AIRCRAFT GEARING

ANALYSIS
of
Test and Service Data

J. O. Almen and J. C. Straub

RESEARCH LABORATORIES DIVISION GENERAL MOTORS CORPORATION
Detroit, Michigan

FOREWORD

Since we rarely know the magnitude of loads that are applied to gears in service or the extent to which the stresses from the applied loads are concentrated by elastic deflections, thermal changes of dimensions, or by dimensional errors; and since we do not know the strength of our materials in their operating environments, or the effect of processing upon metal strength--to mention some of the more important unknowns and unknowables--any prediction of performance based upon assumed knowledge of such variables must necessarily be wide of the mark.

When gears must be designed in accordance with textbook procedures, which procedures are based upon many assumptions and misconceptions, they will inevitably be overdesigned and unbalanced with respect to the several kinds of failures that afflict gears. Most aircraft gears are greatly overdesigned against tooth breakage, with the result that overcaution against one kind of failure actually produces inferior gears because of their proneness to failures from causes not considered by the designer.

The only known way to produce well-balanced designs of machine parts, where intensive use of material is essential, is by the construction of empirical formulas from service records of large statistical samples. To accomplish this objective for aircraft engine gears, the Research Laboratories Division of the General Motors Corporation asked and obtained the cooperation of the major aircraft engine manufacturers during the war in the accumulating of suitable data.

The data are taken from service reports of many thousands of engines used in military and airline operations. They are concerned only with gears or gear combinations that failed from any cause attributable to design, material, or workmanship. It should be understood that "failure," as here used, does not necessarily mean chronic failure or catastrophic failure, but includes also occasional damage occurring to a few gears at some time during the service operation of the engine.

Suitable data were obtained on seventy-three gear combinations run under a variety of loads, speeds, and temperatures. External as well as internal gears were included. The torque ranged from two pound-feet to 6500 pound-feet; the speeds ranged from 1260 rpm to 28,000 rpm; and pitch line velocity ranged from 785 feet per minute to 19,100 feet per minute.

We wish to thank the following aircraft engine manufacturers, whose contribution of data made this study possible: Allison Division of

the General Motors Corporation, Continental Aviation & Engineering Corporation, Packard Motor Car Company, Pratt & Whitney Aircraft Division of United Aircraft Corporation, and Wright Aeronautical Corporation.

TABLE OF CONTENTS

INTRODUCTION

The loads that may safely be applied to automotive gear teeth exceed the loads that may be applied to gears made for any other service, regardless of precision or cost of manufacture. Automotive gears are not only capable of transmitting greater loads, but they also meet higher standards of quietness than highly loaded gears in any other service.

Automotive gears were developed by processes of trial and error over a period of many years, processes that closely resemble the evolutionary processes in nature. Numerous designers, working independently, contributed to the development by making various modifications to meet the dictates of space, tool equipment, or last-minute ratio changes. Frequently, design modifications were made with the hope that improvement in capacity or quietness would result. The automotive gears that we use today represent only a small fraction of the numerous design and material variations that were tried. These gears survived because actual service in literally millions of automobiles, used in all kinds of service by all kinds of drivers, proved them to be the best fitted for their jobs.

Since automotive gears were not developed by the methods that we are pleased to consider as rational applications of engineering and metallurgical fundamentals, they do not conform in many particulars to "good" engineering practices. Conventional design formulas were usually used by automotive gear designers, but, having learned by experience that these formulas were not reliable, each designer applied numerous modifications based upon his own experience, which, together with a sixth sense of fitness, enabled him to produce gears of light weight and low cost that were usually successful. Sometimes new designs would prove to be regressive, and occasionally, but often without intent, a new design proved to be a mutation which enabled a particular designer to advance, for a time, ahead of his competitors.

There is nothing wrong with applying evolutionary processes in machine design. In fact, until we have a much better understanding of the nature of our structural materials, and until we learn how our machine parts are loaded in service, no other process is available whereby we may progress toward more intense use of materials of construction. The process is unsatisfactory, however, in that, being based upon individual experience, the designers can seldom record their methods in mathematical formulas or other precise statements that will enable their successors to build directly upon past experience.

"Know-how" can be empirically expressed

This difficulty can be overcome if service data on a sufficient number of machine parts of a given type are available whereby empirical formulas can be constructed. By empirical methods, we may consolidate the experiences of all designers, manufacturers, and users and thus make immediately available that intangible, but essential, ingredient of modern production called "know-how." By empirical means, we may evaluate materials, processes, and design details to the end that any product for any particular use will give the best customer service for the least cost.

Empirical engineering formulas are in extensive use. They are the sole guides for determining the capacity of ball and roller bearings, and, more recently, empirical formulas have been constructed that have taken most of the guess work out of the design of bevel gears for automobile rear axle use and helical and spur gearing for automobile transmission use.

The construction of empirical formulas for any machine element is simple in principle but difficult in practice. It requires extensive data on the behavior of the type of machine part in question in actual user service. Laboratory data alone will not suffice, but data from laboratory tests that are properly correlated with service performance may be used to support service data. The empirical formula that was developed for automobile spiral bevel gears, now in universal use, is based upon the service records of many thousands of automobiles in actual user service. These data are supported by laboratory fatigue tests on several hundred complete automobile rear axles, in which test conditions were selected such that the type of failure and the locations of the failures were the same as failures that occurred in user service. Since service failures were relatively rare, and since it was necessary to differentiate between failures that were primarily the fault of the gears and failures in which the gears were damaged by some accident or as the result of the failure of related parts, such as failure of the supporting bearings, a very large statistical sample had to be examined.

Empirical formulas used in automotive gear design

The manner in which the empirical formulas for rear axle spiral bevel gears and transmission helical and spur gears were constructed is described in detail in the papers "Factors Influencing the Durability of Spiral-Bevel Gears for Automobiles," by J. O. Almen (*Automotive Industries*, November 16 and 23, 1935), and "Factors Influencing the Durability of Automobile Transmission Gears," by J. O. Almen and J. C. Straub (*Automotive Industries*, September 25 and October 9, 1937). Since, for these gears, the service data were supported by laboratory fatigue tests, the data could be presented in the form of fatigue curves. Since the stress was calculated by empirical formulas, the stress units are *not* pounds per square inch and the stress units calculated for spiral bevel gears do *not* have the same value as the stress units calculated for helical and spur gears. The stress cycles shown in the fatigue plots in the papers are, of course, the actual number of load applications to failure of each gear.

The permissible stress, as calculated for carburized spiral bevel gears used in passenger automobiles in which the pinions are overhung, as shown in Fig. 10 and the first four illustrations in Fig. 12 of the paper on spiral bevel gears, is 42,000 stress units. If the pinion is straddle-mounted, as shown in Fig. 11 and the last illustration of Fig. 12, the permissible stress is increased to 51,000 stress units because of the reduced deflection and consequent reduced stress concentration.

For truck service, the permissible stress for overhung pinions is 33,000 stress units and 43,000 stress units for straddle-mounted pinions. For bus service, the permissible stress for overhung pinions is 30,000 stress units and for straddle-mounted pinions the permissible stress is 40,000 stress units. The permissible stress may be further increased by approximately ten per cent by shot peening as a final operation.

By "permissible stress," is meant that spiral bevel gears stressed to not more than the amounts indicated above would not fail by breakage during the life of the vehicle, but at greater stresses, occasional tooth breakage might be expected.

Clutches influence strength of gears

At the time the spiral bevel gear formula was constructed, all automobiles were equipped with mechanical friction clutches, and because of harsh action of many clutches, the gear loads were often greater than the maximum engine torque multiplied by the transmission ratio. Such overloads would necessarily appear in the empirical formula and would help to establish the permissible stress. It is to be expected that automobiles equipped with hydraulic clutches may safely operate at greater calculated stress because harsh action is completely avoided, but the form of the empirical formula would probably not be altered.

Empirical formulas for predicting the fatigue strength by breakage of automotive hypoid gears have not yet been constructed for lack of sufficient authenticated service failures. The few service failures by breakage of teeth that have been recorded, together with a limited number of laboratory fatigue tests on hypoid rear axles; provide sufficient data to justify the use, at least temporarily, of the spiral bevel gear formula. However, for gear sets having ring gears of the same diameters, the permissible stress may be ten per cent greater for hypoids than for spiral bevels.

Machine parts subject to several kinds of failure

Many machine parts, including gears, are subject to several kinds of failures. As discussed above, gear teeth may fail by breakage, which is bending fatigue of the teeth loaded as cantilever beams. They may also fail by pitting, which is fatigue failure of the surfaces of the teeth by tensile stresses induced by the compressive loads; or they may fail by abrasion, one form of which results from welding of small areas of contacting teeth because of the heat of friction and intense pressure.

Since these three forms of failure are controllable by design, it is necessary, to assure the success of highly loaded, high-speed gears, that adequate precautions are taken against each form of failure. It is also necessary to avoid overdesign against one kind of failure at the expense of another. This requires that empirical formulas be constructed to establish design limits for each form of failure.

Given sufficient data, empirical formulas can be constructed to establish design limits for each form of failure for any particular type of service. It is probable that any adequate collection of data can be successfully expressed by more than one empirical formula, but no formula can be successful, except by rare chance, until adequate statistical data have been accumulated. It is desirable that an empirical formula should be rational; that is, that the formula should be based upon the conventions used in ordinary design calculations. This is not essential, however, since a formula may serve its purpose even though the terms used appear to be completely irrational.

Statistical data on aircraft gears

Prior to our entry into World War II, there were not enough airplane engines in use to provide sound bases for statistical studies of service failures that might lead to the construction of empirical formulas. The great increase in production and the extensive use of these engines in severe military service indicated that studies of service records might be profitable in supplying useful data for future designs.

The Research Laboratories Division of the General Motors Corporation initiated such a study in 1943 by asking the cooperation of several of the large manufacturers of airplane engines in supplying data on gear tooth failures from service records in airline and military operations. Data were requested on any gear used in production that had developed one or more of the several forms of distress enumerated above; that is, pitting, breakage, or severe wear. Data were also requested on the type of service, the time that the gears had been in service, and design details of both gears of a pair. The requested cooperation was freely given in every case, and useful data were supplied on one hundred and forty-six designs, ranging from main reduction gears to high-speed supercharger gears.

Service failures by breakage and by pitting were not numerous enough to be significant for the purpose of establishing definite design limits, but comparisons of bending stress and surface compressive stress of the one hundred and forty-six gear designs that had been submitted, with the bending and compressive stresses in automobile gears previously analyzed, showed that the aircraft engine gears should be relatively immune to breakage and to pitting.

"Scoring" of gear teeth

The wear-damaged gears were not available for examination, but since severe wear is almost always caused by local friction welding, hereafter called "scoring," this was assumed to be the cause of the wear distress in most of the one hundred and forty-six gears that were reported. This assumption was later supported by the high values of sliding and unit loading that were found in the course of the analysis. Scoring of automobile rear axle gears, lubricated with ordinary mineral oil, had previously been found to occur in gear sets in which the product of the Hertz compressive stress, in psi, and the tooth sliding velocity, in feet per second, exceeded 1,500,000. This simple method of predicting scoring in rear axle gears was successful, probably because the service, when scoring occurred, was approximately the same as to load, temperature, and sliding velocity, since the axle gears covered a relatively narrow range of size.

It was not to be expected that this simple method would suffice for predicting scoring in aircraft engine gears, for which the temperature, unit loading, and velocity of sliding varied over a much greater range. Actual trials had shown that for certain aircraft gears, PV values greater than 4,000,000 did not score (see "Dimensional Value of Lubricants in Gear Design," J. O. Almen, SAE Journal, Vol. 50, 1942). It was hoped that the new and more extensive data supplied by the cooperating aircraft engine manufacturers would aid in clearing up this discrepancy.

As in all empirical formula constructions, a considerable number of methods were tried to find numerical values for borderline scoring conditions in service. These included several trials of the general form P^mV^n, none of which were successful, as will be seen in the discussion of this report. Another geometric factor, T, was then added, as is shown in Fig. 1, which resulted in establishing a very good empirical limit, PVT, applicable to all gear sets (except as noted in the discussion) for scoring of aircraft gears lubricated with normal mineral oil.

Design conventions not applicable

It is not too difficult to accept the product PV as a measure of friction heat for conditions of borderline lubrication, but the new factor T is foreign to our normal concepts. The real and only reason for the factors used in any formula, empirical or otherwise, is to enable the designers to accurately predict the behavior of their designs in actual service. Whenever possible, the factors used should conform to established conventions, but the success of the formula should never be jeopardized for the sake of custom. The factor T may be said to imperfectly represent the accumulated heat--the ambient temperature--of the gear teeth. It is better, however, to accept the factor T, as well as P and V, for what they are; that is, parts of a measuring system--a go and no-go gauge.

It should be remembered that we cannot express incipient welding in rational terms because we do not know the coefficient of friction of the rubbing teeth, nor do we know the temperature required to form a weld under the conditions of pressure and surface contamination that prevail in the welded areas. Unless these things are known, none of the factors used to express scoring can have individual, quantitative meaning. Until measurements have been made of the actual temperatures, the actual pressure, the actual service loads, the distribution of the load, and the manner in which these and many other factors influence weldability, the methods that we usually consider to be rational must be classed as mere conventions.

CONCLUSIONS

A method of calculating the scoring resistance of spur gears was developed which checks quite well with the test data. The method is based on the total heat generated per cycle by the sliding action under pressure of the mating teeth. This method is given herein. The results of this method as applied to the test data are shown in Fig. 1 which shows good correlation. The safe limit of the scoring factor is 1,500,000 with mineral oil.

The G.M.R. method of calculation of bending stress on spur and helical gears was applied to the data but it was found that the data on bending failures were insufficient to appraise the validity of the method.

A method of calculating compressive stress as a factor of pitting resistance was also applied to the test data but here again the information on pitting failures was insufficient to appraise the method.

DISCUSSION

Scoring Factor for Spur and Helical Gears

The method of calculation of the scoring factor, PVT, for spur and helical gears which is presented here has been established by test data. The data were obtained from manufacturers of aircraft engines and the scoring factor was calculated for each gear. The results of these calculations in conjunction with the test data are illustrated in Fig. 1. While the method is set up for spur and helical gears, the test data available are confined to spur gears. The points on this chart are plotted in the following manner:

Each vertical line designates a gear design and is numbered for reference. To the left of the line is plotted the calculated scoring factor at the tip of the pinion tooth and to the right the scoring factor at the tip of the gear tooth. Scoring failures are represented by open circles and gears showing no scoring failure are represented by closed circles. This chart includes gear tests on 73 designs run under a wide variety of conditions.

Reduction gears at moderate speeds and high torques, as well as supercharger gears at speeds up to 28,000 RPM and lower torques, are represented in the chart. Both external and internal gears are included. Several thousands of tests are represented on the chart. Note that gears with a scoring factor in excess of approximately 1,500,000 show failure whereas those with less than 1,500,000 are free from scoring.

It should be mentioned here that a limiting factor (1,500,000) can be used as a scoring criterion because this type of failure is not fatigue in character but rather a "yes or no" type. That is, if gears do not score on the first service application of max. speed and torque, they are not likely to score at all.

With the exception of the pinion of design #6, which shows a high factor without scoring, there is little deviation from the line marked "safe limit." Design #6 is one in which the tooth profile was modified considerably and the end of action is, therefore, in doubt. As will be seen from the method of calculation presented later, this would have a marked effect on the calculated scoring factor.

Fig. 2 shows the results of the same group of tests using a conventional method of calculating a scoring factor. The test results are plotted in the same manner as in Fig. 1. The factor in this case is the commonly used value of PV, in which the unit pressure is simply multiplied by the sliding velocity. The assumptions of load distribution are the same as in the PVT method here presented. Such a calculation is evidently inadequate for a wide range of conditions of speed and torque, even though it apparently shows some correlation in limited ranges of torque and speed. It should be noted that this chart is plotted to 1/3 of the scale in Fig. 1.

Method of Calculation - Nomenclature

The scoring factor used in the chart of Fig. 1 is based on the total heat generated by the contacting surfaces during the cycle of sliding from the pitch line to the beginning or end of action. This is in contrast to the PV factor often used, which gives the instantaneous rate of generation of heat at the area of contact.

The method involves the calculation of the unit pressure based on the Hertz equations. The load is assumed to be distributed uniformly on the average total length of contact lines.

The nomenclature used in the calculation is as follows:

T_p = pinion torque, pound inches

RPM_p = pinion speed, revs. per min. with respect to its own axis

N_p, N_g = number of teeth on pinion and gear respectively

F = face length, inches (of pinion or gear, whichever is smaller)

α = normal pressure angle, degrees

Δ = helix angle at pitch line, degrees

ϕ = pressure angle in plane of rotation, degrees

$\tan \phi = \tan \alpha \sec \Delta$

CD = operating center distance, inches

PR_p, PR_g = operating pitch radii of pinion and gear respectively, inches

OR_p, OR_g = outside radii of pinion and gear respectively, inches

IR_g = inside radius of internal gear, inches

Calculation of PVT - external gears

Referring to Fig. 3

$$A_p = \sqrt{OR_p^2 - PR_p^2 \cos^2 \phi}$$

$$A_g = \sqrt{OR_g^2 - PR_g^2 \cos^2 \phi}$$

$$Na = A_p + A_g - CD \sin \phi$$

The unit pressure for external gears is calculated by the expressions

$$P_p = 2290 \sqrt{\frac{2\pi \; T_p}{F \; N_a \; N_p} \; \frac{CD \sin \alpha}{A_p \; (CD \sin \phi - A_p)}}$$ at the tip of the pinion tooth

and

$$P_g = 2290 \sqrt{\frac{2\pi \; T_p}{F \; N_a \; N_p} \; \frac{CD \sin \alpha}{A_g \; (CD \sin \phi - A_g)}}$$ at the tip of the gear tooth

Having determined the unit pressure, the scoring factor, PVT, is calculated by the expressions

$$PVT_p = \frac{\pi \; RPM_p}{360} \left(1 + \frac{N_p}{N_g}\right) (A_p - PR_p \sin \phi)^2 \; P_p$$ at the tip of the pinion tooth

and

$$PVT_g = \frac{\pi \; RPM_p}{360} \left(1 + \frac{N_p}{N_g}\right) (A_g - PR_g \sin \phi)^2 \; P_g$$ at the tip of the gear tooth

In the case of spur gears the helix angle is zero and the pressure angle in the plane of rotation is the normal pressure angle and therefore $\phi = \alpha$ for spur gears.

With mineral oil as a lubricant, the safe limit of PVT is 1,500,000 as shown by the chart Fig. 1.

The derivation of this method of calculation is given in the Appendix.

Calculation of PVT - Internal Helical Gears

Referring to Fig. 4

$$A'_p = \sqrt{OR_p^2 - PR_p^2 \cos^2 \phi}$$

$$A'_g = \sqrt{IR_g^2 - PR_g^2 \cos^2 \phi}$$

$$N'_a = A'_p - A'_g + CD \sin \phi$$

The unit pressure for internal gears is calculated by the expressions

$$P'_p = 2290 \sqrt{\frac{2 \pi T_p}{F N'_a N_p} \frac{C D \sin \alpha}{A'_p (A'_p + C D \sin \phi)}} \text{ at O.D. of pinion}$$

and

$$P'_g = 2290 \sqrt{\frac{2 \pi T_p}{F N'_a N_p} \frac{C D \sin \alpha}{A'_g (A'_g - C D \sin \phi)}} \text{ at I.D. of gear}$$

Having determined the unit pressure, the scoring factor, PVT, is calculated by the expressions:

$$PVT'_p = \frac{\pi RPM_p}{360} \left(1 - \frac{N_p}{N_g}\right) (A'_p - PR_p \sin \phi)^2 P'_p \text{ at O.D. of pinion}$$

and

$$PVT'_g = \frac{\pi RPM_p}{360} \left(1 - \frac{N_p}{N_g}\right) (PR_g \sin \phi - A_g)^2 P'_g \text{ at I.D. of gear}$$

For spur gears $\phi = \alpha$, the same as for external gears. The safe limit is 1,500,000 with mineral oil as a lubricant as is the case with external gears. The derivation of these equations is given in the Appendix.

Factors Influencing Scoring Resistance

In many gear designs in use today, scoring difficulties could be reduced or even eliminated without additional cost if proper attention were paid to the tooth design. This applies also to pitting difficulties, and in most cases there is no conflict between remedies for pitting and scoring.

One factor that is often overlooked is the proportion of tooth height with respect to the pitch line, in relation to the tooth ratio. In a gear design of a ratio nearly one to one, no special scoring difficulties are liable to result from the use of tooth proportions with equal addenda on gear and pinion.

But as the tooth ratio increases it becomes increasingly important to proportion the addenda in order to avoid high unit pressures at the tip of the gear. Further, this condition becomes accentuated in coarse pitch gears. The reason for the necessity of such considerations is quite simple. The unit pressure varies roughly as the reciprocal of the smaller radius of curvature of the teeth, and that radius is theoretically zero at the base circle. Therefore, to avoid high unit pressures and high scoring factors, it is necessary to avoid action near the base circle. Fig. 7 shows the unit pressure and scoring factor for a pair of gears of 3.8 ratio. The variation in unit pressure along the line of action is shown immediately above the diagram of the gears. Note that where the line of action approaches the base circle of the pinion, at the left, the unit pressure rises asymptotically. The gears shown have equal addenda on gear and pinion with the result that at the tip of the gear tooth the unit pressure is more than twice as great as that at the tip of the pinion tooth. The sliding velocity along the line of action varies directly as the distance from the pitch line as shown above the unit pressure chart. Next above the sliding velocity is shown the variation of PV or the instantaneous rate of heat generated. It is an intermediate step in determining the scoring factor PVT which is illustrated in the top curve of Fig. 7. The values of PVT plotted vertically in this curve are obtained as the area under the PV curve, starting from the pitch line. For instance, at the tip of the gear the PVT value represents the approximate area of the PV curve to the left of the pitch line. It is approximate in that the PV curve is considered as a straight line from the pitch line to the tip of the gear.

Note again that at the tip of the gear the scoring factor is very much higher than at the tip of the pinion for the same reason that the unit pressure is excessive there. It should be emphasized that the ratio of the gears illustrated here is rather moderate in comparison to many designs which require ratios of 8 to 1 or higher. In such cases, the situation becomes even worse, because with increasing ratios the pitch line moves more and more to the left, or towards the pinion base circle. The ratio chosen for the illustration was selected because the factors involved can be pointed out more clearly. Designs such as that shown in Fig. 7 are not uncommon even in higher ratios. Such a design can be substantially improved at little, if any, additional cost by simply changing the tooth proportions.

Fig. 8 shows the same design as shown in Fig. 7 except that the gear addendum is decreased and the pinion addendum increased. Note that this change results in a marked decrease in the maximum unit pressure and also a decrease in the maximum scoring factor.

A striking example of such an improvement in actual practice is shown by comparing designs 38 and 39 in the scoring factor chart of Fig. 1. Design 38 is one in which the ratio is approximately 2 to 1. The pinion addendum in this pair is slightly greater than that of the gear but the gears are of coarse pitch, and consequently of long addenda. The result is that action takes place rather close to the base circle. As shown in Fig. 1, the scoring factor at the tip of the gear in this design is 2,650,000. Approximately 400 gear sets of this design were tested, of which 33% scored at the tip of the gear.

Design 39 is identical to 38 just mentioned except that the gear addendum is .070" shorter than that in 38. Referring again to Fig. 1, this change reduced the PVT to 1,750,000 in design 39, and the scoring dropped to less than 2% in 3000 tests. This comparison is even more striking in the fact that the pinion addendum is not increased to maintain the same working depth. In other words, this improvement was made in the scoring resistance in spite of the fact that the contact ratio was materially reduced.

Another example is shown by comparing designs 35 and 36 in Fig. 1. In this case the scoring failures were reduced from almost 5% of gears tested in 35 to 1/3 of 1% in 36. This improvement is also the result of reducing the gear addendum, thereby reducing the PVT factor from 1,780,000 to 1,550,000.

In the design of a pair of gears the selection of the diametral pitch is of great importance unless weight of the gear box is of no consequence. The diametral pitch should be a compromise between bending strength on the one hand and resistance to pitting and scoring on the other. Low bending stresses are obtained by the use of a coarse pitch because the tooth thickness is large. However, as the pitch becomes coarse, it is necessary to increase the tooth height in order to maintain a reasonable contact ratio. This means that action approaches closer to the base circles of the gears, which in turn means high compressive stresses which may cause pitting. Further, the increased tooth height increases the sliding velocity which in combination with the high compressive stress may cause scoring. From the standpoint of scoring and pitting this increase of tooth height to accomodate a coarse pitch becomes increasingly important as the tooth ratio increases. A good design, then, is one in which these factors are balanced.

The pressure angle is also important in the strength of gear teeth, but in this case there is no conflict between bending stress and unit pressure or PVT factor. Higher pressure angles tend to decrease the bending stress as well as the unit pressure and PVT factor. To cite an example of such improvement in scoring resistance, we can refer again to Fig. 1, designs 35, 36 and 37. It will be recalled that the scoring failures were reduced in this design from 5% in design 35 to 1/3 of 1% in design 36 by decreasing the gear addendum.

A second change was made by increasing the pressure angle from 20° to 25° in design 37. As shown in Fig. 1, design 37 has a reduced PVT factor as compared to 36. Over 2000 gear sets were tested after this change was made and failure was completely eliminated.

Some designers place great emphasis on the contact ratio as a factor influencing the strength of gears. But if a high contact ratio is accompanied by high unit pressures, its advantages may be more than offset by pitting and scoring difficulties. The examples cited above bear out the fact that a high contact ratio of itself is not necessarily beneficial. Design 37, just mentioned, is one which has a contact ratio of less than 1.2. In spite of the fact that this appears very low, no failures have occurred, whereas designs 35 and 36, both of which had definitely higher contact ratios, failed by scoring.

Pitting Factor, P

Pitting, in contrast to scoring, is a fatigue type of failure and therefore more detailed data are required to appraise a method of calculation. In the appraisal of a scoring factor it is sufficient to know the torque and speed of the gears tested and whether or not scoring occurred under these conditions. The appraisal of a pitting factor on the other hand requires data on the actual life of the gears before such failure, if any, occurred. Such data could then be plotted on a log-log fatigue chart with life plotted against the calculated factor. The factor thus calculated would be appraised on the basis of whether the results fall reasonably close to a fatigue line.

The available data on pitting is incomplete from this standpoint. However, a "yes or no" chart is plotted in Fig. 5 to determine whether such a chart would give any limiting values of a pitting factor. As can be seen from the chart, no such limiting value is indicated.

The factor used in calculating the pitting resistance of spur and helical gears is the unit pressure, P, as calculated in the PVT factor given above. That is, for external gears, the unit pressure is

$$P_p = 2290\sqrt{\frac{2\,\pi\,T_p}{F\,N_a\,N_p}\;\frac{CD\sin\alpha}{A_p\,(CD\sin\phi - A_p)}}$$ at the tip of the pinion tooth

and

$$P_g = 2290\sqrt{\frac{2\,\pi\,T_p}{F\,N_a\,N_p}\;\frac{CD\sin\alpha}{A_g\,(CD\sin\phi - A_g)}}$$ at the tip of the gear tooth

For internal gears the unit pressure

$$P'_p = 2290\sqrt{\frac{2\,\pi\,T_p}{F\,N_a\,N_p}\;\frac{CD\sin\alpha}{A'_p\,(A'_p + CD\sin\phi)}}$$ at O.D. of pinion

and

$$P'_g = 2290\sqrt{\frac{2\,\pi\,T_p}{F\,N_a\,N_p}\;\frac{CD\sin\alpha}{A'_g\,(A'_g - CD\sin\phi)}}$$ at I.D. of gear

Bending Stress Factor

The third type of gear failure, bending or tooth breakage, is similar to pitting failure in that it is fatigue in character. Therefore, data on the life of gears tested are necessary for appraisal of a method. This information is lacking in the available data as was the case in pitting failures and, for the same reason as pointed out under pitting, the data are not sufficiently complete to appraise a bending stress formula. As was the case in pitting, the bending stress factor is plotted in Fig. 6 in an effort to show limiting values if they exist. As can be seen from the chart, no limiting value is apparent. The bending stresses were calculated by the G.M.R. method described in Report D-314.

Factors Other Than Tooth Design

Aside from the design of the gear teeth themselves there are other factors which must be considered as part of the overall design of a set of gears. The housing, shafts, bearings, etc. should be designed in such a way as to allow the teeth to mesh uniformly for the full length of the teeth as nearly as possible when the load is applied. A heavy tooth bearing on either end of the teeth inevitably results in load concentrations at that end. This load concentration is undesirable from every standpoint. It cannot be entirely eliminated because spur or helical gears have straight lines of contact and it is impossible to maintain parallelism of the potential lines of contact when high loads are applied to the gears. Such load concentration can be reduced, however, by proper design of the gear mounting. It is, of course, desirable to support each gear between two bearings whenever possible, in order to avoid more deflection at one end of the gears than at the other. The case and shafts should be sufficiently rigid to avoid excessive deflections. However, this should not be interpreted as a statement that all gear mountings should be as rigid as possible. In some cases it is more practical to balance deflections in one sense with others of the opposite sense. This is a matter of study of the individual case.

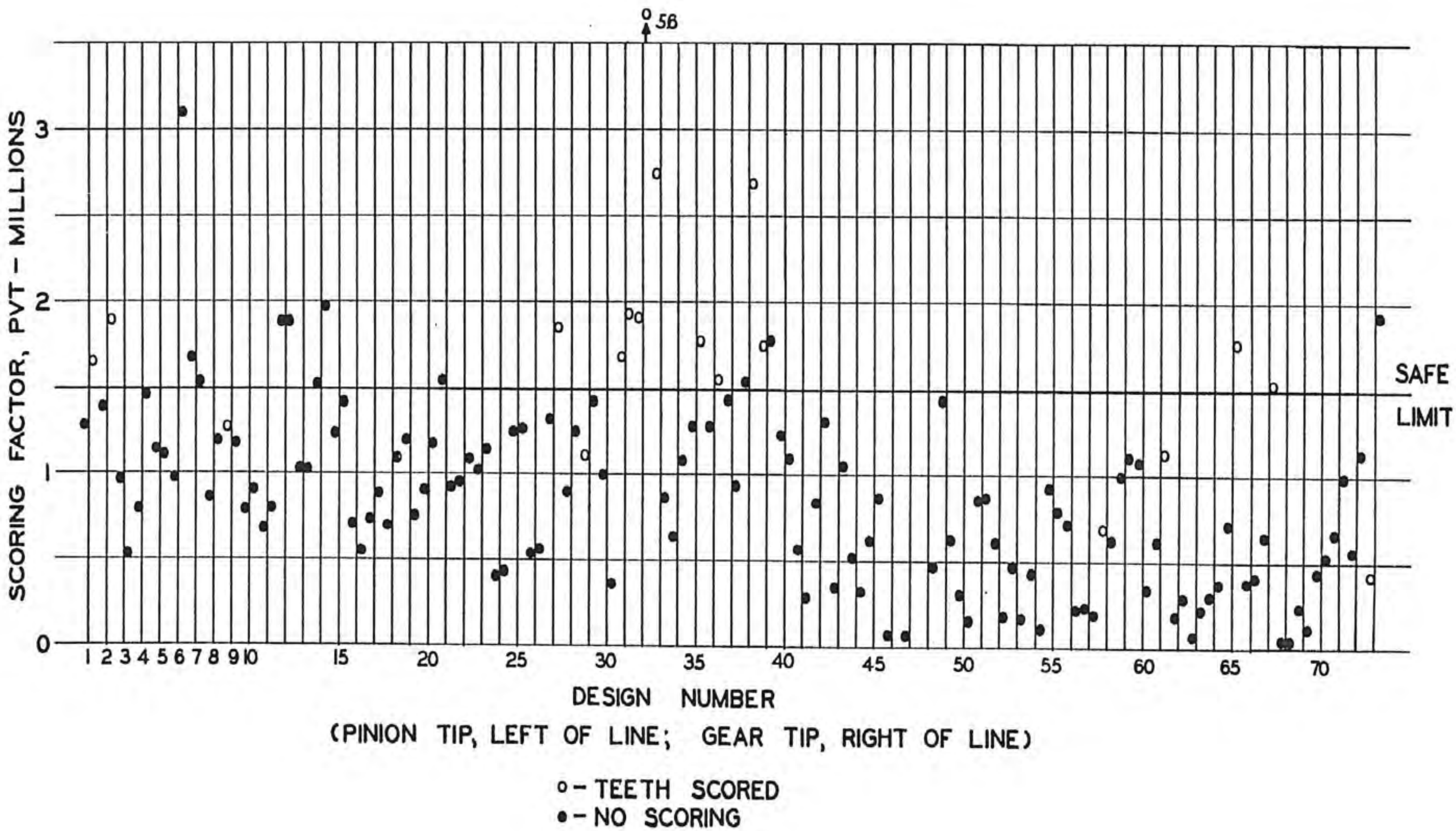
The material used in gears is important with respect to its hardness. The data presented in Fig. 1 and 2 deal entirely with hardened steel gears. That is, gears with a file hard surface. It is very important in gears which are highly loaded to maintain a hard surface. This is true for all three types of failure. Where the surface hardness is low, the bending stress, unit pressure and scoring factor must be correspondingly low, if failure is to be avoided.

Another factor which must be considered in a gear design is the cooling system. This varies with the job that the gears are expected to do. Where the power to be transmitted is high, more cooling capacity must be provided than if the gears are to run at light load and moderate speed. In some cases it is difficult to provide adequate cooling for the gear box. In such instances it may be necessary to operate at higher temperatures. Here again a lower scoring factor can be tolerated than if cooling were sufficient to maintain a reasonably low oil temperature. Fig. 9 is a qualitative chart showing the influence of surface hardness and temperature upon the unit pressure and sliding velocity which can be tolerated. This is a three-dimensional chart in which the curved surfaces within the cube represent the influence of pressure, sliding velocity and oil temperature on the scoring characteristics. Note that for hard material, higher values of load, speed and oil temperature can be tolerated than for soft material. As the oil temperature increases, using a given hardness of material, the pressure or velocity or both must decrease in order to be safe. Finally, assuming a given oil temperature, lower pressure will tolerate a higher sliding velocity and vice versa.

Some gear designs are required to carry extremely high loads and speeds under adverse conditions. In such cases, the use of extreme pressure lubricants may be necessary. With the use of such lubricants the scoring resistance of gears can be definitely increased. The use of extreme pressure lubricants, however, is usually limited to isolated gear boxes because such lubricants are likely to attack non-ferrous bearing materials. The effect of an extreme pressure lubricant on the scoring resistance is illustrated in Fig. 10, which is another three-dimensional chart similar to Fig. 9. In this case the chart shows that with an extreme pressure lubricant, higher loads and speeds can be used than with mineral oil. These characteristics are summarized in Fig. 11, which shows the influence of the factors mentioned above. The higher range of pressure-velocity combinations is for E.P. oils, while the lower range is for mineral oils. These two groups are in turn influenced by surface hardness and oil temperature.

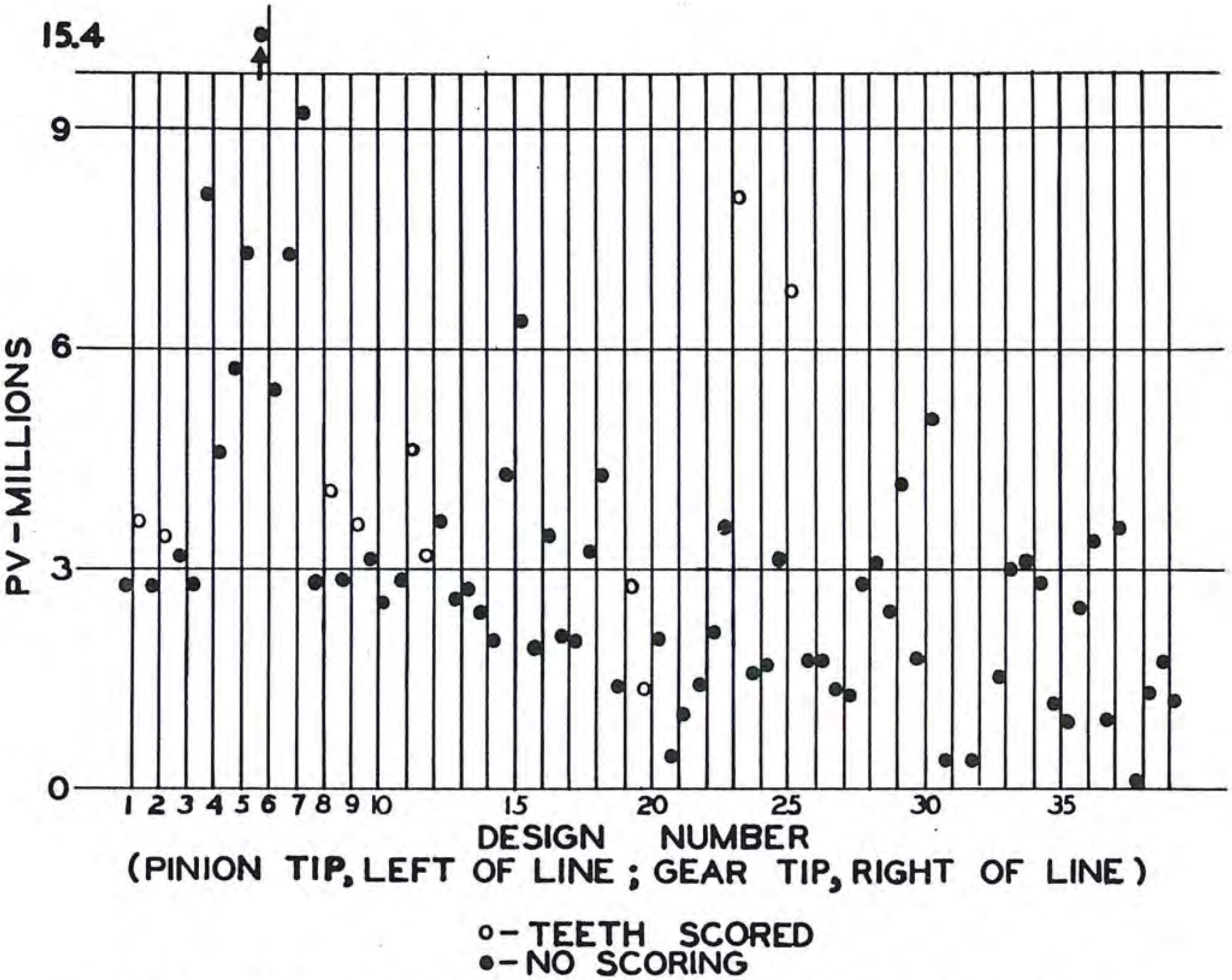
The method of lubrication is another factor influencing the scoring resistance of gears. It is important to remember that the function of a lubricant is one of cooling as well as lubrication. The oil carries the heat from the gears to the housing or heat exchanger as the case may be, and then to the outside. Where the gears are lubricated by passing through a sump of oil, excessive oil must be avoided as well as an inadequate supply. With excessive oil, where the gears are submerged in a large volume of oil, the churning losses may be so great as to heat the oil, and excessive temperatures may develop.

The method of lubrication to be preferred, if possible, is that of using an oil pump and a jet of oil on the teeth as they come out of mesh. The reason for the jet being preferred on the out-of-mesh side rather than on the entering side is because the teeth are at their maximum temperature after meshing and, therefore, the cooling is more effective. The oil remaining on the teeth when sprayed on the leaving side is ample for lubrication, because only a thin film of oil is required for that purpose. A further advantage of this type of lubrication is that dirt or any other foreign material is thrown off and thus excluded from the gear teeth in action.



CORRELATION OF SCORING FACTOR, PVT, WITH ACTUAL TEST DATA ON 73 GEAR DESIGNS

FIG. 1



FAILURE OF "PV" TO CORRELATE WITH ACTUAL TEST DATA OF 39 AIRCRAFT ENGINE SPUR GEAR DESIGNS

FIG. 2

RADIUS OF CURVATURE
EXTERNAL GEARS

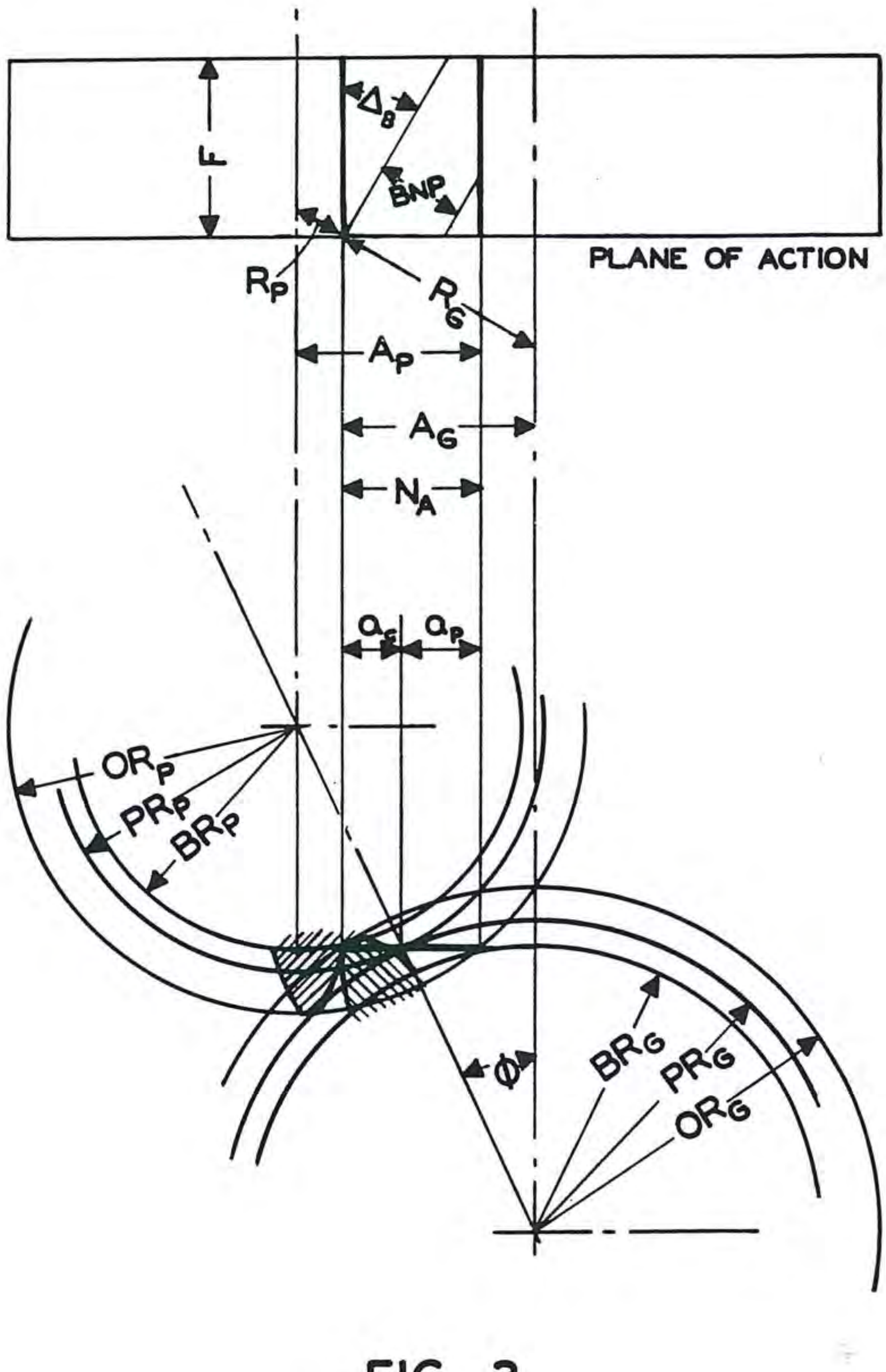


FIG. 3

RADIUS OF CURVATURE
INTERNAL GEARS

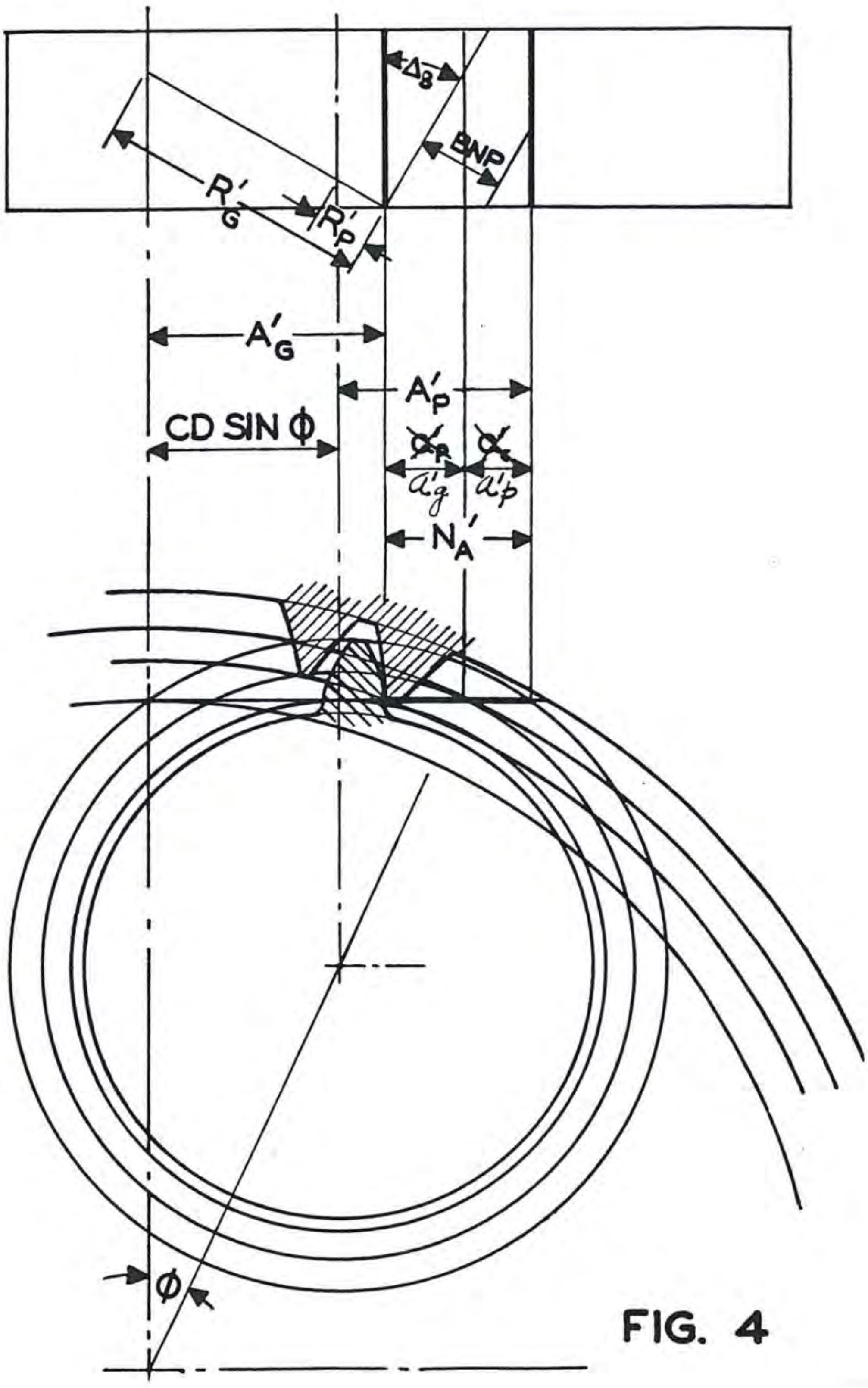
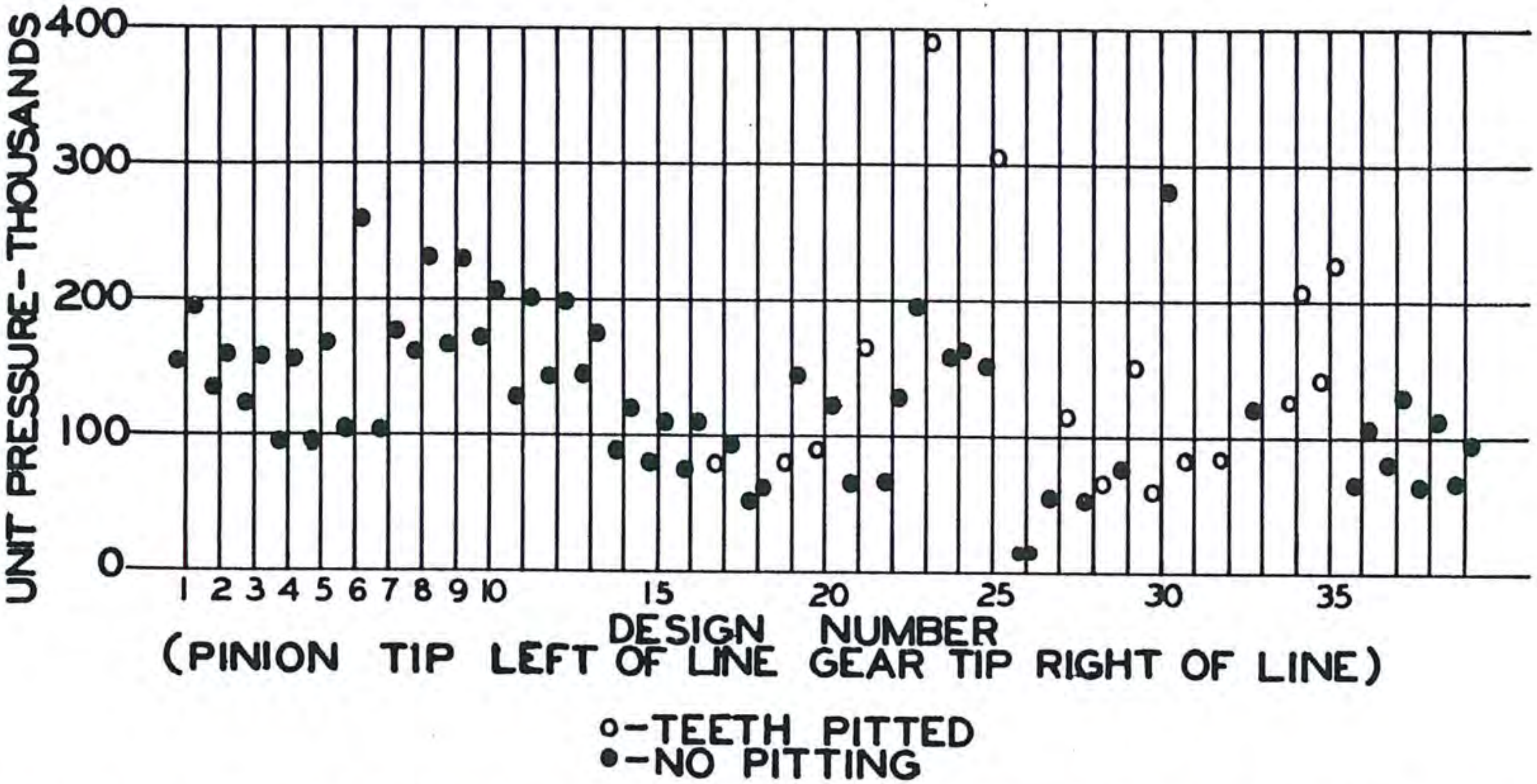
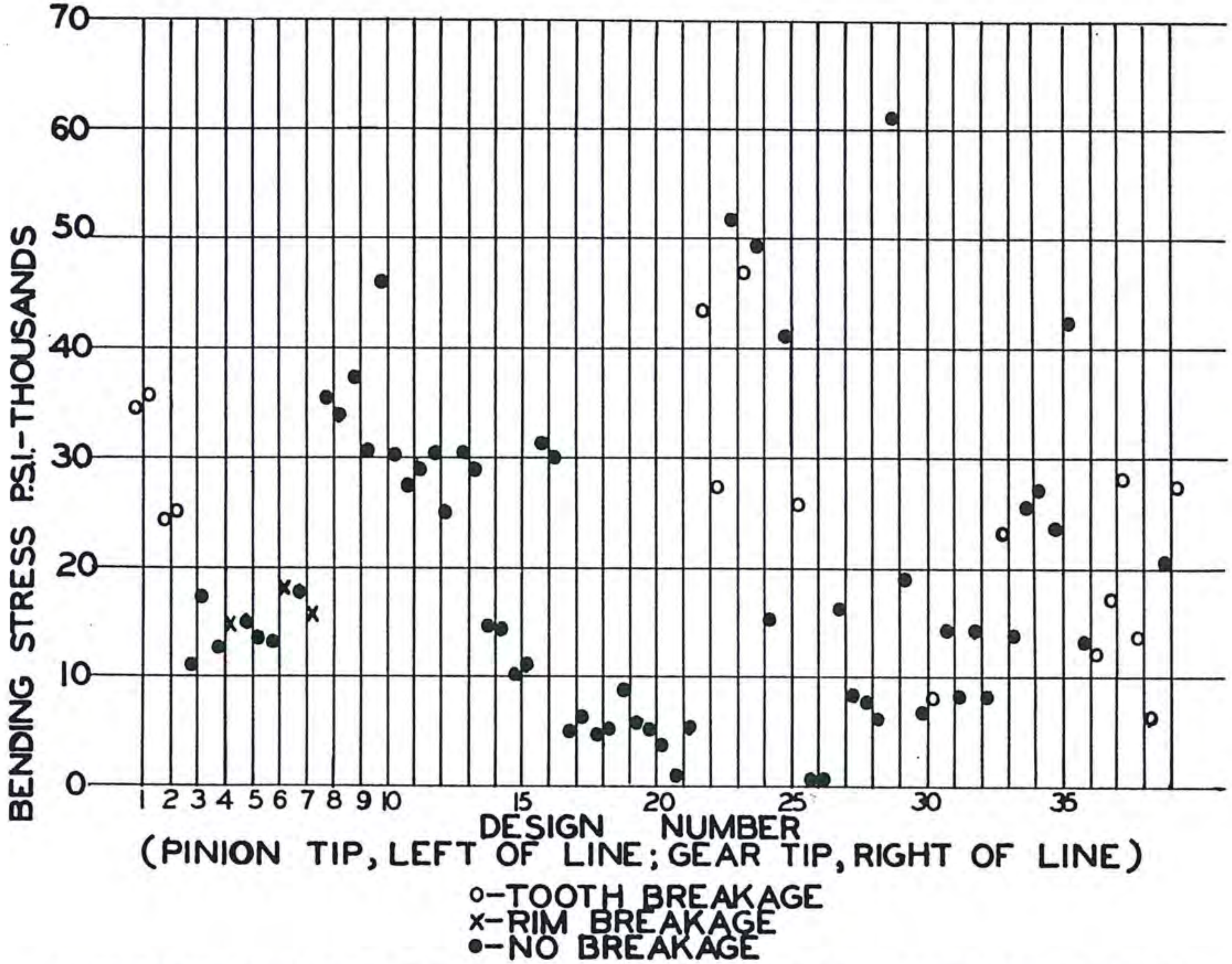


FIG. 4



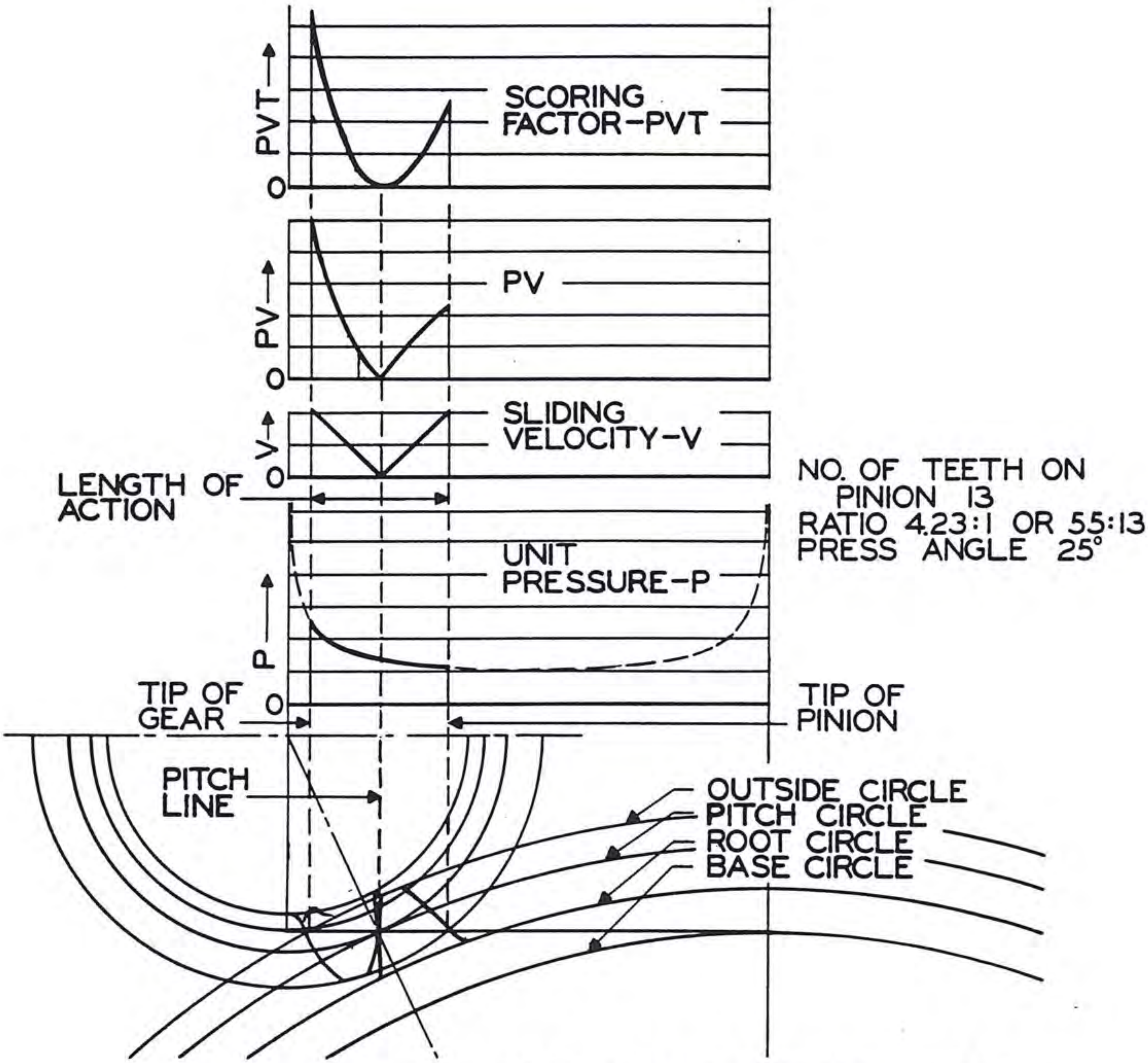
LACK OF CORRELATION OF UNIT PRESSURE WITH ACTUAL TEST DATA ON 39 AIRCRAFT ENGINE SPUR GEAR DESIGNS

FIG. 5



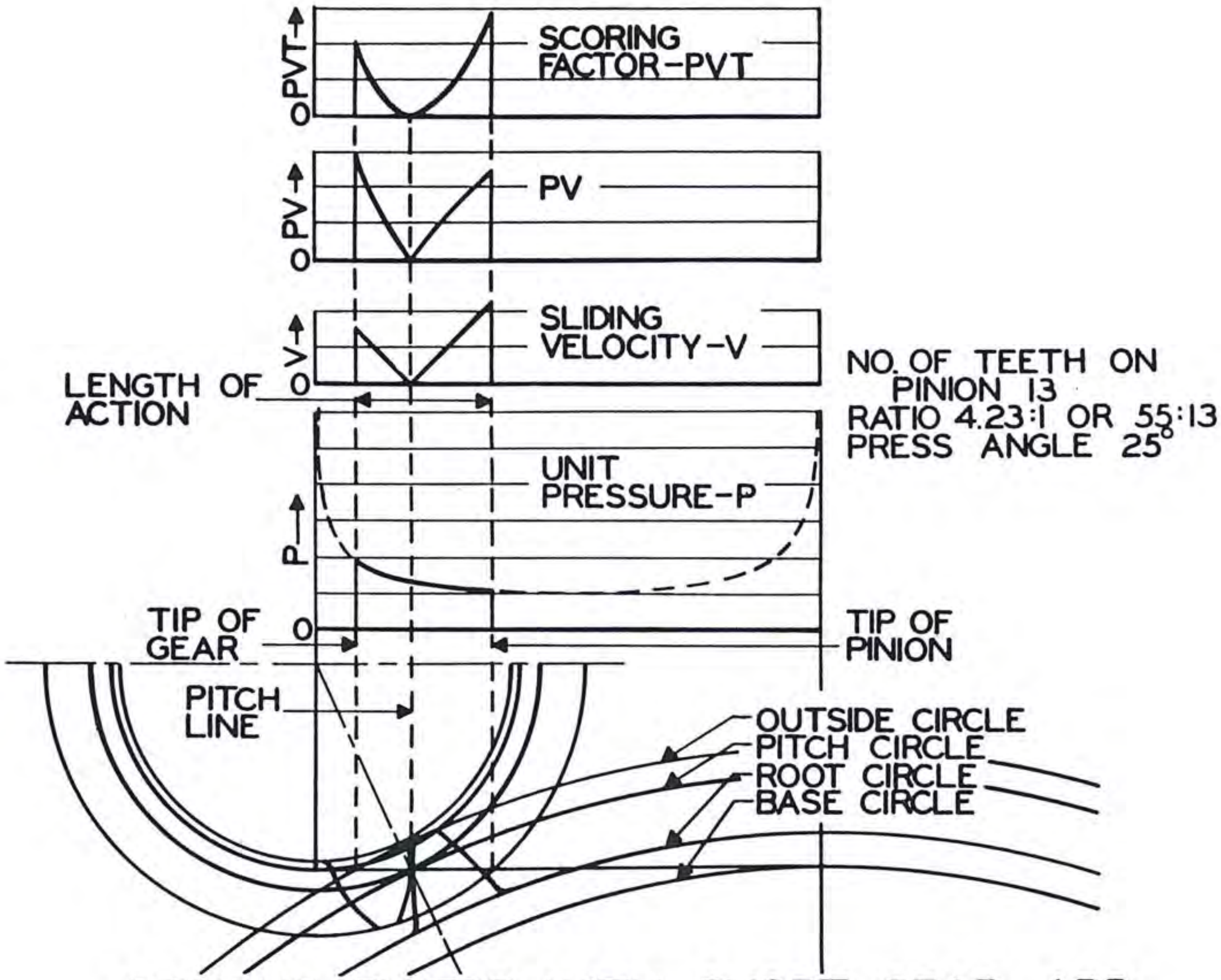
BENDING FAILURES IN ACTUAL TEST DATA
AIRCRAFT ENGINE SPUR GEAR DESIGNS

FIG. 6



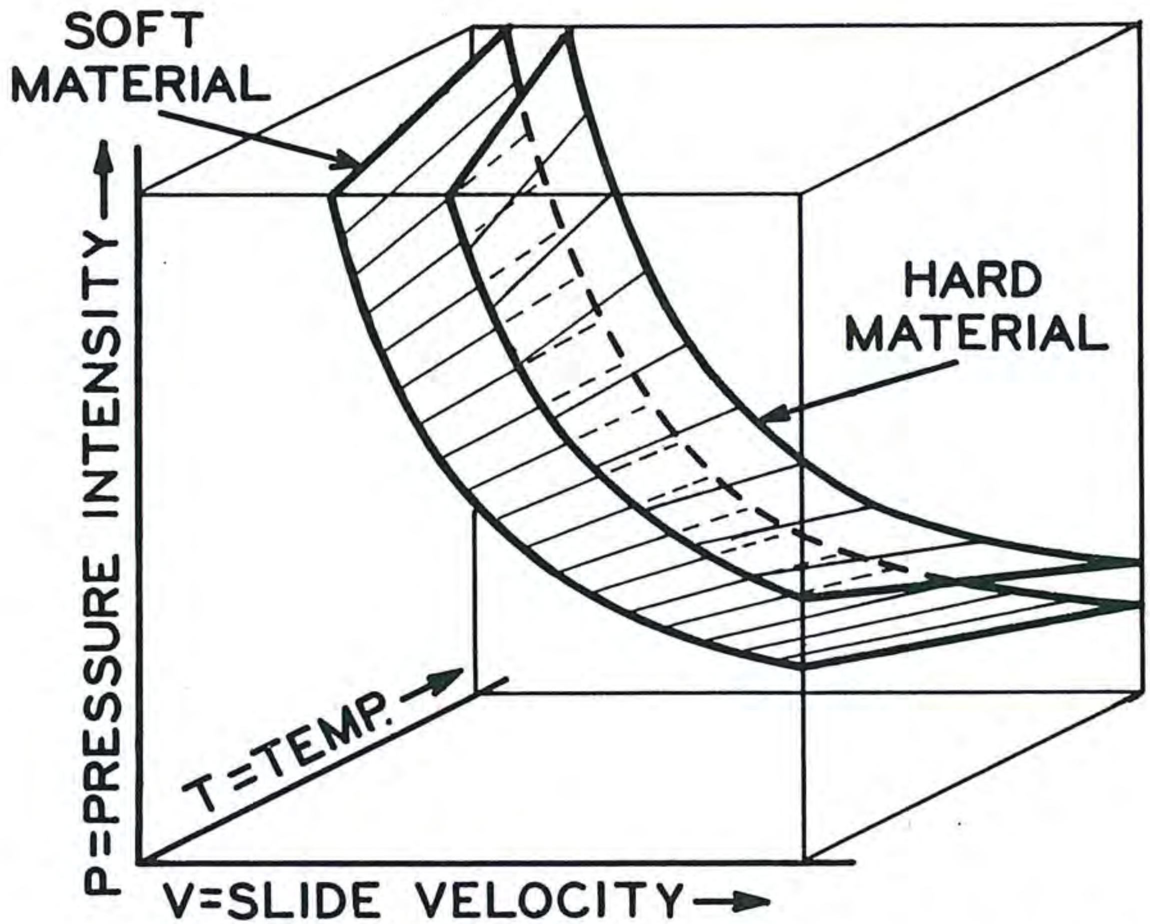
SCORING FACTOR WITH EQUAL ADD.

FIG. 7



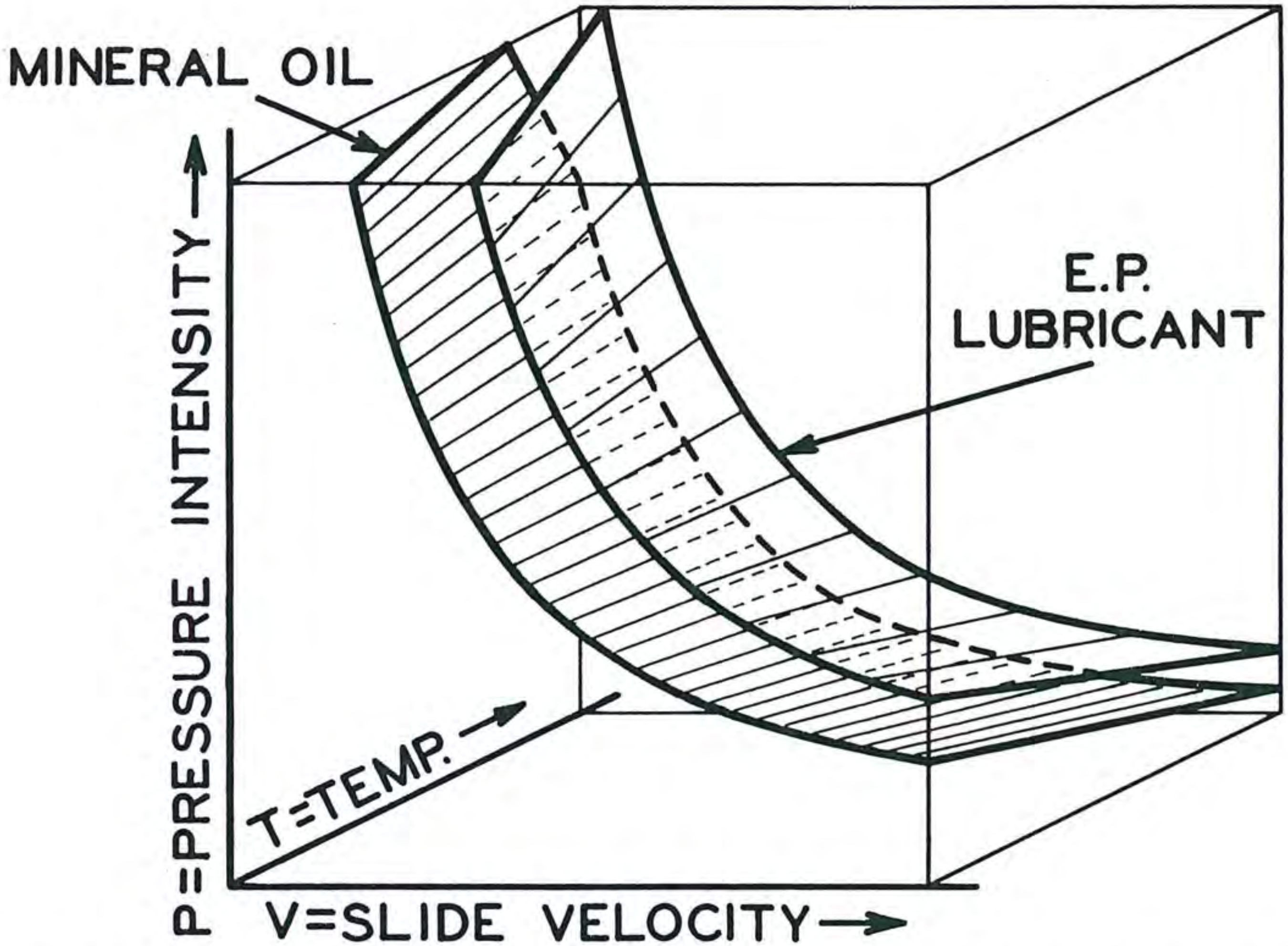
SCORING FACTOR WITH SHORT GEAR ADD. AND LONG PINION ADD.

FIG. 8



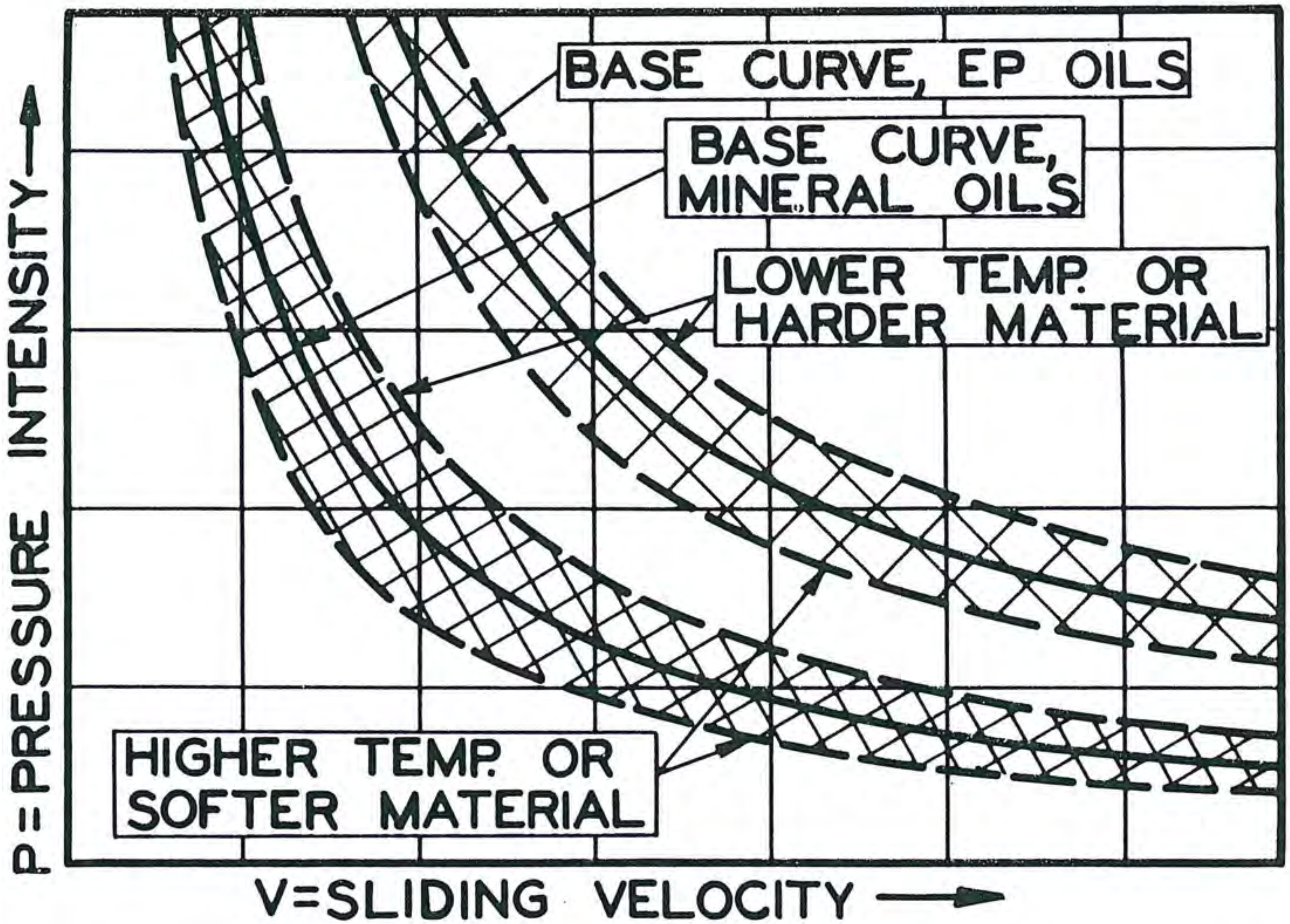
EFFECT OF PRESSURE, TEMP., SLIDING VEL. & HARDNESS ON SCORING LIMIT

FIG. 9



EFFECT OF PRESSURE, TEMP., SLIDING VELOCITY & LUB. ON SCORING LIMIT

FIG. 10



EFFECT OF HARDNESS, TEMP., AND LUBRICANT ON SCORING LIMIT

FIG. 11

APPENDIX I

Derivation of Method of Calculation of PVT (External Gears)

Unit Pressure

As mentioned previously, the unit pressure as calculated is based on the Hertz equations. Of necessity certain assumptions must be made as to the distribution of the load on the teeth in contact. This method assumes that the load is distributed uniformly on the average total length of contact line. The term average is used rather loosely here for want of a better term. The actual length over which the load is assumed to be distributed is calculated as L in the following expression:

$$L = \frac{F\ N_a}{\beta\ N\ P} \qquad \text{(see Fig. 3)} \tag{1}$$

in which $\overline{\beta\ N\ P}$ is the basic normal pitch and the other symbols correspond with the nomenclature given on page 6.

The basic normal pitch is equal to

$$\beta\ N\ P = \frac{2\ \pi\ PR_p\ \cos\phi\ \cos\Delta\beta}{N_p}$$

in which Δ_β = helix angle at the base circle.

Substituting β N P in the expression for L:

$$L = \frac{F\ N_a\ N_p}{2\ \pi\ PR_p\ \cos\phi\ \cos\Delta_\beta} \tag{2}$$

The total load normal to the teeth is

$$P = \frac{T_p}{PR_p\ \cos\phi\ \cos\Delta\beta} \tag{3}$$

The load per inch of contact line then becomes

$$p = \frac{P}{L} \tag{4}$$

$$= \frac{T_p}{PR_p\ \cos\phi\ \cos\Delta_\beta} \cdot \frac{2\ \pi\ PR_p\ \cos\phi\ \cos\Delta_\beta}{F\ N_a\ N_p}$$

$$p = \frac{2\ \pi\ T_p}{F\ N_a\ N_p} \tag{5}$$

Having thus determined the load per inch on the contact lines, the unit pressure is calculated, assuming the tooth surfaces as elements of cylinders contacting each other.

In general, the maximum compressive stress occurs at the tip of the gear tooth or that of the pinion tooth, depending upon the radii of curvature of the tooth flanks.

When the contact is at the tip of the gear tooth, the normal radius of curvature of the gear tooth at that point is

$$R_g = \frac{A_g}{\cos \Delta_\beta}$$

and that of the pinion tooth in the corresponding position is

$$R_p = \frac{CD \sin \phi - A_g}{\cos \Delta_\beta}$$

The Hertz equation for unit pressure at the tip of the gear tooth using steel gears is

$$S_g = \sqrt{5,250,000\ p\ \frac{R_p + R_g}{R_p\ R_g}} \tag{6}$$

The expression $\frac{R_p + R_g}{R_p\ R_g}$ can be simplified as follows:

$$\frac{R_p + R_g}{R_p\ R_g} = \frac{A_g + CD \sin \phi - A_g}{\cos \Delta_\beta} \cdot \frac{\cos^2 \Delta_\beta}{A_g\ (CD \sin \phi - A_g)}$$

$$= \frac{CD \sin \phi \cos \Delta_\beta}{A_g\ (CD \sin \phi - A_g)}$$

But

$$\cos \Delta_\beta = \frac{\cos \Delta \cos \alpha}{\cos \phi}$$

$$\frac{R_p + R_g}{R_p\ R_g} = \frac{CD \sin \phi \cos \Delta \cos \alpha}{\cos \phi\ A_g\ (CD \sin \phi - A_g)}$$

$$= \frac{CD \tan \phi \cos \Delta \cos \alpha}{A_g\ (CD \sin \phi - A_g)}$$

Further

$$\tan \phi = \frac{\tan \alpha}{\cos \Delta}$$

Finally

$$\frac{R_p + R_g}{R_p\ R_g} = \frac{CD\ \frac{\tan \alpha}{\cos \Delta} \cos \Delta \cos \alpha}{A_g\ (CD \sin \phi - A_g)}$$

which simplifies to

$$\frac{R_p + R_g}{R_p\ R_g} = \frac{CD \sin \phi}{A_g\ (CD \sin \phi - A_g)} \tag{7}$$

Substituting (5) and (7) in (6)

$$S_g = \sqrt{5,250,000\ p\ \frac{R_p + R_g}{R_p\ R_g}} \tag{6}$$

$$= 2290 \sqrt{\frac{2\ \pi\ T_p}{F\ N_a\ N_p}\ \frac{CD \sin \phi}{A_g\ (CD \sin \phi - A_g)}} \tag{7a}$$

This gives the unit pressure at the tip of the gear tooth. It is apparent that at the tip of the pinion tooth the unit pressure is

$$S_p = 2290 \sqrt{\frac{2\ \pi\ T_p}{F\ N_a\ N_p}\ \frac{CD \sin \phi}{A_p\ (CD \sin \phi - A_p)}} \tag{7b}$$

S_p and S_g are used to determine the unit pressure on external gears as shown on the chart, Fig. 7. S_p and S_g as calculated by these expressions are plotted as "P" at the tip of gear and tip of pinion in Fig. 7.

Sliding Velocity (external gears)

The sliding velocity of the tooth surfaces in contact is determined as follows:

The relative angular velocity of one gear with respect to the other is

$$w = w_p + w_g \tag{8}$$

in which $w_p = \frac{2\pi}{60} RPM_p$ = angular velocity of pinion, radians/sec.

and $w_g = \frac{2\pi}{60} RPM_g$ = angular velocity of gear, radians/sec.

RPM_p and RPM_g = speed of pinion and gear respectively, revs./min.

Substituting for w_p and w_g in (8)

$$w = \frac{\pi}{30}(RPM_p + RPM_g)$$

But $RPM_g = RPM_p \frac{N_p}{N_g}$

$$\therefore \quad w = \frac{\pi}{30} RPM_p \left(1 + \frac{N_p}{N_g}\right) \text{ rad./sec.}$$

The distance from the instantaneous center of rotation (pitch point) is

$$a_g = \frac{A_g - PR_g \sin\phi}{12} \quad \text{at tip of gear tooth, feet}$$

$$a_p = \frac{A_p - PR_p \sin\phi}{12} \quad \text{at tip of pinion tooth, feet}$$

The linear sliding velocity is then

$$V_g = w\, a_g = \frac{\pi}{30} RPM_p \left(1 + \frac{N_p}{N_g}\right) \frac{A_g - PR_g \sin\phi}{12} \quad \text{at tip of gear tooth, ft./sec.} \qquad (8a)$$

or

$$V_p = w\, a_p = \frac{\pi}{30} RPM_p \left(1 + \frac{N_p}{N_g}\right) \frac{A_p - PR_p \sin\phi}{12} \quad \text{at tip of pinion tooth, ft./sec.} \qquad (8b)$$

The values of V at the tip of the gear and the tip of the pinion in Fig. 7 are calculated by these expressions.

It has been more or less common practice to use a "PV" factor for a measure of scoring resistance. That is, the product of unit pressure P and sliding velocity V is referred to as the "PV" factor.

Such a PV factor is obtained by the product of equations (7a) and (8a) or (7b) and (8b):

$$PV_g = \frac{\pi \ RPM_p}{360} \left(1 + \frac{N_p}{N_g}\right)(A_g - PR_g \sin \phi) \ S_g \qquad \text{at tip of gear} \qquad (9a)$$

$$PV_p = \frac{\pi \ RPM_p}{360} \left(1 + \frac{N_p}{N_g}\right)(A_p - PR_p \sin \phi) \ S_p \qquad \text{at tip of pinion} \qquad (9b)$$

The values of PV at the tip of the gear and tip of the pinion in Fig. 7 are calculated by these expressions. The PV factor as calculated by this method is not a true measure of the scoring resistance, as demonstrated in Fig. 2.

Scoring Factor PVT (external gears)

The PVT factor which shows correlation with test data and does serve as a measure of scoring resistance is obtained as the approximate area under the PV curve from the pitch line to the tip of gear or tip of pinion along the line of action.

The area under the PV curve is approximate in that the PV curve is considered a straight line, from zero at the pitch line to the value of PV at the tip of gear or pinion.

Considering the tip of the gear, the area under the PV curve to the left of the pitch line in Fig. 7 is simply the area of the triangle in which the vertical side is PV_g and the horizontal side is a_g. The area of this triangle is

$$\text{Area} = \frac{PV_g \ a_g}{2}$$

But $$a_g = \frac{A_g - PR_g \sin \phi}{12}$$

$$\text{Area} = \frac{PV_g \ (A_g - PR_g \sin \phi)}{24}$$

Since the calculated values are only relative, the denominator of the above expression is eliminated for ease of calculation, and the scoring factor is expressed as

$$PVT_g = PV_g \ (A_g - PR_g \sin \phi)$$

Substituting for PV_g, the scoring factor becomes

$$PVT_g = \frac{\pi \ RPM_p}{360}\left(1 + \frac{N_p}{N_g}\right)(A_g - PR_g \sin\phi)^2 \ S_g \qquad \text{at tip of gear} \qquad (10a)$$

By similar procedure

$$PVT_p = \frac{\pi \ RPM_p}{360}\left(1 + \frac{N_p}{N_g}\right)(A_p - PR_p \sin\phi)^2 \ S_p \qquad \text{at tip of pinion} \qquad (10b)$$

These are the expressions used in the calculation of the scoring factors for external gears at the gear and pinion tips as shown in Fig. 7. The intermediate points of the PVT curve as shown are plotted only for illustration. A word of explanation should be given, however, as to the derivation of the PVT curve. At any point along the line of action, the value of PVT is the area under the PV curve (considered as a straight line) from the pitch point to the point in question. The question might be asked as to why the PVT factor is not plotted as zero at the beginning of action, increasing to a maximum at the pitch line for the approach action. The reason for the curve being plotted as shown is due to the fact that the designs which had scoring failures were scored, in general, at the tip of the gear or pinion regardless of whether it happened to be the beginning or end of action. The actual value of the scoring factor is not affected by the chart because the area under the total PV curve is the same in either case.

APPENDIX II

Derivation of Method of Calculation of PVT (Internal Gears)

The factors involved in the PVT factor for internal helical gear are similar to those for external helical gears.

It is apparent that the load per inch of contact line can be calculated in the same way as for external gears. That is

$$p = \frac{2 \pi T_p}{F N_a N_p} \tag{11}$$

Unit Pressure (tip of gear)

The determination of the radii of curvature, however, requires somewhat different expressions. Referring to Fig. 4, when the contact is at the tip of the gear tooth (I.D.), the radius of curvature of the gear tooth at that point is

$$R'_g = \frac{A'_g}{\cos \Delta_\beta}$$

and that of the pinion tooth is

$$R'_p = \frac{A'_g - \bar{CD} \sin \phi}{\cos \Delta_\beta}$$

For steel internal gears the Hertz equation for unit pressure at the tip of the gear tooth is

$$S'_g = \sqrt{5{,}250{,}000\ p\ \frac{R'_g - R'_p}{R'_p\ R'_g}} \tag{12}$$

The term $(R'_g - R'_p)$ is the difference between the radii of curvature rather than their sum because the curvature is in the same sense for internal gears.

Simplifying the expression $\frac{R'_g - R'_p}{R_p R_g}$

$$\frac{R'_g - R'_p}{R'_g R'_p} = \frac{A'_g - A'_p + CD \sin\phi}{\cos\Delta_\beta} \quad \frac{\cos^2\Delta_\beta}{A'_g\,(A'_g - CD \sin\phi)}$$

$$= \frac{CD \sin\phi \cos\Delta_\beta}{A'_g\,(A'_g - CD \sin\phi)}$$

But $\quad \cos\Delta_\beta = \frac{\cos\Delta \cos\alpha}{\cos\phi}$

$$\frac{R'_g - R'_p}{R'_p R'_g} = \frac{CD \sin\phi \cos\Delta \cos\alpha}{A'_g \cos\phi\,(A'_g - CD \sin\phi)} = \frac{CD \tan\phi \cos\Delta \cos\alpha}{A'_g\,(A'_g - CD \sin\phi)}$$

Further $\quad \tan\phi = \frac{\tan\alpha}{\cos\Delta}$

$$\frac{R'_g - R'_p}{R'_p R'_g} = \frac{CD \sin\alpha}{A'_g\,(A'_g - CD \sin\phi)} \tag{13}$$

Substituting (11) and (13) in (12)

$$S'_g = 2290 \sqrt{\frac{2\,\pi\,T_p}{F\,N_a\,N_p} \quad \frac{CD \sin\alpha}{A'_g\,(A'_g - CD \sin\phi)}}$$

This expression gives the unit pressure at the tip (I.D.) of the internal gear and is the maximum value of unit pressure on the basis of the foregoing assumptions.

Unit Pressure (tip of pinion)

When the contact is at the tip of the pinion tooth, the radius of the pinion tooth is (see Fig. 4)

$$R'_p = \frac{A'_p}{\cos\Delta_\beta}$$

and that of the gear tooth is

$$R'_g = \frac{A'_p + CD \sin\phi}{\cos\Delta_\beta}$$

$$\frac{R'_g - R'_p}{R'_g\, R'_p} = \frac{A'_p + CD \sin\phi - A'_p}{\cos\Delta_\beta} \quad \frac{\cos^2\Delta_\beta}{A'_p\,(A'_p + CD\sin\phi)}$$

$$\frac{R'_g - R'_p}{R'_g\, R'_p} = \frac{CD \sin\phi \,\cos\Delta_\beta}{A'_p\,(A'_p + CD\sin\phi)}$$

But $\cos\Delta_\beta = \dfrac{\cos\Delta\,\cos\alpha}{\cos\phi}$

$$\frac{R'_g - R'_p}{R'_g\, R'_p} = \frac{CD \sin\phi\,\cos\Delta\,\cos\alpha}{\cos\phi\,A'_p\,(A'_p + CD\sin\phi)} = \frac{CD \tan\phi\,\cos\Delta\,\cos\alpha}{A'_p\,(A'_p + CD\sin\phi)}$$

Further

$$\tan\phi = \tan\alpha\,\sec\Delta$$

$$\frac{R'_g - R'_p}{R'_g\, R'_p} = \frac{CD \tan\alpha\,\sec\Delta\,\cos\Delta\,\cos\alpha}{A'_p\,(A'_p + CD\sin\phi)}$$

$$= \frac{CD \sin\alpha}{A'_p\,(A'_p + CD\sin\phi)} \tag{14}$$

Substituting (11) and (14) in the expression for unit pressure, as before

$$S'_p = 2290\sqrt{\frac{2\,\pi\,T_p}{F\,N_a\,N_p}\;\frac{CD\sin\alpha}{A'_p\,(A'_p + CD\sin\phi)}}$$

This is the unit pressure at the tip of the pinion tooth.

<u>Sliding Velocity</u>

In the case of internal gears the relative angular velocity is

$$w = w_p - w_g$$

in which w_p and w_g are the angular velocities of pinion and gear. Using the same nomenclature as for external gears:

$$w_p = \frac{2\ \pi}{60}\ RPM_p$$

$$w_g = \frac{2\ \pi}{60}\ RPM_g = \frac{\pi}{60}\ RPM_p\ \frac{N_p}{N_g}$$

$$w = \frac{\pi}{30}\ RPM_p \left(1 - \frac{N_p}{N_g}\right) \qquad \text{rad./sec.}$$

The distance from the instantaneous center of rotation (pitch point)

$$a'_g = \frac{PR_g \sin \phi - A'_g}{12} \qquad \text{at tip of gear tooth (I.D.)} \quad \text{feet}$$

and

$$a'_p = \frac{A'_p - PR_p \sin \phi}{12} \qquad \text{at tip of pinion} \quad \text{feet}$$

The sliding velocity is

$$V_g = w\ a'_g = \frac{\pi}{30}\ RPM_p \left(1 - \frac{N_p}{N_g}\right) \frac{PR_g \sin \phi - A'_g}{12}$$

$$= \frac{\pi}{360}\ RPM_p \left(1 - \frac{N_p}{N_g}\right)(PR_g \sin \phi - A'_g) \qquad \text{at tip of gear tooth (I.D.), ft./sec.}$$

or

$$V_p = w\ a'_p = \frac{\pi}{360}\ RPM_p \left(1 - \frac{N_p}{N_g}\right)(A'_p - PR_p \sin \phi) \qquad \text{at tip of pinion tooth, ft./sec.}$$

Scoring Factor PVT

Finally, the scoring factor PVT is obtained by

$$PVT_g = \frac{\pi}{360} RPM_p \left(1 - \frac{N_p}{N_g}\right) (PR_g \sin \phi - A'_g)^2 \; S'_g \qquad \text{at tip of gear tooth (I.D.)}$$

or

$$PVT_p = \frac{\pi}{360} RPM_p \left(1 - \frac{N_p}{N_g}\right) (A'_p - PR_p \sin \phi)^2 \; S'_p \qquad \text{at tip of pinion tooth}$$