DESIGN ABSTRACTS

Balanced Gear-Tooth Design

How shot peening increases bending strength and permits design for greater scoring resistance

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WHEN a pair of gears is required to operate at high loads or high speeds or both, particularly where weight and size are at a premium, it becomes important to consider shot peening in the design. The greater the required horsepower per pound of transmission equipment, the more vital becomes the design of the gears called upon to do the job.

This discussion is concerned primarily with the design of the gear teeth themselves, particularly on spur and helical gears.

TYPES OF FAILURE: In general, gear tooth failure may be grouped into three classifications:

1. *Tooth breakage:* An entire tooth or large portion of a tooth is broken out due to bending stress. It is commonly known as a fatigue type of failure
2. *Pitting:* Characterized by pits or small craters in the contacting surface of the tooth. It is usually found on the pinion tooth at or somewhat below the pitch line. This is also a fatigue type of failure
3. *Scoring:* Sometimes referred to as spalling, scuffing or galling, this type of failure is distinctly different from the first two and is evidenced by a decided roughness on the working tooth flank as though the mating surfaces had seized. It is generally accepted that scoring is actually the result of welding the two surfaces together and then tearing them apart. It is caused by high compressive stress in combination with high sliding velocity of the tooth surfaces upon each other. Whereas bending and pitting failures are most likely to occur after a considerable amount of service, scoring is most likely to occur in the early stages of operation, if at all.

A pair of gears may be subject to any one or all of the foregoing types of failure, depending upon the type of service. In some cases this may result in a compromise between bending strength on one hand and scoring resistance on the other, because in some respects a design factor may favor one at the expense of the other. However, in highly stressed gears, scoring resistance may be increased by proper design consideration which utilizes greater bending strength.

BENDING STRENGTH: In computing the bending strength of a pair of gears, one of the first questions that arises concerns the load on the teeth. A mathematical analysis does not yield a direct answer. Another way would be actually to test a large number of gears of different designs and different load carrying capacities under conditions as nearly as possible like those obtained in the actual service for which the gears have been designed. With complete data on the test procedure, cycles to failure, and design information, the results can be analyzed statistically. This latter method was used in selecting a method of bending stress calculation involved in this discussion.

The procedure used in this selection was as follows: Data were obtained direct from different manufacturers on a large number of dynamometer fatigue tests on spur and helical automotive type transmission gears. Complete information was obtained on the design and test results of each pair of gears.

Several methods of stress computation were then set up, each on the basis of a different set of assumptions. In order to determine which, if any, of these methods would give consistent results, a "stress" value was computed for every gear included in the tests. The stress values obtained by each method were plotted on a log-log chart against the average number of cycles at which fatigue failure occurred. Some of the methods of stress calculation were immediately disqualified because no consistent relationship was obtained between the "stress," as calculated and the fatigue life from the test data. Results as calculated from other methods, however, did show a definite relationship between the calculated stress values and the average fatigue life from the test data, and it was not a difficult matter to select that method which showed the most consistent relationship.

Finally the method of calculation

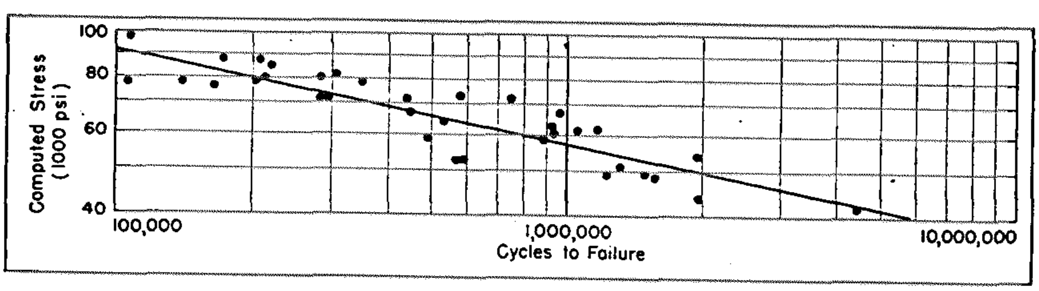

Fig. 1—Fatigue chart of 155 helical automobile transmission gears

selected was based on the assumption that the load is distributed uniformly on the average total length of contact lines. The tooth strength factor is obtained from a layout in the section normal to the tooth, with the load applied at the tip of the tooth in the same manner as originally described by Wilfred Lewis. A complete description of this method of calculating bending strength was published some years ago.

An appraisal of the effectiveness of this method can be obtained from the chart of *Fig.* 1, which shows the relationship between the calculated stress and the average life of the gears as determined by the dynamometer tests. Included in the chart are 155 tests on a variety of designs. All tests represented in *Fig.* 1 were made on gears of approximately 60 Rockwell C surface hardness, with no surface treatment other than the heat treatment itself.

Data May Suggest Redesign

By this method, the design of a proposed pair of gears can be analyzed, and in some cases an appreciable gain in fatigue strength can be obtained by changes in the tooth proportions. For example, the requirements might permit a decrease in the diametral pitch or an increase in pressure angle, either of which would decrease the bending stress.

It is quite evident, of course, that any strength calculation cannot be expected to overcome poor manufacturing practice. A number of factors in processing can seriously affect the life of the gears. One such factor is the nature of the tooth fillet, with regard to stress concentration resulting from deep tool marks. Deep grooves in the tooth fillet can be more serious than the stress concentration because of the fillet itself. To minimize this condition it is good practice to use a chamfer, or better still, a rounded corner on the hob or cutter.

SHOT PEENING: In gears which are intended to carry high stresses, a pronounced increase in fatigue strength can be obtained by shot peening the teeth. Effectiveness of shot peening in overcoming fatigue failures in a given design of a machine part is well known. But its advantages in designing for greater fatigue strength and greater utilization of material are often overlooked. The increase in allowable stress for a given fatigue life requirement will vary with that life requirement. That is, the greater the required life, the greater will be the benefit derived from shot peening. However, even under severe requirements, an increase of 10 per cent in allowable stress is conservative.

In *Fig.* 2 is shown a fatigue line, similar to that of *Fig.* 1, based on the same method of calculation, but from considerably more data than those originally accumulated. The lower line is comparable to that of *Fig.* 1 except that it represents carburized gears only and is somewhat higher. The original data in *Fig.* 1 included carburized gears as well as through-hardened and cyanided gears. Although the test points for the through-hardened gears were somewhat lower than for those which were carburized, there were not sufficient data at that time to distinguish between these two groups. More recent data on carburized gears have indicated that there is a distinct advantage in carburized gears as compared to those which are through-hardened and cyanided. It would therefore be expected that results on through-hardened gears would fall below the average line. The upper line of *Fig.* 2 shows the average life in relation to the calculated stress for shot-peened carburized gears. In all cases, peening was the last operation insofar as the teeth are concerned, and no attempt was made to protect the tooth flanks from the blast.

Strength and Life Increased

It can be seen from the chart that, for example, an average life of 800,000 cycles is obtained on nonpeened gears at a calculated stress of 71,000 psi. For the same average life, the allowable stress for shot-peened gears is 88,000 psi, or an increase of more than 24 per cent in allowable stress. From the standpoint of increase in life at a given calculated stress, it can be seen that at a calculated stress of 80,000 psi, the average life of nonpeened gears is 300,000 cycles, and that of shot-peened gears 3,000,000 cycles, or an increase of 1000 per cent in life.

Shot peening is equally effective on gears of lower hardness and, of course, on other types of gears such as spiral bevel, hypoid, etc. Sufficient data are not available for stress-life charts on the other types but an increase of 10 per cent in allowable stress is quite conservative.

Although shot peening is usually the last operation, the slight roughening of the surface of the tooth flanks has no detrimental effect. Actually, some manufacturers feel that shot-peened gears are more quiet running than nonpeened gears of the same design and manufacture. In some cases, however, it is desired to have a smooth finish on the tooth flanks. This can be accomplished by cutting the teeth with a protuberance hob, which produces an effect similar to undercutting at the root of the tooth. The gears are then hardened, shot peened and ground. This procedure allows the tooth flanks to be ground without removing any of the shot-peened surface in the fillet where the bending stress is maximum.

PITTING RESISTANCE: Data on pitting resistance are not as plentiful as in the case for bending strength, primarily because of the difficulty in obtaining quantitative data. Bending failures can be easily recognized while the gears are running, because of the sudden noise. Pitting on the other hand is a slow progressive failure, starting with extremely small craters in the tooth profile. In some cases these craters grow larger and more numerous until, finally, enough material is removed to cause complete failure. In other cases it may develop as very small pits which progress to a moderate degree and then stop without any real damage.

Wickendon, Brophy and Miller have been successful in establishing some quantitative data, and in determining the life of gears at which pitting begins. It is rather generally accepted that pitting is the result of high compressive stress, but to my knowledge there are not sufficient data available to establish the validity of any method of calculation on the basis of actual fatigue tests. A value of

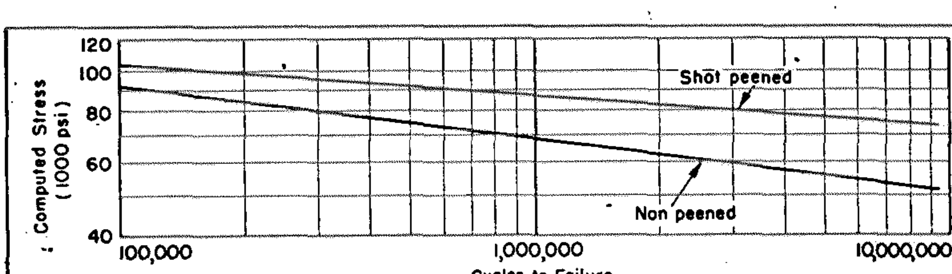

Fig. 2—Fatigue chart of carburized automotive type spur and helical gears, shot-peened and nonpeened

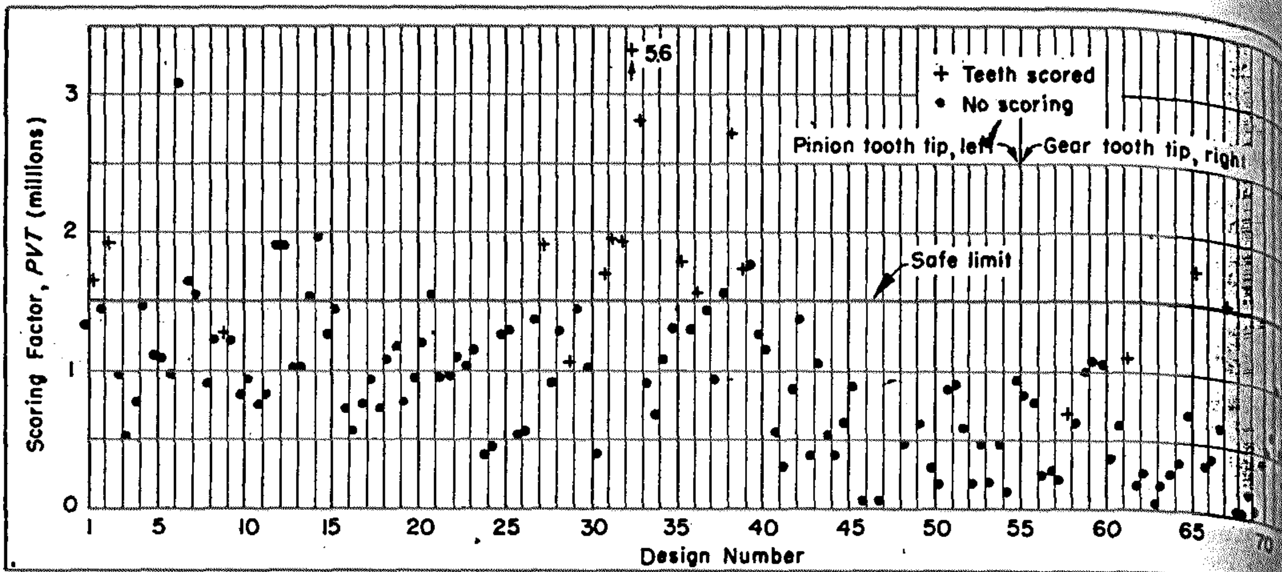

Fig. 3—Correlation of scoring factor, *PVT*, with actual tests on 73 gear designs

maximum compressive stress can be obtained by a method described later in this discussion. A limiting value of 200,000 psi has been used with some success. This is a qualitative value, but it is believed to be on the conservative side.

SCORING RESISTANCE: Scoring is caused by a combination of high compressive stress and high sliding velocity on the contacting tooth surfaces. By means of an approach similar to that described in appraising the bending strength formula, a method of calculation of scoring resistance in spur and helical gears was developed by the author, under the direction of J. O. Almen at the Research Laboratories Division, General Motors Corp. Dynamometer test data were accumulated on a large number of gears, along with complete design information. With these data at hand, various assumptions were made until good correlation was obtained between calculated values of scoring resistance and the actual test data. Briefly, the method selected is based upon the same assumption, with regard to distribution of the transmitted load, as that used in the bending strength calculation discussed previously.

The method consists of the calculation of the product, *PVT*. Here, *P* is the maximum compressive stress as determined by the Hertz equation for cylindrical surfaces, for a point located at the tip of the gear tooth or pinion tooth, and based upon the total tooth load derived from torque, the average total length of lines of contact, and the curvatures of the tooth surfaces in the plane normal to the line of contact at the selected point; *V* is the sliding velocity of the surfaces at the selected point; and *T* is the distance in the plane of rotation from the pitch point to the selected point. Simplified formulas for the calculation of *PVT* are given in the appendix.

The degree of correlation of the calculated values with actual test results can be seen from the chart of *Fig.* 3. Data shown in this chart represent actual test results on well over 50,000 pairs of aircraft gears. The gears tested covered a wide range of requirements. The torque requirements varied from a few pound-feet up to several thousand pound-feet, and the speed requirements ranged from a few hundred rpm up to 28,000 rpm. All test data accumulated were on fully hardened spur gears, lubricated with mineral oil. External as well as internal gears were included.

Safe PVT Limit Shown

Note that, by and large, those gears which have a calculated *PVT* factor in excess of 1,500,000 failed by scoring, whereas those with a *PVT* factor of less than 1,500,000 had no scoring failure. It should be mentioned that with extreme pressure lubricants, a value of *PVT* in excess of 1,500,000 may be used.

BALANCED DESIGN WITH SHOT PEENING: A study of the design considerations from the standpoint of both bending strength and scoring resistance reveals that there is some conflict between high bending strength on the one hand and high scoring resistance on the other. For example, a coarse pitch is desirable for bending strength because of the greater thickness at the root of the tooth. However, in some cases this would be impractical because a coarse pitch would necessitate long teeth for continuous action. This, of course, would mean an increase in the length of action and, consequently, higher sliding ve- locity. Furthermore, with long te- particularly in high ratio gears, t- action approaches the base circle the pinion which in turn results in high compressive stress because the small radius of curvature of pinion tooth in that region. In such case, an increase in bending strength may lead to scoring tendencies. There- fore, in order to obtain satisfactory operation with minimum weight, may be necessary to compromise be- tween bending strength and scoring resistance. This leads to a logical question as to the usefulness of shot peening relative to scoring resistance. For a given design, experience indi- cates that shot peening has little di- rect influence on the scoring tendency of gears of the same design and op- erating conditions. However, since balanced design may involve a com- promise between bending strength and scoring resistance, a distinct ad- vantage can be gained by designing the gears with a finer pitch in favor of scoring resistance. This would re- sult in decreased bending strength which can be restored by shot peen- ing. By this reasoning it can be seen that shot peening can be used directly for increasing bending fatigue strength, or indirectly, by proper de- sign consideration, for increasing scoring resistance.

APPENDIX: The nomenclature used in the calculations is as follows:

C = Center distance, inches
F = Face width, inches
n_p = Pinion speed with respect to its own axis, rpm
N_P, N_G = Number of teeth in pinion and gear
P_P, P_G = Maximum compressive

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stress for pinion and gear tooth respectively, psi

r, R = Operating pitch radius of pinion and gear respectively, inches

r_o, R_o = Outside radius of pinion and external gear respectively, inches

R_i = Inside radius of internal gear, inches

T_p = Pinion torque, pound-inches

Z = Length of line of action, inches

ρ_P, ρ_G = Radius of curvature at tip of pinion and gear tooth, inches

ϕ_n = Normal pressure angle, degrees

ϕ_t = Transverse pressure angle, degrees ($\tan \phi_t = \tan \phi_n \sec \psi$)

ψ = Helix angle, degrees

Calculation of the *PVT* factor for external gears involves the following formulas:

Radius of curvature in the transverse plane, at the tip of the pinion tooth,

$$\rho_P = \sqrt{r_o^2 - r^2 \cos^2 \phi_t}$$

and at the tip of the gear tooth,

$$\rho_G = \sqrt{R_o^2 - R^2 \cos^2 \phi_t}$$

Length of action,

$$Z = \rho_P + \rho_G - C \sin \phi_t$$

Maximum compressive stress, at the tip of the pinion tooth,

$$P_P = 2290 \sqrt{\frac{2\pi T_P C \sin \phi_n}{F Z N_P \rho_P (C \sin \phi_t - \rho_P)}}$$

and at the tip of the gear tooth,

$$P_G = 2290 \sqrt{\frac{2\pi T_P C \sin \phi_n}{F Z N_P \rho_G (C \sin \phi_t - \rho_G)}}$$

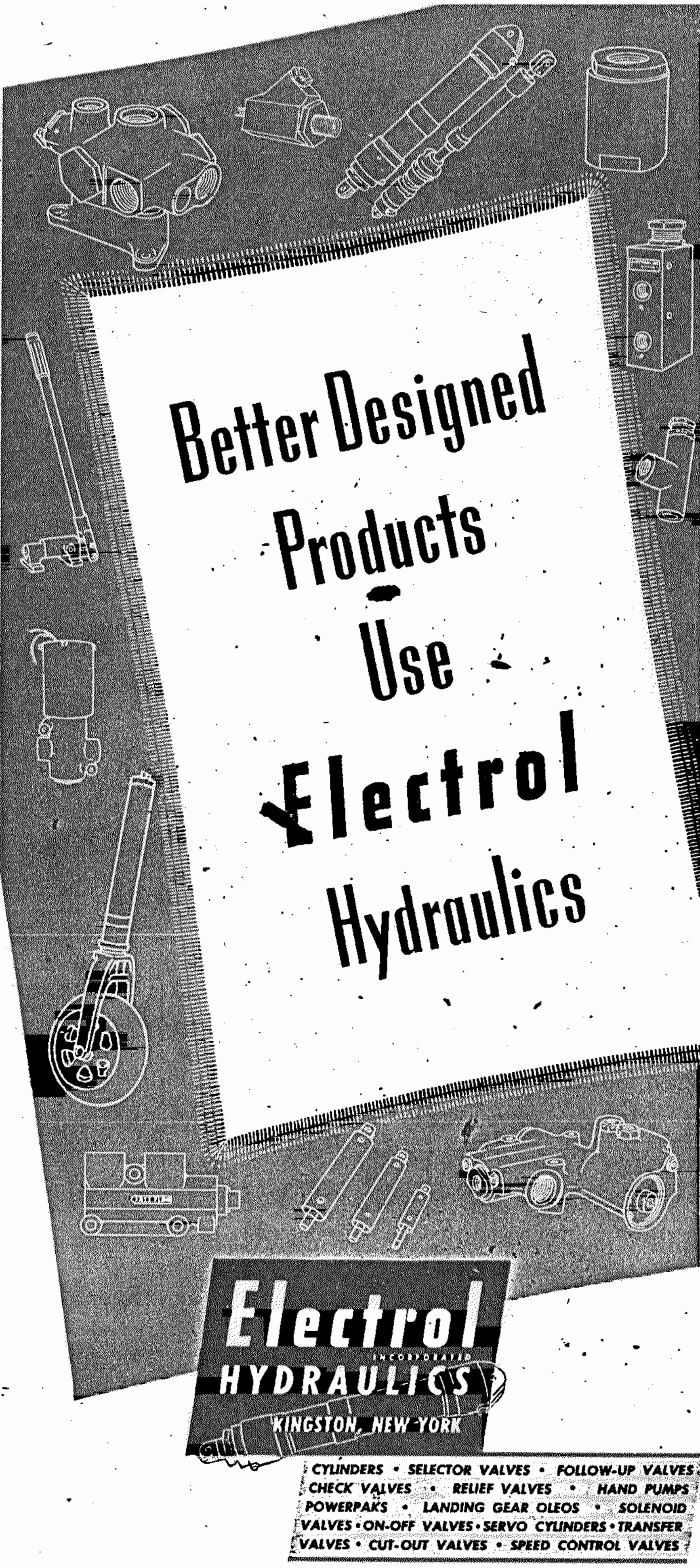
Scoring factor, at the tip of the pinion tooth,

$$(PVT)_P = \frac{\pi n_p}{360} \left(1 + \frac{N_P}{N_G}\right) (\rho_P - r \sin \phi_t)^2 P_P$$

and at the tip of the gear tooth,

$$(PVT)_G = \frac{\pi n_p}{360} \left(1 + \frac{N_P}{N_G}\right) (\rho_G - R \sin \phi_t)^2 P_G$$

The following formulas apply in the



calculation of the *PVT* factor, for internal gears:

Radius of curvature in the transverse plane, at the tip of the pinion tooth,

$$\rho_P' = \sqrt{r_o^2 - r^2 \cos^2 \phi_t}$$

and at the tip of the gear tooth,

$$\rho_G' = \sqrt{R_i^2 - R^2 \cos^2 \phi_t}$$

Length of action,

$$Z' = \rho_P' - \rho_G' + C \sin \phi_t$$

Maximum compressive stress, at the tip of the pinion tooth,

$$P_P' = 2290 \sqrt{\frac{2\pi T_P C \sin \phi_n}{F Z' N_P \rho_P' (\rho_P' + C \sin \phi_t)}}$$

and at the tip of the gear tooth,

$$P_G' = 2290 \sqrt{\frac{2\pi T_P C \sin \phi_n}{F Z' N_P \rho_G' (\rho_G' - C \sin \phi_t)}}$$

Scoring factor, at the tip of the pinion tooth,

$$(PVT)_{P'} = \frac{\pi n_p}{360} \left(1 - \frac{N_P}{N_G}\right) (\rho_P' - r \sin \phi_t)^2 P_P'$$

and at the tip of the gear tooth,

$$(PVT)_{G'} = \frac{\pi n_p}{360} \left(1 - \frac{N_P}{N_G}\right) (R \sin \phi_t - \rho_G')^2 P_G'$$

From a paper entitled "Shot Peening as a Factor in the Design of Gears," presented at the ASME Spring Meeting in Atlanta, April 2-5, 1951. Complete copies may be obtained from ASME, 29 W. 39th St., New York 18; $0.25 each to members, $0.50 to nonmembers.

Improved Machine Design

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IT CAN be said that machine design may be improved along three lines: those of function, cost, and appearance.

FUNCTION: Function relates to what the machine is and what it does, the performance that it gives, and its handling characteristics. From the point of view of selling the machine, it is its specification which is of prime importance. In comparing one machine with another, it is the specification which determines those factors which appeal most to the buyer and will influence his choice.

The second and, from some points