that the gear material and the supporting structure are inflexible. Not only are these assumptions not realized in practice, but other factors invariably present localize stresses in an unpredictable manner. Various modifications of the Lewis formula are in use, presumably representing efforts to take into account the effect of these variable.

The method of comparing the reliability of stress formulae used in this investigation is similar to that used by the General Motors Research Laboratory on spiral bevel gears reported to this association in a paper presented at Niagara Falls, Canada, Oct. 15, 1935,* in which a fatigue curve was established for use in the design and study of spiral bevel gears. The method is based on the well-known logarithmic

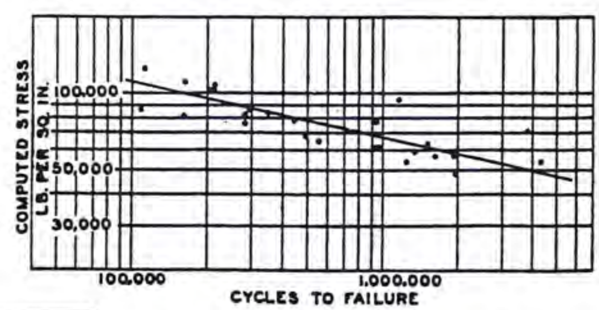


Fig. 2—Fatigue chart of 138 helical automobile transmission gears stress calculated by a method in extensive use.

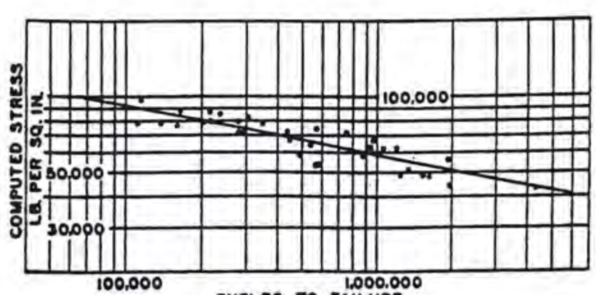


Fig. 1—Fatigue chart of 155 helical automobile transmission gears using the preferred method of calculating tooth stress.

*AUTOMOTIVE INDUSTRIES, Nov. 16, 1935, and Nov. 23, 1935, issues.

fatigue chart in which the number of stress cycles required to produce failure is plotted against the calculated applied stress. The plotted points in this chart lie on a straight line if the test specimen is of a form that permits accurate computation of actual or relative stress. Conversely, if the plotted points are widely scattered, it may be concluded that the stress calculations are in error.

To apply this method of analysis to stresses in transmission gear teeth, it is necessary to have a reasonably large number of breakdown tests of actual gears assembled in actual transmissions in which the applied load is held constant throughout the test and in which the duration of the test is known. Reliable breakdown test data on some 200 helical transmission gears were made available by the General Motors Manufacturing Divisions. All of these gears were run under the maximum torque for which they were designed. The test was continued until, in most casas, one of the operating gears failed by tooth breakage. The gears in this group of tests consist of 28 designs differing in such particulars as diameter, pitch, pressure angle, helix angle, face width, etc. Six types of alloy steel with as many heat treatment were used.

The stress for each gear design was calculated by nine different methods and the resulting stresses for each method were plotted against the actual life of the gears as determined by test. Figs. 1 to 3 show typical plots that were obtained. In these charts, each point represents from 1 to 35 gears, since each point is the average life of all of the gears of each design tested at one load. In the chart Fig. 1, all of the plotted points lie reasonably close to a straight line as compared to the wide scattering of points shown in Fig. 3. These two charts are, respectively, the best and the poorest obtained from the nine methods of calculation that have been tried. It may be, however, that another formula can be found that will give better results than have been obtained thus far. The method used to calculate the stress for plot Fig. 1 was proposed by C. H. Logue and has been in use for some years by one of the General Motors' divisions. The plot Fig. 2 shows the somewhat more scattered results that were obtained from a method of stress calculation used by another division of General Motors. Gear tests by both of these divisions are included in the plots. It is apparent that the method used in Fig. 1 is the more reliable.

The plot Fig. 3 is interesting because the method used for calculating stress for this chart is the same that gives the best results when applied to spiral bevel gears as is shown by Fig. 4. This is a plot of a large number of spiral bevel gear durability tests in which the stress was calculated by a method proposed by McMullen and Durkan.† The comparison shows the danger of using a single method of stress determination for all types of gears, for it is obviously impossible to predict the durability of helical gears by the latter method. The application of the Logue method used for Fig. 1 to spiral bevel gears would result in chaos similar to that shown in Fig. 3. The rational fatigue curves are sufficient justification for these two methods of calculating stress for their respective cases, even though it may be difficult to accept some of the assumptions that are made in constructing the formulae.

The values of stresses shown in these plots are not true stresses but are relative values for the gears of each chart only. The stress values for helical gears shown in Fig. 1 are twice as great as the stress values for spiral bevel gears shown in Fig. 4. This, however, does not mean that the actual stress is in this ratio. In fact, there are indications that the actual stress is greater in helical gears than in spiral bevel gears. The fact that true stress is not shown by these charts does not detract from the value of either method, since the only purpose of making a stress calculation is to enable the designer to pre-

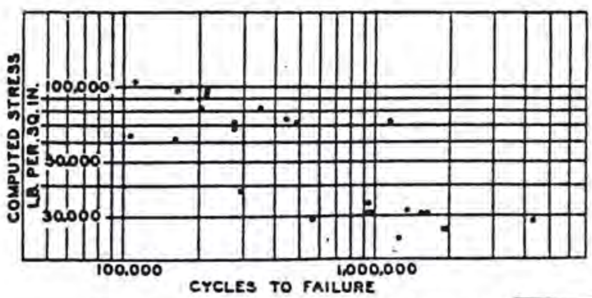


Fig. 3—Fatigue chart 138 helical automobile transmission gears stress calculated by the McMullen-Durkan method.

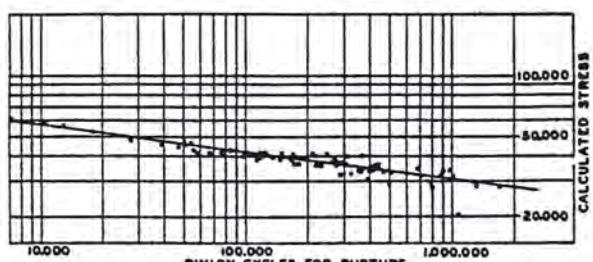


Fig. 4—Fatigue chart of 250 spiral bevel rear axle gears stress calculated by the McMullen-Durkan method.

† *Machinery*, June, 1922.

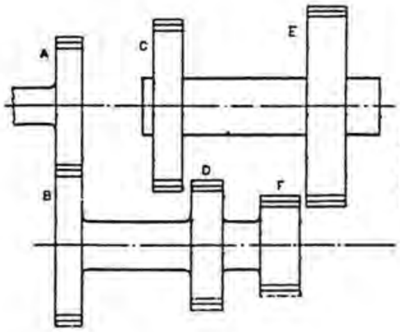


Fig. 5—Diagrammatic sketch of a transmission showing gears tested.

dict the performance of the gears in service. This requirement is met by the consistency obtained between the calculated and actual durability regardless of the numerical value of the stress.

Automobile gear service differs from industrial service in that the total time of operation at maximum load is relatively short. It is, therefore, not necessary that automobile gears be designed to run at maximum load to the fatigue endurance limit. As in rear axle gears, the minimum life for the pair of gears *E* and *F*, Fig. 5, that are used in low-gear ratio only, need not exceed 100,000 cycles at maximum engine torque to be free from breakage trouble during the entire life of the car. Gears *C* and *D*, used in second gear only, require somewhat greater life, approximately 300,000 cycles, since the accumulated operation in second gear at maximum load during the lifetime of the car is greater than for low-gear ratio. The gears *A* and *B* are in use during the accumulated time of low-gear operation as well as the accumulated time in second gear and, therefore, the minimum life for this pair should be the sum of the other two sets, or 400,000 cycles. It should be understood that the above limits are minimum values and that as a safety factor the average durability should be at least twice as great, since a variation in life of 200 per cent to 300 per cent in presumably identical gears may be expected.

The chart Fig. 6 is for the same gears calculated by the same method as for Fig. 1 except that each point represents a single gear instead of an average of all gears of one design. The variation in life shown in this chart is as much as ten to one for certain gears that were under development. This is interesting in showing that experimental gears are usually more variable in life than production gears. The transmission in which the gear life variation is the greatest is designed with a large safety factor, the average life for the second gear train being well over one million cycles.

Fig. 7 is a plot of a group of gears calculated by the same method used for Fig. 1, that were not run to destruction; that is, the test was stopped before failure occurred. Some of these points represent the remaining three gears in the train of four gears after one had failed; others represent tests that were stopped without any failures. This chart further emphasizes the large variation in the life of gears that are presumably identical in design, material and heat treatment. This group of unfailed gears, if run to failure, would show an average life curve at least 250 per cent greater than the average life curve shown in Fig. 1. It may, therefore, be said that the potential life of automobile transmission gears is much greater than is now obtained in practice. With an understanding of the factors that are responsible for this variation in life, it should be possible to increase average gear life to the upper curve shown in Fig. 7.

The large difference in life of helical transmission gears is the result of small variations in the contacts between mating teeth. The ideal contact conditions in which the load is distributed over the face of the teeth, as is promised by the layout and hoped for by the designer, is approached only by rare accident. If it is assumed that the teeth can be made to mate perfectly (which may be approached by careful lapping) the perfection is destroyed immediately the load is changed. When load is applied to the gears the teeth are deformed, the shafts bend, the case deflects, clearances are taken up, and the fancied perfection is impossible except under single load conditions. A clearer conception of the deformations that occure can be had by imagining all parts of the transmission to be made of rubber. With such a transmission before him, the designer would have a much better appreciation of his problem.

Compare the fatigue chart Fig. 1 for helical transmission gears and the fatigue chart Fig. 4 for spiral bevel gears. Note the greater scattering of the plotted points from the average fatigue curve in Fig. 1. Also bear in

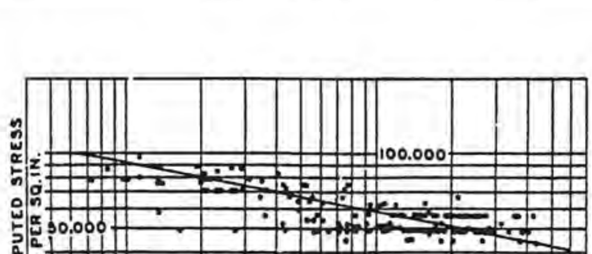


Fig. 6—Fatigue chart of 155 helical automobile transmission gears. This chart is the same as Fig. 1 except that individual points are plotted.

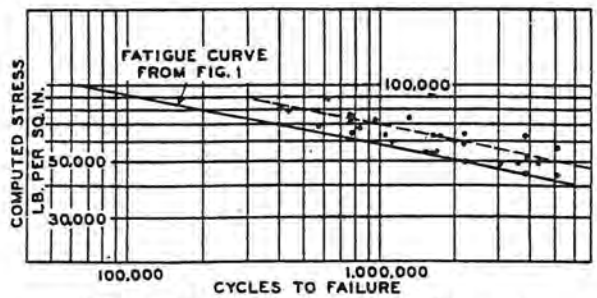


Fig. 7—Not a fatigue chart. The 132 plotted points represent tests that were stopped without gear failure.

mind the still greater discrepancy if the unfailed gears plotted in Fig. 7 were included in Fig. 1, as would be quite proper for this purpose. The more uniform durability of spiral bevel gears is not accidental but is inherent in the design of the gears. In spiral bevel gears the impossibility of attaining ideal contacts is recognized, and the teeth are deliberately cut to mismate in order to avoid the greater contact error that would result if the ideal contact were attempted. Fig. 8 shows four exaggerated sketches of mating teeth of a spiral bevel pair. Sketch *G* illustrates an ideal mating, the teeth resembling the type of contact sought for in spur and helical gears. Sketch *H* illustrates the contact conditions as they would be under increased load. The deflections of the pinion shaft and supporting bearings are such as to concentrate the load at the large ends of the teeth, the result being highly concentrated stress and early failure. Sketch *J* illustrates the contact in spiral bevel gears as actually cut. The radius of curvature of the pinion tooth is greater than the radius of curvature of the gear tooth by an amount sufficient to permit the deflections illustrated in *K* without the severe concentration of load at the large ends of the teeth as illustrated in *H*. This rocking-chair principle not only increases the resistance to fatigue by reducing stress concentrations, but also is necessary to avoid gear noise.

In contrast to spiral-bevel-gear practice, spur and helical gears are cut to mate in the manner illustrated in *G* and in consequence actually mate as illustrated in *H*, due not only to deflections but also to unavoidable errors in helix angle as a result of warpage, manufacturing tolerances, etc. Small variations in helix angle from whatever source result in a large increase in stress in localized areas, and greater variations in life, since the life of the gear varies approximately as the fifth power of the stress as determined by the slope of the curve shown in Fig. 1.

This slope also is additional evidence that stress concentrations are greater in helical than in spiral bevel gears. It is well known that, other things being equal, the slope of fatigue curves increases as the stress concentrations are increased. From Fig. 4 it will be seen that the life of spiral bevel gears varies approximately as the seventh power of the stress.

Although this paper is primarily concerned with the breakage of gear teeth, it is evident that localized loading will produce other types of failure, such as wear and pitting. In automobile transmissions, however, pitting and wear are of secondary importance.

Numerous methods have been proposed for cutting spur and helical gears to obtain a tolerance for helix angle error as is done in spiral bevel and hypoid gears, but none have come into extensive commercial use. The desired result may be obtained by running straight cut teeth against slightly curved teeth, or both sets of teeth may be curved on slightly different radii. The relative curvature of the mating teeth should be proportional to the relative change in helix angle through the load range and to the initial error of gear alignment. The less the change in total helix angle, the less need be the difference in curvature. If commercial means can be found to cut gears in this manner, the result will be to increase the fatigue strength of the gears and reduce gear noise.

Throughout the fatigue tests of helical transmission gears, it has been observed that in a pair of mating identical gears the gear that will break depends upon which ends of the teeth are most heavily loaded. This is as would be expected from the shape of the teeth, as is shown in Fig. 9, in which *L* and *M* illustrate identical pairs of gears ex-

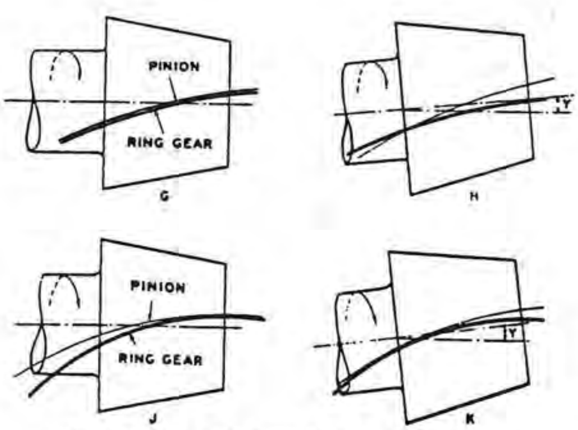


Fig. 8—Sketches showing drastic load concentration if perfect contact is attempted in spiral bevel gears contrasted with the method used in practice to avoid this concentration.

cept for the direction of helix angle error. Assuming that the upper gear in each case drives in a clockwise direction, the helix angle error in *L* is such as to concentrate the load on the left-hand end of the teeth, in *M* on the right-hand end. For the mating error shown in *L*, the lower gear will break; for the mating error shown in *M*, the upper gear will break. This is because the tooth flank forms an acute angle with the tooth end, and the load, which is normal to the tooth, is applied on an overhung portion of the tooth. The mating tooth carries its load adjacent to the obtuse angled end, and is, therefore, stronger. This order of failure will hold for either direction of rotation and for either direction of load.

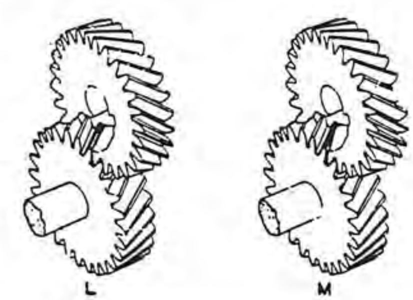


Fig. 9 — Helical gears in mesh, showing concentrated load on teeth due to helix angle errors.

If the contact error is always on one end of the teeth, partial compensation is possible by increasing the thickness of the disadvantageously loaded tooth and correspondingly decreasing the thickness of the advantageously loaded teeth. This expedient is resorted to in automobile spiral bevel gears and results in a large increase in life for forward drive, but since in such gears the contact concentration is on the same end of the teeth for forward and reverse drive, the ring gear life is reduced when driven in reverse. This is permissible, however, since the life requirements for reverse drive are less than for forward drive. The advantage of this compensation in spiral bevel gears is often confused by "four-square" test fixtures in which gears are loaded in both forward and reverse. In such tests it is usually found that the pinion of the gears that are driven forward will fail, but that earlier failure will occur in the ring gear for the gears that are run in reverse.

Another method of avoiding stress concentration on the acute-angled end of helical gear teeth is shown in Fig. 10 *N*. The mating gears are displaced in an axial direction an amount sufficient to prevent contact on the overhung portion of the teeth. This method is useful only for one direction of drive. If the acute-angled ends of the teeth of both gears are relieved, as shown in Fig. 10 *P*, the resistance to breakage is increased for either direction of drive and for load concentration at either end of the teeth. This is an approach to cutting the gears for large helix-angle tolerance, as is practiced in rear-axle gears, and will result in greater durability than the offset method shown in Fig. 10 *N*. In addition to the compensations shown in Fig. 10 *N* and *P*, it is usually found that mating errors due to deflections are sufficiently great to require that the mating gears be cut with different helix angles. Not only do these corrective measures increase gear life, but they are also effective in reducing gear noise.

The durability of helical gears is greatly influenced by the finish of the teeth. The sharp edges, particularly on the acute-angled ends of the teeth, should be removed, even though the gears are offset or relieved. The removal of the sharp edges is sometimes done without benefit and perhaps additional damage to gear life, when the chamfering operation leaves notches near the roots of the teeth. All sharp edges or irregularities, such as cutter marks, are points of stress concentration. The extent to which such concentrations should be avoided depends upon the cost of their removal to obtain the required durability as compared to the cost of increased gear size.

Six varieties of alloy steel, each with its own heat treatment, are represented in the lot of gears plotted in Figs. 1

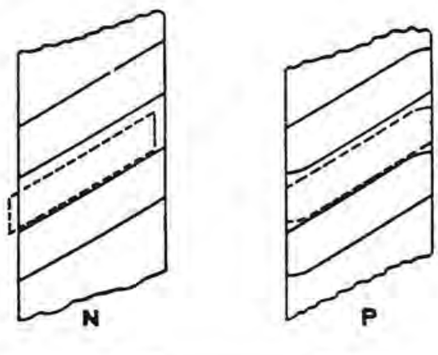


Fig. 10 — Sketch illustrating two methods of avoiding concentrated load at the weak end of the teeth.

Although the varieties of steels used in these tests and the number of gears represented by some of the steels tested is not large, there is little indication that any one is better or worse than any other in resistance to fatigue. This is in agreement with the findings for spiral bevel gears, in which the same comparison was possible on a larger number of steels and heat treatments, and justifies the conclusion that steel for transmission gears should be selected for uniformity of warpage in heat treatment, machinability and cost. Although experience with spiral bevel gears had indicated that no appreciable difference could be expected among ordinary carburizing gear steels, the present tests permit comparison of gears made of carburized and uncarburized steels. Three of the steels used were carburized and three were uncarburized. There is a very slight difference in the position of these steel types relative to the average fatigue curve, the carburized gears being somewhat higher. This, however, cannot be regarded as evidence of the superiority of carburized gears, since these materials were used in a different transmission and were produced by a different technique.

The large variations of automobile gears in resistance to fatigue are due to mechanical differences and not to metallurgical differences, assuming that good metallurgical practice has been followed. It is customary for the engineer to look upon the metallurgist as a sort of medicine man who can cure his design ills by metallurgical witchcraft. This belief has been encouraged by a large part of the metallurgical profession by the too literal application of laboratory test data on ideal specimens to roughly finished and irregular machine elements. The engineer has often failed to appreciate that the responsibility for producing good designs is his own, because he has not realized the importance of such "tremendous trifles" as stress concentrations on the durability of machine parts as have been discussed in connection with helical gears.

THE methods for calculating stress in helical automobile transmission gears used for fatigue chart, Fig. 1, and for spiral bevel gears used for fatigue chart, Fig. 4 (see also "Factors Influencing the Durability of Spiral Bevel Gears" AUTOMOTIVE INDUSTRIES, Nov. 16 and Nov. 23, 1935) are described below. The term "bending stress" used in these descriptions is not to be interpreted as actual bending stress, but as a figure proportional to the true stress which gives a measure of the endurance strength in terms of the design factors. The justification for the use of these methods of stress computation lies in the fact that the bending stresses so-called, when plotted on logarithmic graph paper against the actual life of many failed gears, give a straight line, from which it follows that the life of such gears can be predicted with reasonable accuracy.

Fig. 11 (*A*) shows a gear in the virtual plane, that is, the plane of rotation. However, this method of calculating the bending stress on helical gears involves a layout in the normal section which is represented by a plane through *G-G′*, Fig. 11 (*B*). This is a plane normal to the tooth as shown by the section *G-G′*, Fig. 11 (*C*).

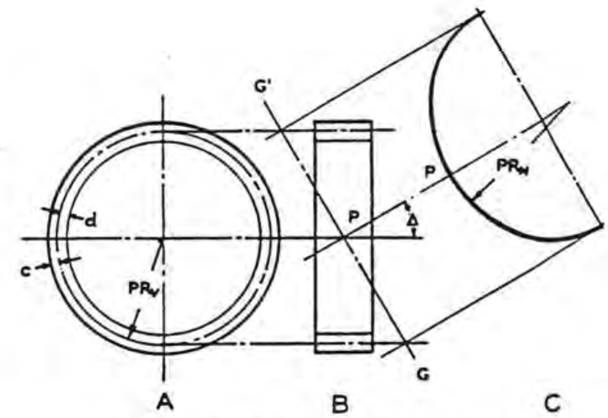


Fig. 11—A. Virtual section or plane of rotation.
B. ----
C. Section G-G′ or normal section.

PR_n, Fig. 11 (*C*), is the pitch radius at the point P in the normal section and is given by

$PR_n = PR_v \div \cos^3\Delta$,

in which

PR_v = pitch radius in the plane of rotation, Fig. 11 (*A*).

Δ = helix angle at the pitch circle Fig. 11 (*B*).

In the normal section, Fig. 11 (*C*), the base radius, *A*, is determined by the expression

$A = PR_n \times \cos\alpha$

α = normal pressure angle.

c and *d* are addendum and dedendum, respectively, as shown in Fig. 11 (*A*). They are unchanged when referred to the normal section.

Fig. 12 shows a layout in the normal section, the same as Fig. 11 (*C*), drawn to an enlarged scale. The construction lines are drawn in the order of the small numerals in Fig. 12. The scale of the layout is made as large as is practical.

(1) and (2) = center lines of the gear.
(3) = pitch circle whose radius = PR_n
(4) = base circle whose radius = *A*
(5) = root circle whose radius = $PR_n - d$
(6) = addendum circle whose radius = $PR_n + c$
(7) = line of action
(8) = normal to the line of action

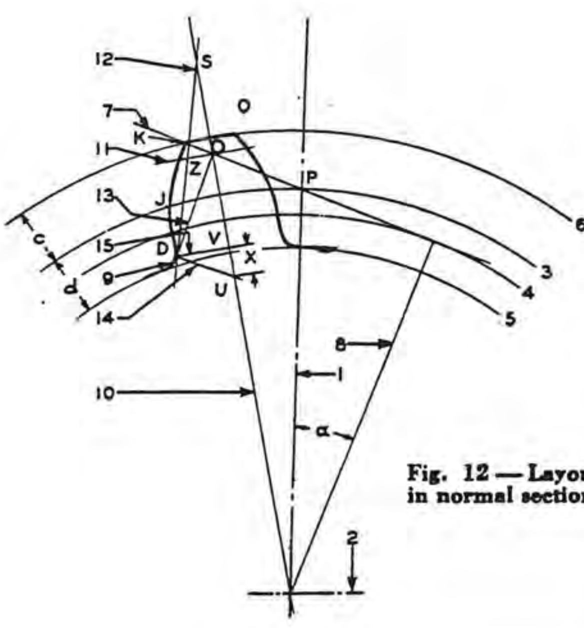


Fig. 12 — Layout in normal section.

The stress is computed with the load applied at the tip of the tooth. Consequently, the intersection K of the line of action with the addendum circle is the top of a tooth flank. The involute profile of the tooth is shown in Fig 12 for the purpose of illustration.

In order to determine the maximum stress in the tooth, it is necessary to construct the fillet (9). The fillet is generated by rolling the cutter rack on the pitch circle of the gear. Fig. 13 shows the process of generating the fillet. The initial position of the rack is shown in heavy lines. The successive positions of the rack are obtained by a slight rotation of the rack pitch line about the point of tangency. The accuracy of the entire calculation depends on the accuracy with which this fillet is generated. After generation, the fillet is drawn by connecting the points marked by (H) on the layout.

Referring again to Fig. 12, from the point J, half of the tooth thickness at the pitch line is laid off, and the tooth centerline 10 is drawn.

Through the intersection O of the tooth centerline and the line of action, the normal 11 is erected. The point of maximum stress is now found by drawing the line 12 tangent to the fillet and cutting the line 11 at Z and line 10 at S, such that $DZ = ZS$. The point of tangency D is the point of maximum stress. From this point DV is drawn normal to the centerline 10. OD is drawn and DU is drawn normal to OD. UV is then the X factor used in the stress equation:

$$S = \frac{3\pi T}{N \times F \times N_a \times X}$$

in which

S = bending stress on the tooth

T = applied torque on driving gear, in lb-in.

N = number of teeth in driving gear

F = face length of gear being calculated

N_a = length of action in the plane of rotation (see Fig. 14). This is not in the plane of the layout shown in Fig. 12

$N_a = \sqrt{OR^2_{pv} - PR^2_{pv} \times \cos^2\phi} + \sqrt{OR^2_{gv} - PR^2_{gv} \times \cos^2\phi} - (PR_{pv} + PR_{gv}) \sin\phi$

OR_{pv} = outside radius of pinion in the plane of rotation

PR_{pv} = pitch radius of pinion in the plane of rotation

OR_{gv} = outside radius of gear in the plane of rotation

PR_{gv} = pitch radius of gear in the plane of rotation

ϕ = pressure angle in the plane of rotation

$\tan\phi = \tan\alpha \div \cos\Delta$

X = tooth-form factor from layout = UV (Fig. 12)

The stress on the mating gear is calculated in the same way.

If one gear is appreciably wider than its mate, it has been found satisfactory to consider it a maximum of ⅛ inch wider than its mate.

In cases where the ends of the teeth are chamfered, the chamfer is neglected.

Note that in the procedure that has been followed, only the point J on the gear tooth flank has been used. Hence there is no need to draw the involute. Point J can be located during the generation of the fillet by marking the point where the flank of the rack tooth passes through the point of tangency of the pitch lines of the gear and rack.

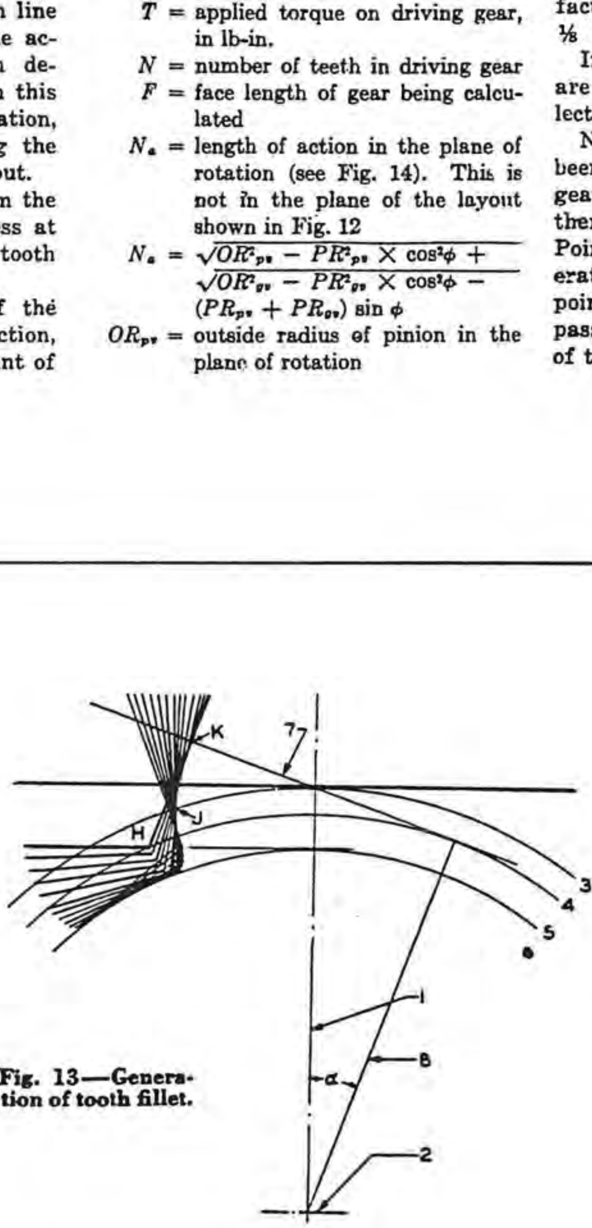


Fig. 13—Generation of tooth fillet.

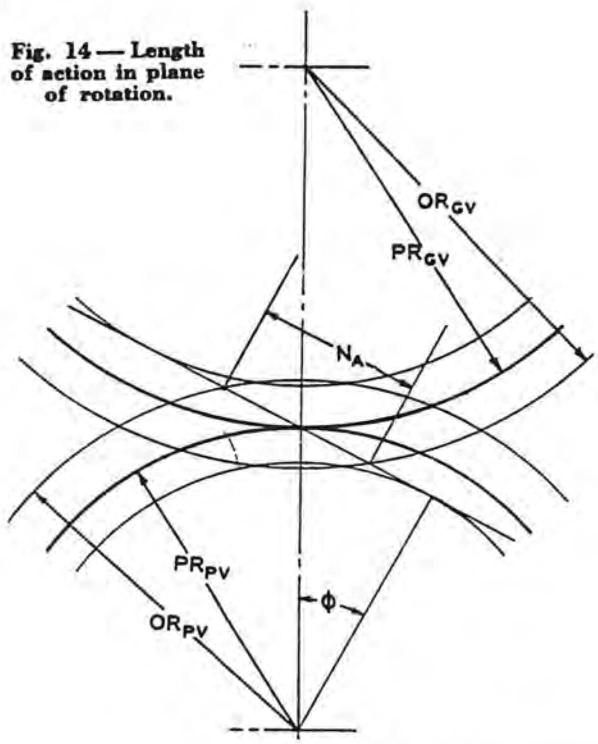


Fig. 14 — Length of action in plane of rotation.

$$R_p = \frac{N_p}{N_g} R_g$$

N_p = number of teeth in pinion

N_g = number of teeth in gear

F = face length of gear and pinion, in.

β = pitch cone angle of pinion

$$\tan\beta = \frac{R_p}{R_g} \text{ or } \tan\beta = \frac{N_p}{N_g}$$

L = pitch cone distance

$$L = \frac{R_p}{\sin\beta} \text{ or } L = \frac{R_g}{\cos\beta}$$

It can be seen from Fig 15 (A) that the center of the gear pitch line in the virtual section lies at the intersection of the gear axis with the line C-C' extended. With bevel-gear ratios common to automobile practice, the virtual pitch radius of the gear is so much greater than that of the pinion that the gear is considered to be a rack.

Fig. 15 (B) is a projection showing the virtual section in which the layout is made. In this layout, the pitch radius is equal to the back cone distance of the pinion. This radius is:

$$R_{pv} = R_p \div \cos\beta$$

In making the layout, it is necessary to determine the following dimensions in the virtual section:

Virtual pressure angle, ϕ

Base radius of the pinion in the virtual section, A_{pv}

Circular pitch, P

Addenda and dedenda of gear and pinion

$\tan\phi = \tan\alpha \div \cos\Delta$

α = normal pressure angle

Δ = spiral angle

$A_{pv} = R_{pv} \cos\phi$

$$P = \frac{2\pi R_p}{N_p}$$

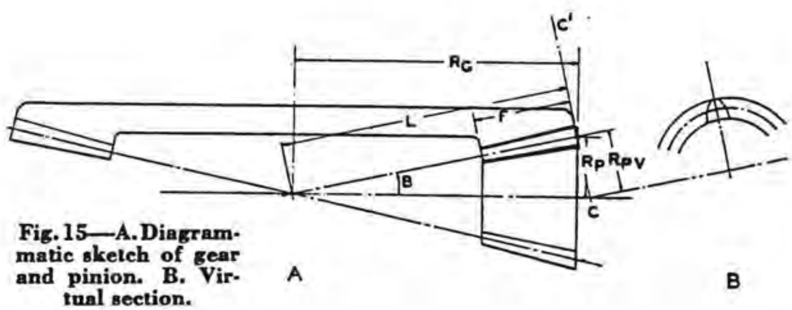


Fig. 15—A. Diagrammatic sketch of gear and pinion. B. Virtual section.

If the tooth thickness is given at a position other than at the pitch line, a short section of the tooth flank can easily be drawn by striking an arc through the point J, using as a center the point of tangency of a line through J tangent to the base circle.

In this method of computing the stress on helical gears, the load is assumed to be distributed uniformly on all teeth in contact.

The method of computing the bending stress in spiral bevel gears used for the fatigue chart, Fig. 4, involves a layout in the virtual section of the gear and pinion. The virtual section is a plane normal to the coincident pitch cone elements of the gear and pinion. The layout is made at the large end of the gears using the nominal spiral angle; that is, the spiral angle at mid-face. Usually, the dimensions are given at the large end, with the possible exception of tooth thickness, which will be discussed later in this report.

In Fig. 15, the virtual section passes through the line C-C', and is normal to the plane of the paper. C-C' is normal to the coinciding elements of the gear and pinion.

Referring to Fig. 15 (A):

R_g = normal pitch radius of gear at the large end

R_p = normal pitch radius of pinion at the large end

The circular pitch, addenda and dedena, are unchanged when converted into the virtual section.

A convenient method of laying out the involute profiles of the pinion tooth is to use an involute template. The template is generated from an arbitrary base circle (say 10 inch base radius) and can be used for practically any spiral bevel layout.

In order to convert the scale of the layout into the scale of the involute template, it is necessary to divide the base radius of the involute template by the virtual base radius of the pinion to be calculated. This gives the magnification factor, M.

$$M = \frac{\text{Base radius of template}}{A_{pv}}$$

All the dimensions of the layout must be multiplied by this magnification factor to convert them into the scale of the layout.

Fig. 16 shows a layout of the gears shown in Fig. 15. All of the dimensions are the same as in the projection in Fig. 15 (B) except that they have been enlarged by the magnification factor, M. The construction of the lines in the layout is made in the order of the small numerals in Fig. 16.

(1) and (2) = center lines of the pinion
(3) = pitch circle whose radius = R_{pv}
(4) = base circle whose radius = A_{pv}
(5) = root circle of pinion whose radius = R_{pv} − dedendum of pinion
(6) = addendum circle of pinion whose radius = R_{pv} + addendum of pinion
(7) = line of action
(8) = normal to the line of action
(9) = root line of gear
(10) = pitch line of gear
(11) = addendum line of gear

The position of the teeth is determined by the intersection G of the line of action 7 and the addendum line of the gear, 11. This point is the beginning of action, or the point at which one pinion tooth is just starting to share the load with the preceding tooth. The flank 12 of the gear rack is now drawn through G, normal to the line of action. The distance to the next rack tooth flank, 13, is equal to the circular pitch measured along the pitch line 10 of the rack.

Now it is necessary to know something about the thickness of the teeth. A convenient way of finding tooth thickness is to use the width of the tip of the gear cutter. This dimension is usually given on the detail print of the gear. It is referred to as "spread" or "finishing tool point width". Let us call it B_1. From this dimension the width of the gear tooth space at the root of the tooth is found. Since the dimension is given as of the cutter, it is normal to the tooth, and consequently it must be converted into the virtual section. Fig. 17 shows a diagrammatic view of a portion of the gear showing a tooth space at the root. The normal width of the space is shown as B_1. In the virtual section the width of this space is B_2.

$$B_2 = B_1 \div \cos \Delta$$

This gives the width of space at the middle of face or mean virtual section. Since the layout is made at the large end, this thickness must be transferred to conform with the other dimensions. This is done by increasing the space proportional to the distance from the apex.

The space at the large end then becomes:

$$B = B_2 \times \frac{L}{L - \frac{F}{2}}$$

or, in terms of B_1 :

$$B = \frac{B_1}{\frac{(1 - F)}{2L} \cos \Delta}$$

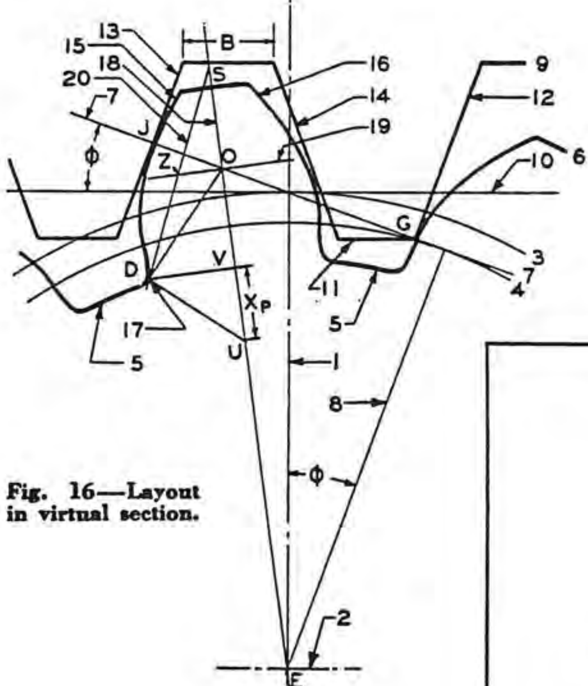


Fig. 16—Layout in virtual section.

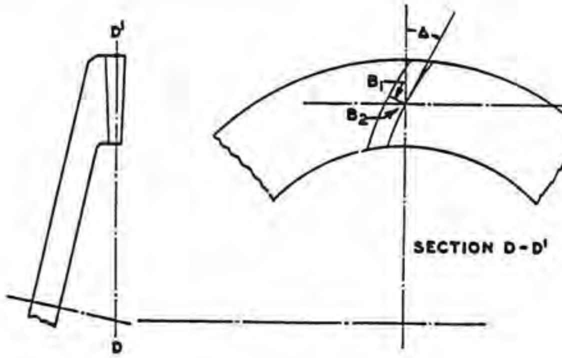


Fig. 17—Conversion of gear tooth space into virtual section.

This dimension, after being converted into the scale of the layout, is laid off at the root of the gear rack as shown in Fig. 16. The opposite side of the gear tooth space, 14, is now drawn.

The contacting face 15 of the pinion tooth is drawn tangent to the gear tooth flank 13 at J, where the line of action crosses it. Then the other side of the pinion tooth, 16, is drawn, allowing for backlash.

In order to determine the point of maximum stress in tooth 15-16, it is necessary to construct the fillet 17. The fillet is generated by rolling the cutter rack on the pitch circle of the pinion. Fig. 18 shows the process of generating the fillet. The cutter rack is the same as the gear rack, except that the end of the cutter rack tooth extended is tangent to the root circle of the pinion. The initial position of the cutter rack is shown in heavy lines. The successive positions of the rack are obtained by a slight rotation of the rack pitch line about the point of tangency. This must be done with care because the accuracy of the entire calculation depends on the accuracy with which this fillet is generated. After generation, the fillet is drawn by connecting the points marked by *H* on the layout.

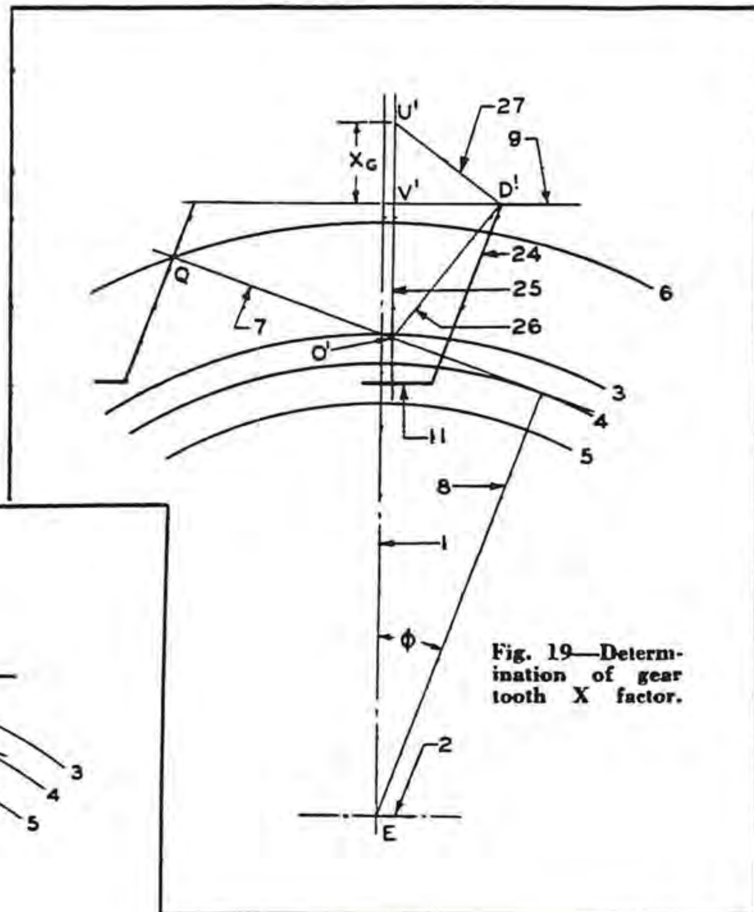


Fig. 19—Determination of gear tooth X factor.

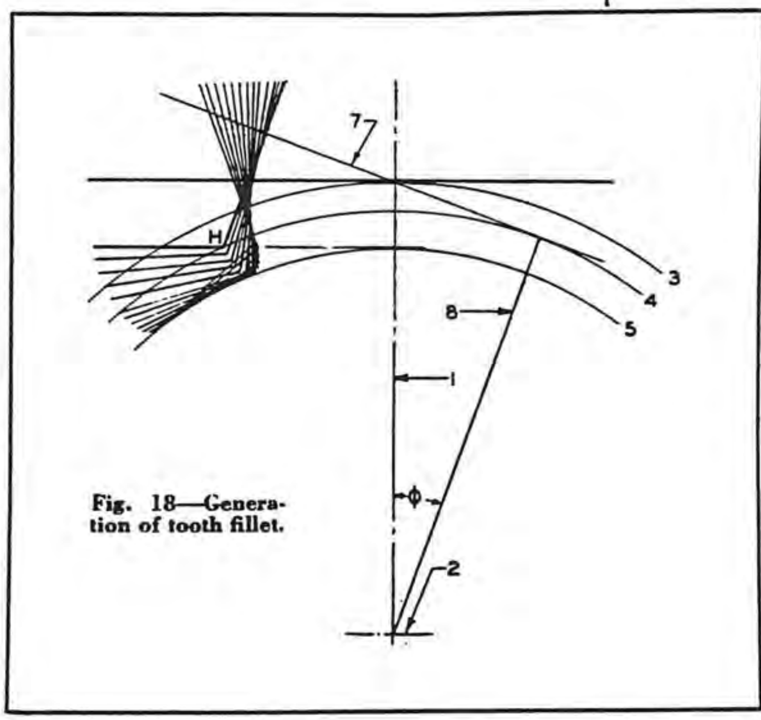


Fig. 18—Generation of tooth fillet.

Referring again to Fig. 16, the centerline 18 of the pinion tooth is now drawn, and through its intersection *O* with the line of action, the normal 19 is erected. The intersection point *O* is the point of application of the load on the tooth centerline.

The point of maximum stress is now found by drawing the line 20 tangent to the fillet and cutting the line 19 at Z and 18 at S such that DZ = ZS. The point of tangency D is the point of maximum stress. From this point DV is drawn normal to the centerline 18. OD is then drawn, and DU is drawn normal to OD. UV is then the X factor used in the stress equation:

$$S_p = \frac{1.5\, W_p}{F \times N \times X_p}$$

in which,

W_p = applied load at the point of application, *O*

$$W_p = \frac{T}{R}$$

T = applied torque on pinion in lb-in.
R = radius to point of load application, converted into the normal plane
$R = OE \times \cos\beta$
OE = distance from center *E* to point of load application *O*
F = face length, in.
N = modification factor for distribution of tooth load for spiral bevel gears

$$N = 1 - \frac{F}{L} + \frac{F^2}{3L^2}$$

X_p = tooth form factor taken from lay-layout

The stress on the gear tooth is calculated on the same basis as that of the pinion tooth, and the *X* factor can be found by using the same layout. Fig. 19 shows the construction for finding the *X* factor. For the purpose of illustration, the construction lines for the pinion stress have been eliminated, but no difficulty should be encountered in the actual layout with all construction lines intact. The position at which one tooth ceases sharing the load is found by the intersection Q of the line of action with the addendum circle of the pinion. The stress is calculated on the next tooth, the contacting flank of which is at a distance of one normal pitch from the point of intersection Q along the line of action. The normal pitch is JG in Fig. 16 and can be transferred to the new position, using a pair of dividers. The tooth flank 24 is drawn normal to the line of action, and cutting the root line at D'. $D'V'$ is made equal to half the tooth thickness at the root, which can be scaled from the gear tooth as shown in Fig. 16.

The centerline 25 of the tooth is now drawn, intersecting the line of action at O'. $O'D'$ is then drawn and $U'D'$ normal to it. $U'V'$ is the X factor for the gear.

The stress on the gear is

$$S_g = \frac{1.5\ W_g}{F \times N \times X_g}$$

The load Wg on the gear tooth is equal to the tangential load at the pitch line, or

$$W_g = \frac{T}{R_p}$$

In some cases, where the gear has a large pressure angle and long teeth, the maximum stress is not at the root of the tooth. The stress is maximum at the point where the tooth thickness is twice the thickness of the tooth at the point of load application on the tooth centerline. If this point falls deeper than the root, of course, the maximum stress is at the root.

In this method of computing the stress on spiral bevel gear teeth, the load is assumed to be carried by one tooth. This is in contrast with the foregoing stress calculation on helical gear teeth, in which the load is assumed to be distributed uniformly on all the teeth in contact.