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FACTORS OF STRESS CONCENTRATION FOR THE BENDING CASE OF FILLETS IN FLAT BARS AND SHAFTS WITH CENTRAL ENLARGED SECTION

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ABSTRACT

The factors of stress concentration for fillets in a field of pure bending (denoted as the limiting case) have been investigated photoelastically for flat bars. Under the condition of pure bending on both sides of the fillets, the maximum stress is a function of the two parameters r/d and D/d, where D is the width of the enlarged section of the bar, d is the width of the bar, and r is the radius of the connecting fillets.

In this investigation, the factors of stress concentration have been determined for the case of a flat bar of width d, containing a central enlarged section of width D and length L and connected by fillets of radius r, when subjected to pure bending in the narrow portion of the bar. Under these conditions, an additional parameter L/D influences the value of the maximum stress obtained at the fillets.

The factors of stress concentration increase with increasing L/D, attaining as an upper bound the value for the limiting case. For $r/d = .025$ this limiting value is reached at about $L/D = 2.0$; while for $r/d = 1.0$, the limiting value is practically reached at $L/D = 0$.

Three-dimensional tests using Fosterite and the "stress freezing" technique have been made to obtain correlation between the two- and three-dimensional factors of stress concentration. A novel method of slicing makes possible the obtaining of a series of tests from a single model.

INTRODUCTION

The factors of stress concentration for fillets in a field of pure bending have been determined photoelastically for flat bars.[1]* The case of pure bending on either side of the fillet shall be referred to as the "limiting case". For the limiting case, the factor of stress concentration is a function of the two parameters r/d and D/d, where d is the width of the narrow portion of the bar, D is the width of the enlarged section, and r is the radius of the connecting fillets (Fig. 1).

In this investigation, the factors of stress concentration, k, have been determined photoelastically for the bending case of a flat bar of width d, containing a centrally enlarged section of width D and length L and connected by fillets of radius r (Fig. 2). The loads were applied at points sufficiently removed from the fillets to produce pure bending in the narrow portion of the bar. Under these conditions, k is influenced by a third parameter, L/D.

SCOPE OF TESTS

Maintaining a constant value of D/d and L/D, models were machined having ratios of r/d varying from about .02 to 1. The value of the stress concentration factor, k, was obtained for each of these models enabling the plotting of one of the curves of k vs. r/d for constant D/d and L/D, as shown in Fig. 2. D/d was kept constant at values of 1.25, 2, and 3, while L/D was maintained at 1/4, 1/2, 1, and 2; resulting in the twelve solid curves shown in Figs. 2, 3 and 4.

The experimental points are indicated in Fig. 2 to show the number of individual tests made and the overall consistency of the results. Values of k obtained by Frocht[1] for the limiting case ($L/D \geqslant 2$) are shown in Fig. 3 to show the agreement with the present investigation.

Presented at the Society's Spring Meeting in Cleveland, Ohio, May 27, 1950.

* Superiors in parentheses pertain to references.

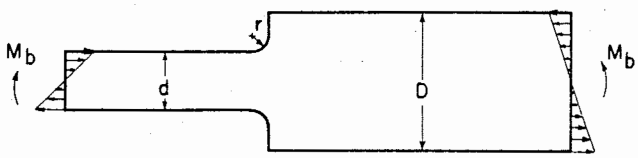

FIG. 1 : FILLETS IN A FIELD OF PURE BENDING (LIMITING CASE)

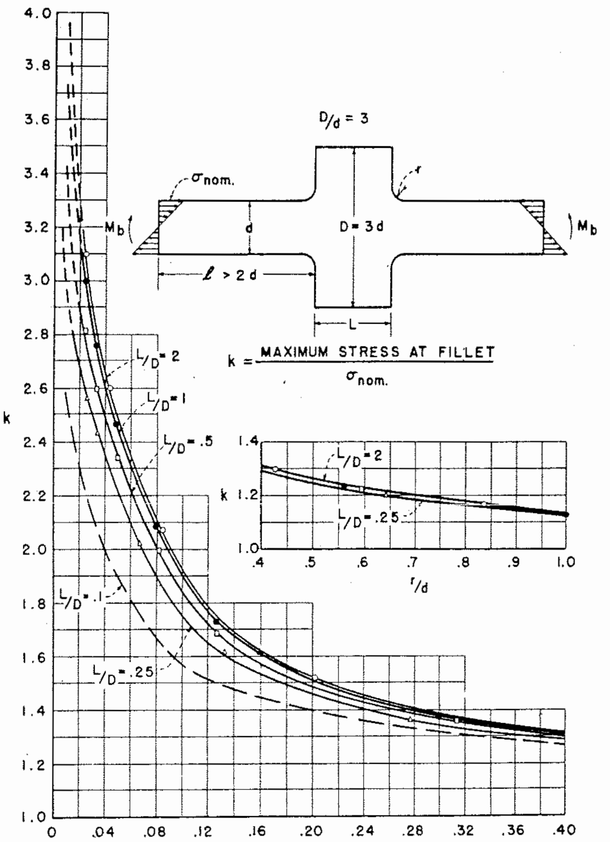

FIG. 2 FACTORS OF STRESS CONCENTRATION FOR THE BENDING CASE OF FILLETS IN A FLAT BAR WITH CENTRAL ENLARGED SECTION. D/d = 3

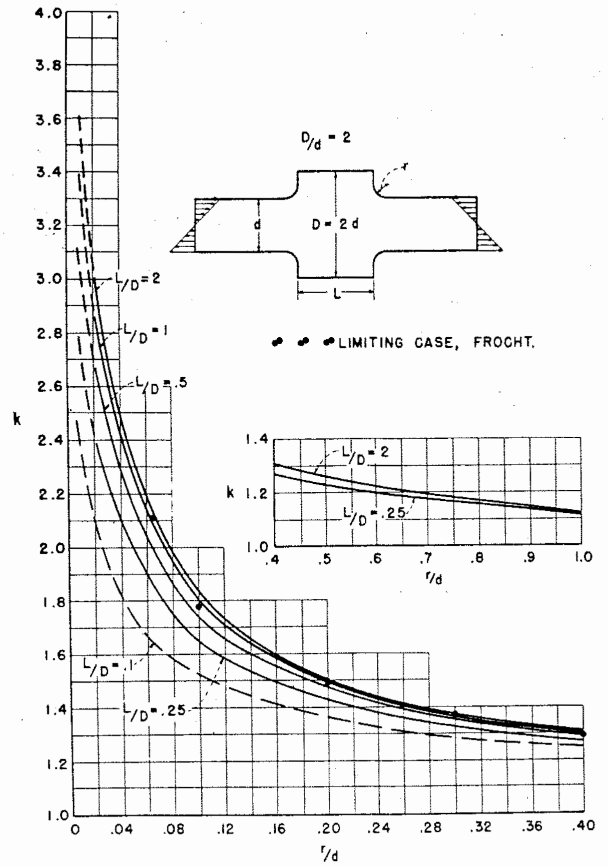

FIG. 3 k vs. r/d FOR VARIOUS VALUES OF L/D. D/d = 2

METHOD OF EVALUATING k.

The factor of stress concentration, k, is defined as the ratio of the maximum stress at the fillet to the nominal stress computed by the flexure formula. Thus

$$k = \frac{\sigma_f}{\sigma_{nom}}, \text{ where} \qquad (1)$$

σ_f = maximum stress of the fillet, and

$$\sigma_{nom} = \frac{M_b c}{I} = \frac{6 M_b}{t d^2}$$

t being the thickness of the bar.

In this investigation k was determined exclusively by the photographic method. Thus

$$k = \frac{n_f}{n}, \text{ where} \qquad (2)$$

n_f is the maximum fringe order at the fillet and n is the fringe order at the outer fiber of the narrow portion of the bar where the conditions of pure bending exist. Thus, referring to Fig. 5,

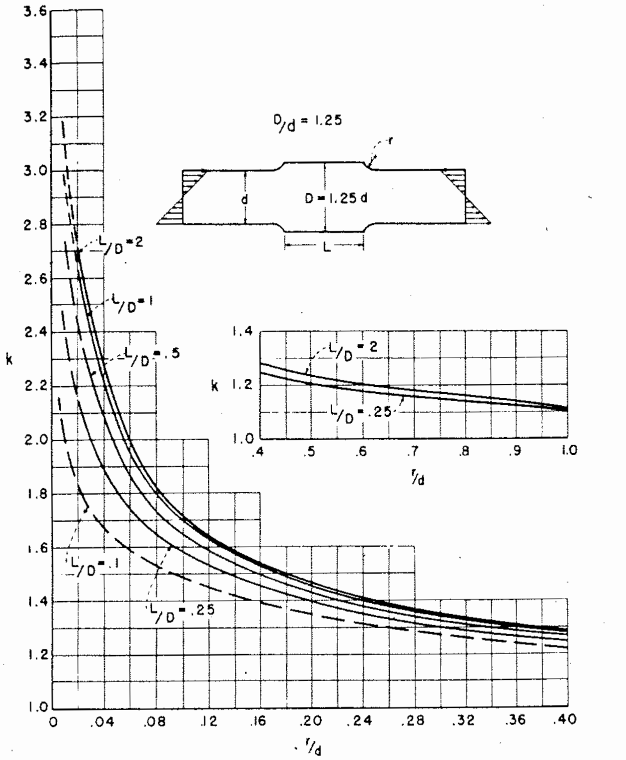

FIG. 4 k vs. r/d FOR VARIOUS VALUES OF L/D. D/d = 1.25.

$$k = \frac{n_f}{n} = \frac{9.5}{4.5} = 2.11 \text{ for the upper fillet.}$$

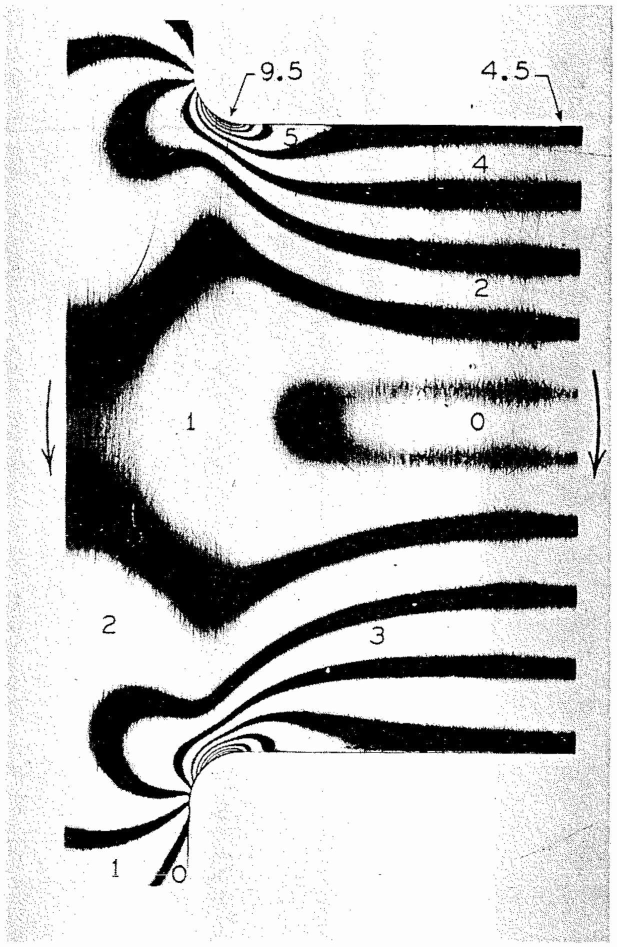

FIG. 5. STRESS PATTERN FOR BENDING CASE OF FILLETS IN A FLAT BAR WITH CENTRAL ENLARGED SECTION.

r = .157 in.	L = 5.908 in.	D/d = 2.97
d = 1.983 in.	t = .117 in.	L/D = 1.002
D = 5.895 in.	r/d = .0792	M_b = 185.0 lb. in.

K(upper fillet) = $9.5/4.5 = 2.11$ MATERIAL: AMBER BAKELITE

For each test, four values of k were obtained: two on the compression side and two on the tension side. Furthermore, at least three different loads were employed for each test. Thus, k for each experimental point was taken as the average of about twelve values.

Fig. 6 is of interest because it shows the small portion of the stress pattern required for the photographic determination of k, a fact which is employed in the slicing of the "frozen stress" models used for the three-dimensional tests of shafts.

RESULTS

In addition to presenting the results as k vs. r/d for constant D/d and L/D as done in Figs. 2, 3 and 4; curves of k vs. L/D for constant r/d and D/d have been plotted as shown in Figs. 7, 8 and 9. Here, the benefit of an additional point is obtained where k is equal to 1 and L/D is negative and given by:

$$L/D = -\frac{1}{D/d}\sqrt{(D/d - 1)(4r/d - D/d + 1)} \quad \text{when}$$

$$2r/d > (D/d - 1) \quad \text{or} \tag{3a}$$

$$L/D = -2(r/d)\left(\frac{1}{D/d}\right) \quad \text{when}$$

$$2r/d \leqslant (D/d - 1) \tag{3b}$$

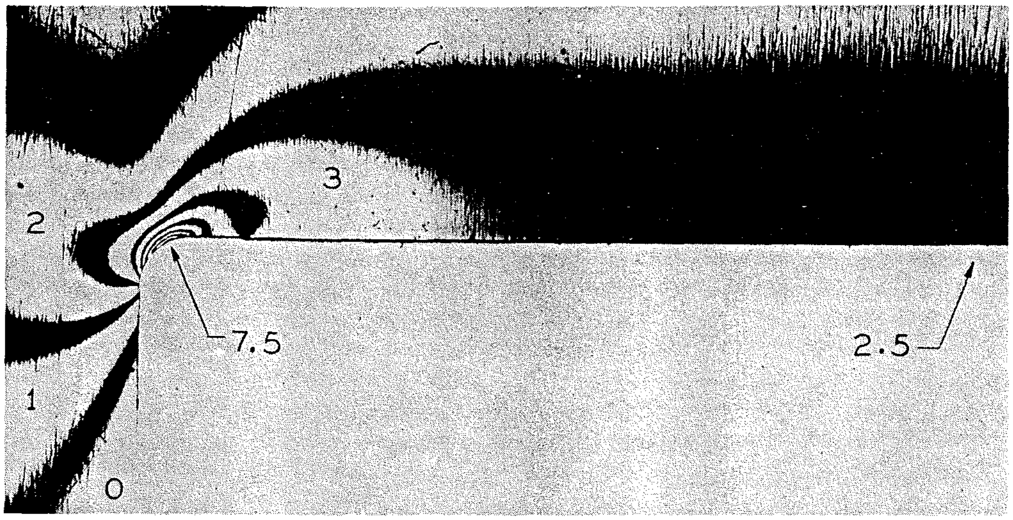

FIG. 6. STRESS PATTERN SHOWING SMALL PORTION NEEDED FOR PHOTOGRAPHIC DETERMINATION OF FACTOR OF STRESS CONCENTRATION.

r = .047 in.	L = 5.895 in.	r/d = .0241
d = 1.951 in.	t = .117 in.	D/d = 3.017
D = 5.891 in.		L/D = 1.00
$K = 7.5/2.5 = 3.00$	MATERIAL: AMBER BAKELITE	

Finally, the results are shown in tabular form in Table I.

In addition, the empirical equation

$$k = 1 + \left\{\tanh^{\frac{1}{2}}\left[2(L/D + r/d)\right]\right\}\left\{\tanh\left[\frac{(D/d - 1)^{\frac{1}{4}}}{1 - r/d}\right]\right\}\left\{\frac{.13 + .65(1 - r/d)^{4}}{(r/d)^{\frac{1}{3}}}\right\}$$

$$\text{for} \quad r/d < 1, \quad D/d > 1, \quad L/D > 0 \qquad (4)$$

has been developed, which is in good agreement with the experimental results and is useful for interpolation and extrapolation within the limits prescribed.

Fig. 10 shows k vs. D/d for various values of r/d, when $L/D \geqslant 2$. Results from empirical equation (4) are shown for $r/d = .025$.

THREE-DIMENSIONAL TESTS

The factors of stress concentration for the three-dimensional case of fillets in a field of pure bending were determined using Fosterite(2) shafts and the "freezing" and "slicing" technique. Instead of slicing the entire shaft, only a small part of the shaft was milled away to expose an outer ridge of the slice as shown in Fig. 11. The factor of stress concentration can be determined photographically as before for the flat bars. Thus, referring to the stress pattern of Fig. 12;

$$k = \frac{n_f}{n} = \frac{5.12}{2.5} = 2.05$$

After partial slicing, the model may be annealed to remove the stresses and remachined to a slightly smaller size so as to remove the ridges left by the partial slicing. In this manner, a series of tests may be obtained from a single model.

Fig. 13 shows a stress pattern obtained by partial slicing for a fillet of 1/16 inch radius in a shaft 2-1/2 inches in diameter, while Fig. 14 shows a "frozen" stress pattern of a conventionally sliced model.

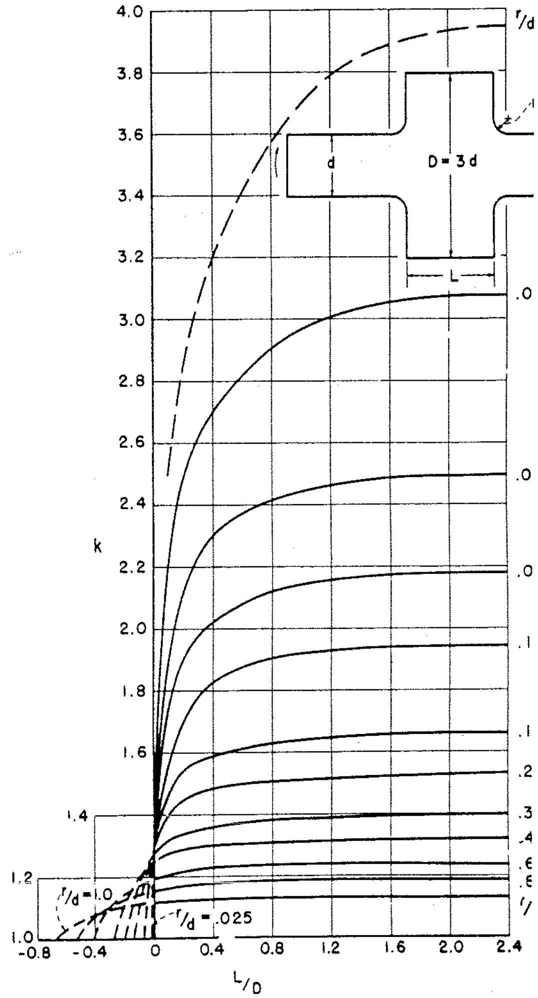

FIG. 7 FACTORS OF STRESS CONCENTRATION, k PLOTTED AGAINST L/D. D/d = 3

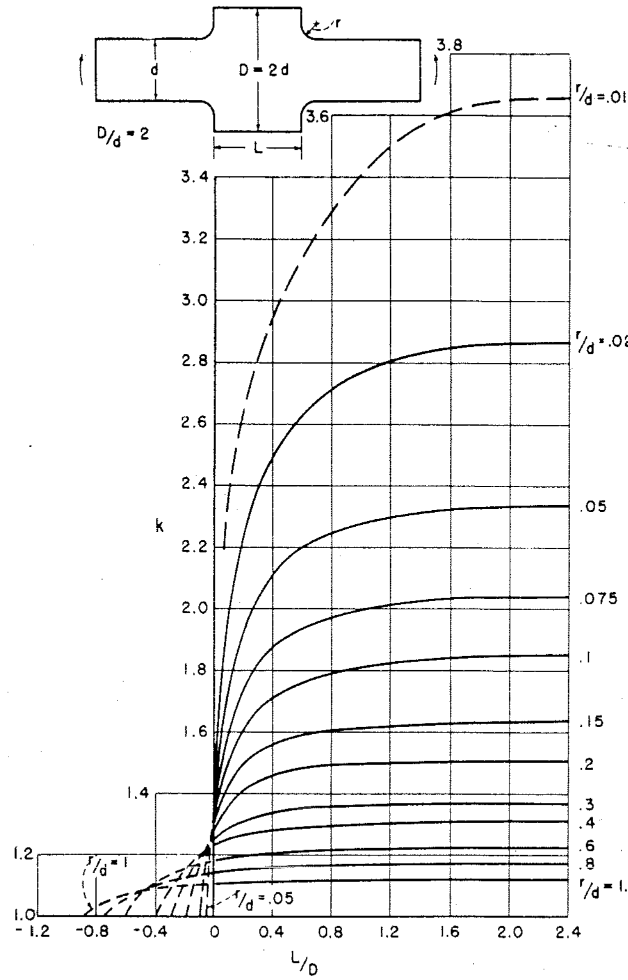

FIG. 8 FACTORS OF STRESS CONCENTRATION, k, PLOTTED AGAINST L/D. D/d = 2

The frozen stress patterns were obtained using a diffusion polariscope(3) and merely coating the models with Halowax oil so that immersion in large tanks was not necessary.

The factors of stress concentration obtained from the "frozen stress" tests for the bending case of fillets in shafts are shown in Fig. 15 together with the two-dimensional results obtained, those of Frocht(1) for flat bars, and the results of Peterson and Wahl(4) obtained on large steel shafts using a mechanical strain gage.

CONCLUSIONS

For the bending case of bars and shafts with central enlarged sections of width D and length L the factor of stress concentration, k, is a function of L/D in addition to the other two parameters r/d and D/d which govern in the pure bending or limiting case. k increases with increasing L/D attaining the maximum value when L/D = 2 for r/d = .025, while for r/d = 1, the limiting value is practically reached at L/D = 0.

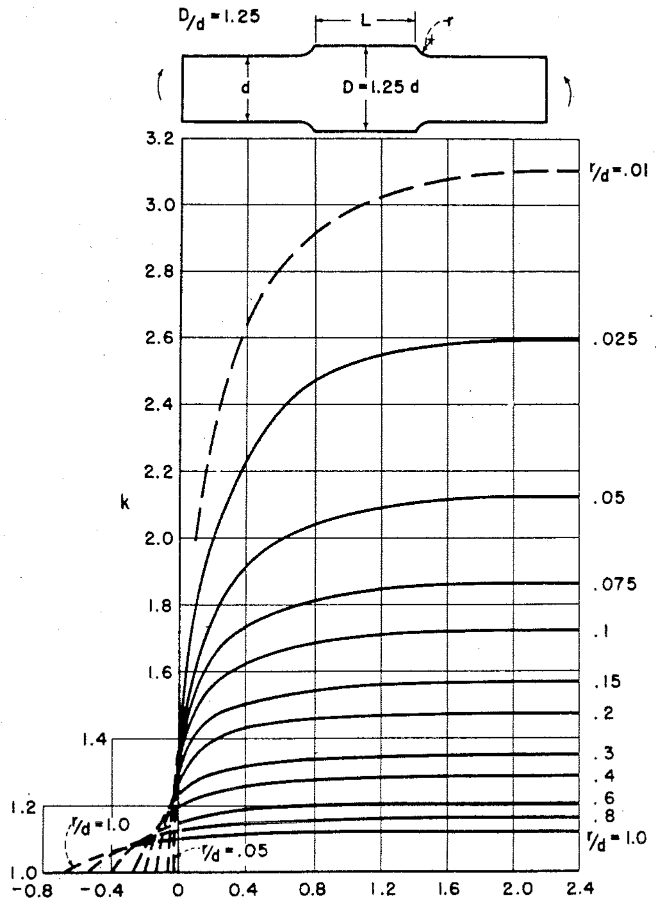

FIG. 9 k vs L/D. D/d = 1.25

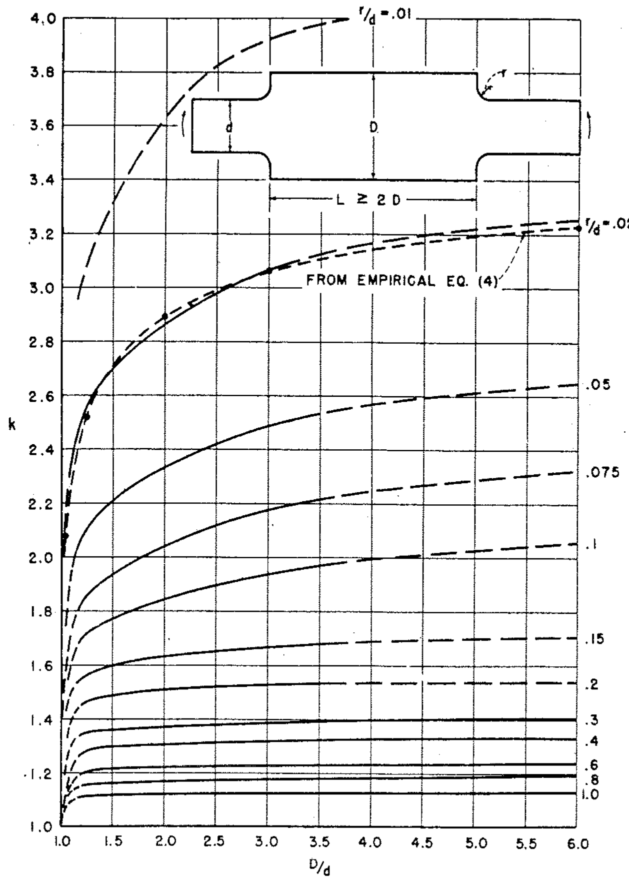

FIG. 10 FACTORS OF STRESS CONCENTRATION, k, FOR THE LIMITING CASE (L/D ≥ 2) OF FILLETS IN BENDING.

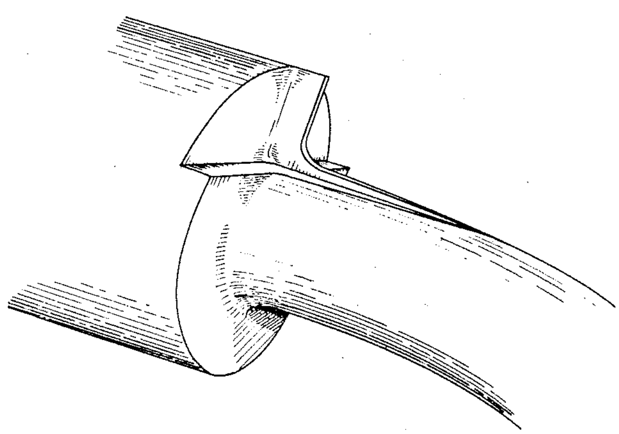

FIG. 11
SKETCH SHOWING METHOD OF PARTIAL SLICING.

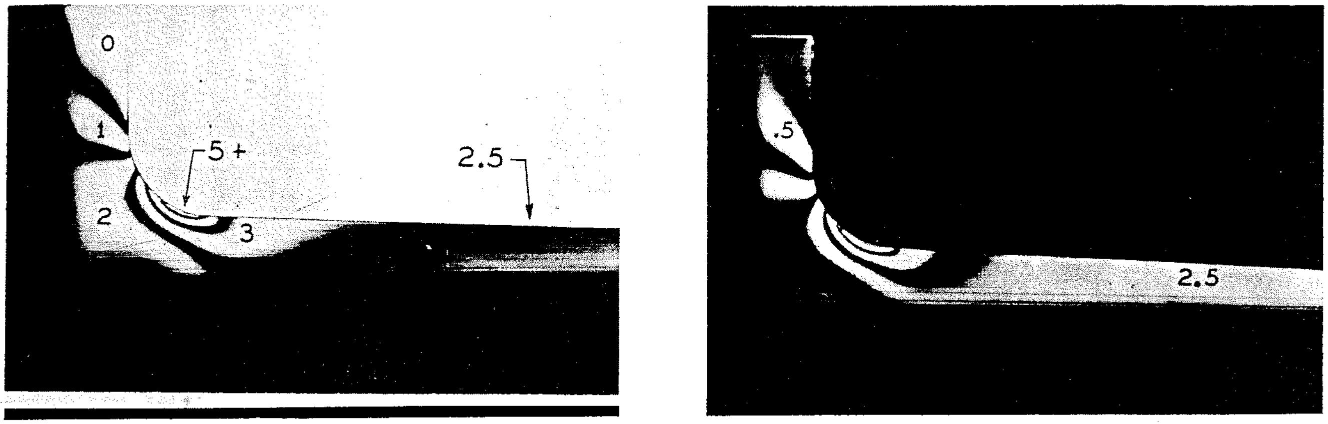

FIG. 12. "FROZEN" STRESS PATTERN FOR FOSTERITE SHAFT WITH FILLET IN PURE BENDING. SHAFT HAS BEEN PARTIALLY SLICED AS SHOWN IN FIG. 11. LIGHT AND DARK BACKGROUND PHOTOGRAPHS OF TENSION SIDE.

INITIAL DIMENSIONS:

r = .188 in.	r/d = .047
d = 4.006 in.	D/d = 1.25
D = 5.008 in.	L/D = 2.01
L = 10.065 in.	M_b = 283.9 lb. in.

FINAL DIMENSIONS:

r_t' = .200 in.	r_t'/d' = .050

THICKNESS OF SLICE = .126 in.

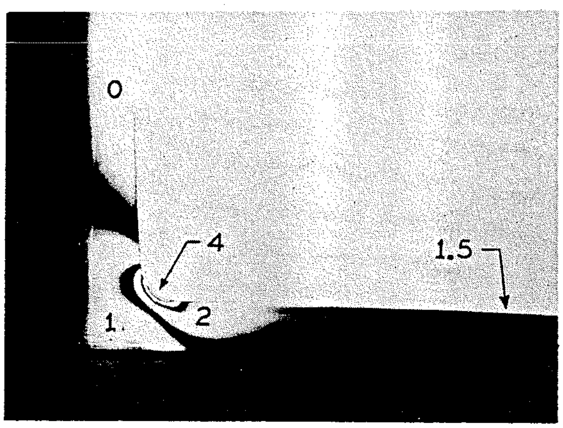

FIG. 13. "FROZEN" STRESS PATTERN FOR PARTIALLY SLICED FOSTERITE SHAFT WITH FILLET IN PURE BENDING. TENSION SIDE.

INITIAL DIMENSIONS:

r = .063 in.	r/d = .0255
d = 2.450 in.	D/d = 2.00
D = 4.900 in.	L/D = 1.98
L = 9.730 in.	M_b = 103.9 lb. in.

FINAL DIMENSIONS:

r_t' = .074 in.	r_t'/d = .0302

THICKNESS OF SLICE: .046 in. $K = 4/1.5 = 2.67$

TABLE I.

FACTORS OF STRESS CONCENTRATION, k, FOR THE BENDING CASE OF FILLETS IN FLAT BARS WITH CENTRAL ENLARGED SECTION

r/d	D/d = 3.00					D/d = 2.00					D/d = 1.25				
	L/D					L/D					L/D				
	2	1	0.5	0.25	0.1*	2	1	0.5	0.25	0.1*	2	1	0.5	0.25	0.1*
0.01*	3.93	3.70	3.32	2.96	2.55	3.65	3.40	3.04	2.72	2.33	3.10	2.97	2.73	2.41	2.00
0.025	3.07	2.97	2.76	2.56	2.21	2.86	2.77	2.57	2.32	2.00	2.59	2.51	2.32	2.06	1.78
0.05	2.49	2.44	2.34	2.21	1.91	2.33	2.27	2.16	1.99	1.73	2.12	2.07	1.96	1.81	1.62
0.075	2.17	2.14	2.05	1.95	1.71	2.04	1.99	1.90	1.79	1.60	1.76	1.83	1.76	1.67	1.54
0.10	1.94	1.91	1.85	1.75	1.57	1.85	1.81	1.74	1.64	1.52	1.72	1.70	1.64	1.58	1.49
0.15	1.66	1.64	1.60	1.56	1.46	1.63	1.61	1.57	1.51	1.43	1.57	1.56	1.52	1.48	1.41
0.2	1.53	1.51	1.49	1.46	1.40	1.51	1.49	1.47	1.43	1.36	1.47	1.46	1.44	1.40	1.35
0.3	1.39	1.38	1.37	1.34	1.31	1.37	1.36+	1.35	1.32	1.28	1.35	1.34	1.32	1.30	1.27
0.4	1.31+	1.31	1.30	1.29	1.27	1.30+	1.30	1.29	1.27	1.25	1.28+	1.28	1.26+	1.24	1.22
0.6	1.23+	1.23	1.22	1.21	1.20	1.22	1.22	1.21	1.20	1.19	1.20+	1.20	1.19	1.18	1.16
0.8	1.18	1.17+	1.17	1.17−	1.16	1.17	1.17−	1.16	1.16−	1.15	1.16	1.15+	1.15	1.14	1.13
1.0	1.12	1.12	1.12	1.12−	1.11+	1.12	1.12	1.12	1.11+	1.11	1.12	1.11+	1.11+	1.11	1.10+

* FROM EXTRAPOLATION

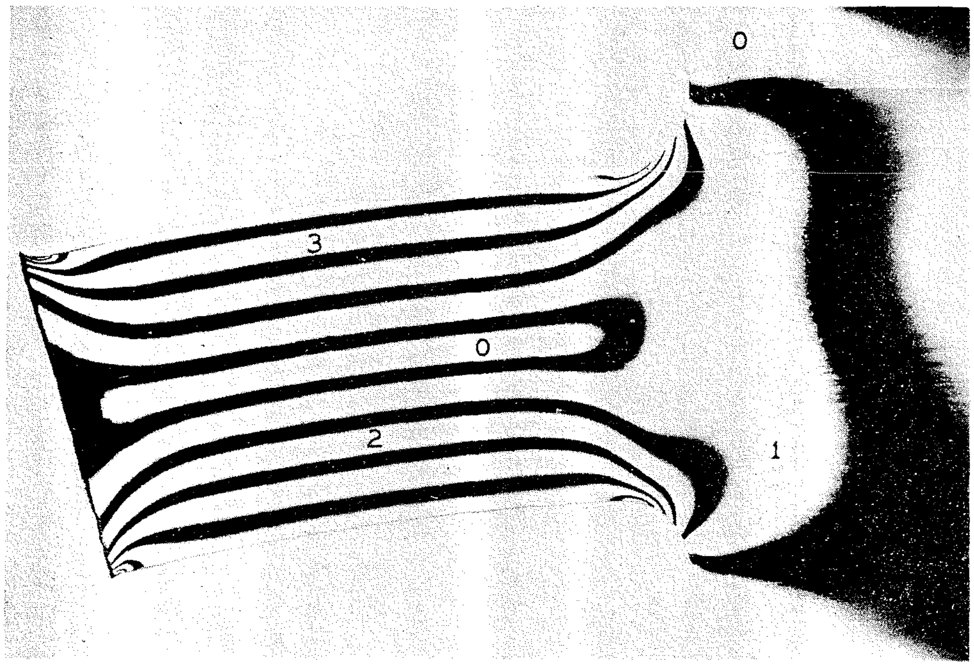

FIG. 14. STRESS PATTERN OF CONVENTIONALLY SLICED MODEL OF SHAFT WITH FILLET IN PURE BENDING.

INITIAL DIMENSIONS:

r = .250 in.	L = 4.430 in.	L/D = 2.00
d = 1.108 in.	r/d = .225	M_b = 9.66 lb. in.
D = 2.216 in.	D/d = 2.00	

THICKNESS OF SLICE = 0.138 in.

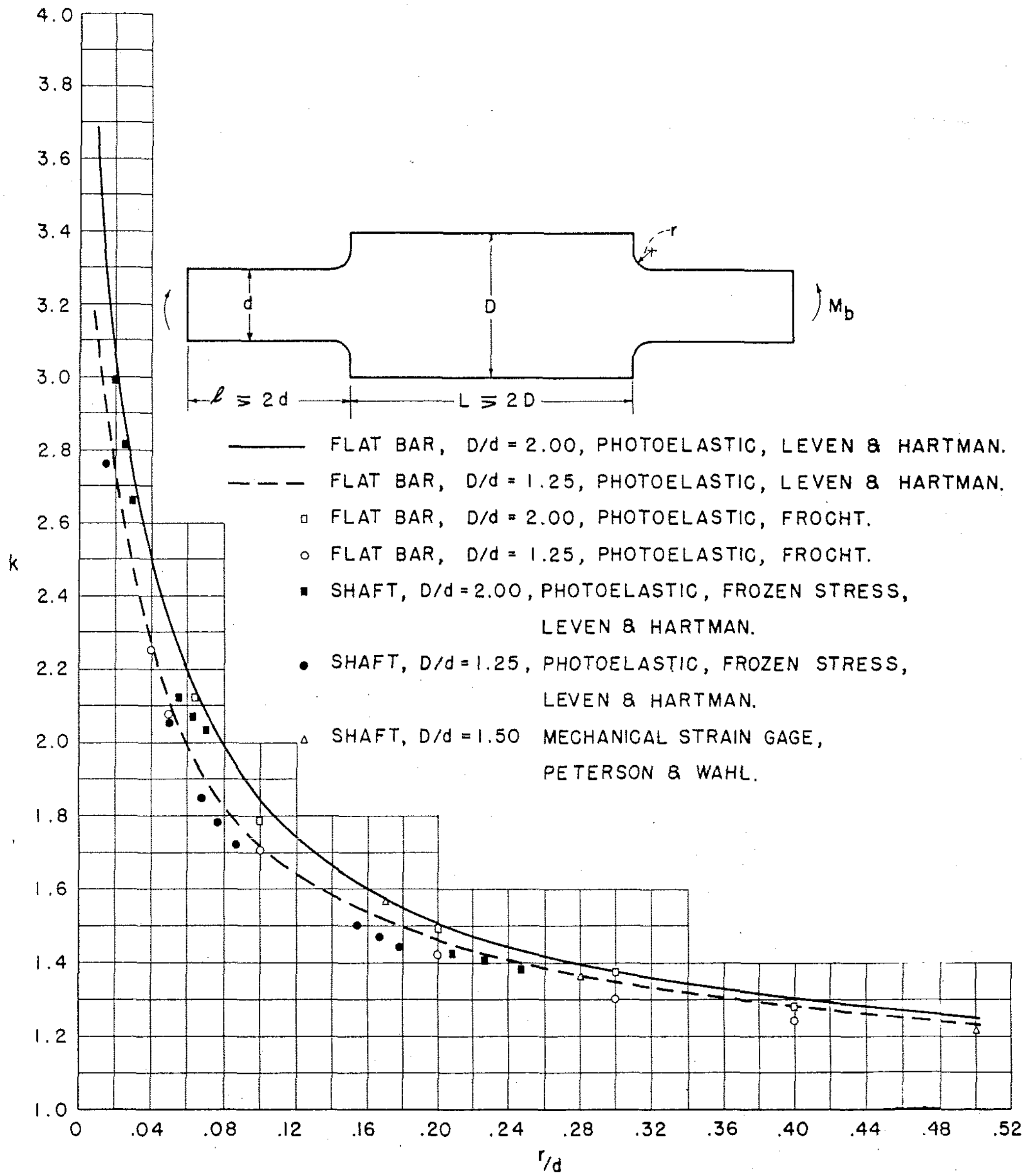

FIG. 15 TWO-AND THREE-DIMENSIONAL FACTORS OF STRESS CONCENTRATION FOR FILLETS IN A FIELD OF PURE BENDING.

In general, the three-dimensional factors of stress concentration are slightly lower than the two-dimensional ones, which is in agreement with the theoretical solution of Neuber[5] for notches in bending. However, the results for steel will be slightly higher than those obtained for Fosterite because of its large value of Poisson's ratio (0.48).[5]

The deformations obtained in the frozen stress tests are quite large, necessitating the measuring of all dimensions after loading. On the basis of the results obtained, however, the large deformations do not appreciably alter the maximum tangential stresses from those which exist in engineering materials.

ACKNOWLEDGEMENT

The authors wish to express their appreciation to Mr. R. E. Peterson, Manager of the Mechanics Department of the Westinghouse Research Laboratories, for his helpful advice.

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