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X-RAY STRESS MEASUREMENTS

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INTRODUCTION

The early work in x-ray stress measurements in this country and in Germany was done on iron and steel with applications to such problems as arise in welding, gun construction, fatigue studies, etc. More recently it has been applied to aluminum alloys in connection with casting and forming problems. Other work has now extended the method to cartridge brass with special reference to the season cracking problem.

Thus the field of application for x-ray residual stress measurements has been increasing. Not only has the number of metals on which measurements are being made increased, but the condition of the metal now presents less of a barrier to useful measurements than it previously did. The measurements on steel are usually made on metal with the grain size for maximum sharpness of the diffracted cones. However, Frommer and Lloyd (1)*** have been able to make successful measurements on cast aluminum with large grains and it has been the experience of the present authors that useful measurements may be made on heavily cold-worked cartridge brass with its very fine grained x-ray structure.

DESCRIPTION OF THE METHOD

The x-ray method of residual stress measurement consists in making a number of measurements by the back reflection technique of a particular interplanar spacing at a point on the surface of a metal specimen, calculating strains from these spacing values, relating these strains to the principal strains in the metal surface by the elastic theory for a homogeneous isotropic body, and then calculating the principal stresses from the principal strains.

The three diffraction patterns usually required to determine the surface strains ϵ_x and ϵ_y are shown in Figure 1. One photogram is made with the incident x-ray beam, I_z, normal to the surface and the other two photograms with the incident beams, I_x and I_y, inclined from the surface normal, OZ, at an angle α. The incident beam I_x lies in the plane XOZ and the beam I_y in the plane YOZ. Using a known specimen-film distance the diffraction ring diameters are measured in the indicated azimuths and interplanar spacings calculated. From these interplanar spacings and a knowledge of the angle α, the strains ϵ_x and ϵ_y (and stresses σ_x and σ_y) may be calculated.

THEORY OF THE MEASUREMENTS

The quantitative development of the x-ray

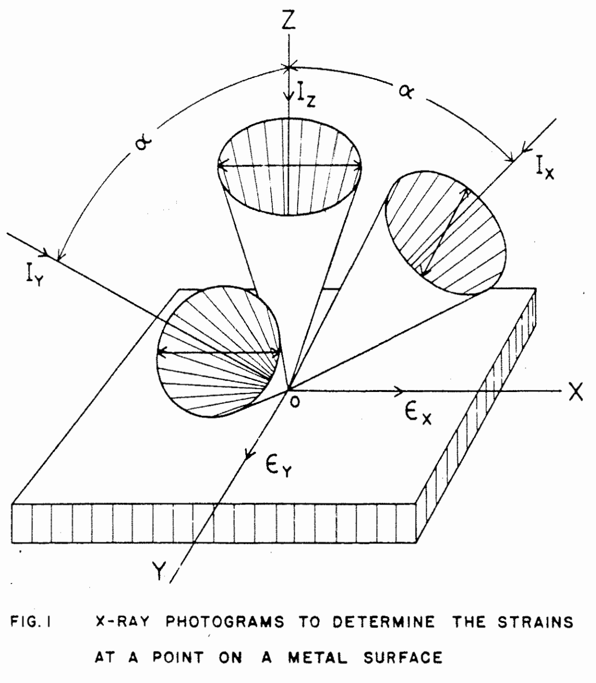

FIG. I X-RAY PHOTOGRAMS TO DETERMINE THE STRAINS AT A POINT ON A METAL SURFACE

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*** Superiors in parentheses refer to bibliography given at the end of this paper.

method of stress measurement is based on certain equations taken from the mathematical theory of elasticity [2] for a homogeneous, isotropic body. This elastic theory is divided into three sections, namely,

a. An analysis of stress based on the two requirements of mechanics for the equilibrium of a body; (1) that the algebraic sum of the forces acting on the body is zero and (2) that the algebraic sum of the moments of these forces is zero.

b. An analysis of strain based on the permissible deformations in a continuous body.

c. The stress-strain equations which connect the theories of stress and strain.

The fundamental equation for the x-ray method is developed in the analysis of strain and is reduced to one less variable by the introduction of the stress-strain relations for a homogeneous isotropic body.

First, consideration is given to an arbitrary vector, $\bar{A}$, connecting two points in an unstrained body. The body is then subjected to a homogeneous deformation by means of a linear transformation of the points representing the body. After analyzing out those parts of the transformation which result in simple translation and rotation of the body, increments in the vector $\bar{A}$ are expressed in terms of the strains in the body and the components of $\bar{A}$. Carrying the analysis farther it is possible to write the strain ($\frac{\Delta A}{A} = \epsilon$) of the vector $\bar{A}$ in terms of the strains (ϵ_{xx}, ϵ_{yy}, ϵ_{zz}, ϵ_{xy}, ϵ_{yz}, and ϵ_{zx}) which characterize the deformation and the directions cosines (l, m, and n) of $\bar{A}$,

$$\epsilon = \epsilon_{xx} l^2 + \epsilon_{yy} m^2 + \epsilon_{zz} n^2 + 2\epsilon_{xy} lm + 2\epsilon_{yz} mn + 2\epsilon_{zx} nl \qquad (1)$$

It can be proven that there is always an orientation of the coordinate axes for which the shear strains (ϵ_{xy}, ϵ_{yz}, and ϵ_{zx}) are zero. The strains for this orientation are known as the principal strains, and the coordinate axes as the principal axes of strain. The orientation of these axes will be assumed known, hence (1) can be written

$$\epsilon = \epsilon_{xx} l^2 + \epsilon_{yy} m^2 + \epsilon_{zz} n^2 \qquad (2)$$

This is the fundamental equation for the x-ray measurements. It gives the strain ϵ in an arbitrary direction defined by the direction cosines l, m, n in terms of the principal strains ϵ_{xx}, ϵ_{yy}, ϵ_{zz}. The central problem of the x-ray measurements is to measure ϵ for various values of l, m, and n. Strictly equation (2) is applicable only to homogeneous strain but the assumption is made that the strain is homogeneous in the small region involved in the x-ray measurements.

Now the x-ray measurements are made at the surface of the metal in a very thin layer. It is assumed that the stress normal to this surface, σ_{zz}, is zero. With this value of σ_{zz} substituted in the stress-strain relations (Hooke's Law) for a homogeneous, isotropic body

$$\left.\begin{aligned} \epsilon_{xx} &= \tfrac{1}{E}[\sigma_{xx} - \nu(\sigma_{yy}+\sigma_{zz})] \\ \epsilon_{yy} &= \tfrac{1}{E}[\sigma_{yy} - \nu(\sigma_{xx}+\sigma_{zz})] \\ \epsilon_{zz} &= \tfrac{1}{E}[\sigma_{zz} - \nu(\sigma_{xx}+\sigma_{yy})] \end{aligned}\right\} \qquad (3)$$

one obtains the relation

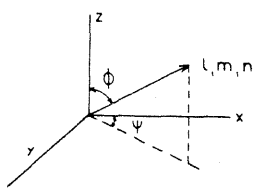
$$\epsilon_{zz} = -\frac{\nu}{1-\nu}(\epsilon_{xx}+\epsilon_{yy}) \qquad (4)$$

With this value of ϵ_{zz} and the relations between the direction cosines l, m, n and the polar angles ϕ and ψ

$l = \cos\psi \sin\phi$

$m = \sin\psi \sin\phi$

$n = \cos\phi$



equation (4) becomes

$$\epsilon = [(\cos^2\psi + \frac{\nu}{1-\nu})\sin^2\phi - \frac{\nu}{1-\nu}]\epsilon_{xx} + [(\sin^2\psi + \frac{\nu}{1-\nu})\sin^2\phi - \frac{\nu}{1-\nu}]\epsilon_{yy} \qquad (5)$$

In this form the fundamental equation is most suitable for practical measurements.

By determining ϵ in two different directions the principal strains, ϵ_{xx} and ϵ_{yy}, may be calculated. The values of ϵ are got from the equation

$$\epsilon = \frac{d - d_0}{d_0} \qquad (6)$$

in which d is the measured interplanar spacing for the strained metal and d_0 the value for the strain-free metal.

Either due to composition uncertainties or difficulties in obtaining truly strain-free specimens, d_0 is not known with sufficient precision for use in equation (6). However, in effect, d_0 may be eliminated from the calculations by dealing with strain differences [3]. Thus for the difference of the two strains ϵ_2 and ϵ_1

$$\epsilon_2 - \epsilon_1 = \frac{d_2 - d_0}{d_0} - \frac{d_1 - d_0}{d_0}$$

or, (7)

$$\epsilon_2 - \epsilon_1 = \frac{d_2 - d_1}{d_0}$$

In the latter equation d_0 need be known only to within 1% of its actual value.

The two strain differences required for the calculation of the principal strains are got from the three interplanar spacing values obtained from the three diffraction ring diameter measurements indicated in Figure 1. To obtain maximum sensitivity the angle α in the figure should be as large as is practicable.

Once the principal strains are known the principal stresses may be calculated from equations (3).

CALCULATION OF SUBSURFACE STRESSES

The fact that the x-ray method yields only the surface stresses is a handicap if the stress distribution throughout the metal is desired. However, frequently the subsurface stresses may be calculated from observations made at various depths after removal of the intervening metal by careful chemical etching. For a plate with an external constraint imposing a constant stress, c, across its thickness, the stress S', observed at the depth x, is related to the stress S, originally present at this depth, by the formula

$$S' = S + \frac{\int_0^x S\,dx}{d-x} - c\,\frac{x}{d-x} \qquad (8)$$

This relationship may be derived in the following manner:

Consider a plate of thickness d which is subjected by external loading to a constant stress c over its cross-sectional area in the direction of the principal stress being measured. It is desired to obtain the stress S which exists at the depth x. The metal is carefully etched away by increments up to the depth x. After each increment the stresses are measured with the final stress at the depth x being S'. The stress S originally present at this depth may be obtained from S' and the stresses measured up to the depth x as a result of the following analysis.

Since the stress integral about the constant stress c is always zero one may write: Before etching

$$\int_0^d (S - c)\,dx = 0 \qquad (9)$$

and after etching to the depth x

$$\int_x^d (S - c + s)\,dx = 0 \qquad (10)$$

where s = S' - S. Since c is assumed constant and s is a constant for a particular depth, equation (10) becomes

$$S = -\frac{\int_x^d S\,dx}{d-x} + c \qquad (11)$$

Using the fact that

$$\int_0^d S\,dx \equiv \int_0^x S\,dx + \int_x^d S\,dx = cd \qquad (12)$$

and the relation S = S' - s, equation (11) becomes

$$S = S' - \frac{\int_0^x S\,dx}{d-x} + c\,\frac{x}{d-x} \qquad (13)$$

The function S' is obtained from the x-ray measurements at the various depths up to x and S is got by a graphical method in which c is obtained by trial and error.

Equation (13) may be applied to curved surfaces when the curvature is small over the region irradiated by the x-rays.

TECHNIQUE

The first step in applying the x-ray method is the selection of an x-radiation and a set of atomic planes which combine to give a reflection with maximum sensitivity to variations in strain. There are three major considerations in this matter.

1. One restriction on the choice may be understood upon differentiation of the Bragg ex-

pression, $n\lambda = 2d \sin\theta$, giving

$$\left(\frac{\delta\theta}{\delta d}\right)_\lambda = -\frac{1}{d}\tan\theta \qquad (14)$$

This equation predicts that maximum sensitivity occurs for small values of d and for values of θ close to 90°. This explains why the back reflection method is used for x-ray stress measurements. For cubic lattices there are graphical methods available for selecting a set of planes and a radiation which combine to provide a reflection with a Bragg angle near 90°.

2. Excepting aluminum and tungsten, most metallic crystals possess appreciable elastic anisotropy. Hence, among several possible planes one may be preferred because Young's modulus for this set of planes is lowest. For highly anisotropic lattices such as copper and brass this factor may improve the sensitivity two or three times.

3. The intensity and resolution of the reflection becomes a matter of great practical importance when dealing with severely cold-worked metals. Experimenting with cold-drawn cartridge brass, for example, it was found that the (331) reflection with nickel $K\alpha$ radiation provides the only reflection of sufficient intensity and resolution for satisfactory measurements on highly worked surface layers.

With the choice of radiation and reflecting planes made, the next decision is concerned with the angle of obliquity for inclined photograms. This angle should be as large as possible and is governed by the size and shape of the specimen being studied. This angle is frequently 45° but angles as low as 30° and as high as 60° have been used.

The interplanar spacing measurements required for calculation of the stresses at a point depend on the measurement of two quantities.

a. the specimen-film distance
b. the diameter of the diffraction rings.

There are two possibilities for the measurement of the specimen-film distance. This quantity may be obtained by (1) using a calibrating material or by (2) maintaining a constant known specimen-film distance. The first method is necessary when dealing with large specimens. The calibrating material is a substance giving a diffraction ring, corresponding to a known interplanar spacing, adjacent to that of the substance in which the stresses are being measured. The powdered calibrating material is spread on the stressed metal to such a density that both its ring and that of the underlying metal may be photographed simultaneously. An alternative procedure is to use a heavier film and photograph the calibrating material and then the underlying metal. By reducing all ring diameters to a standard value for the calibrating material the specimen-film distance is kept constant at a standard value. Figure 2 shows the calibrating material rings of 85-15 brass used for stress measurements on cartridge brass. The second method is best suited for small objects. The specimen is mounted in a holder so designed that the specimen may be rotated through the necessary angle for the inclined photogram without disturbing the specimen-film distance. An example of this type of holder is shown in Figure 3. It is used for measurements on caliber 0.30 cartridge cases and associated draw pieces. Still another method of establishing a

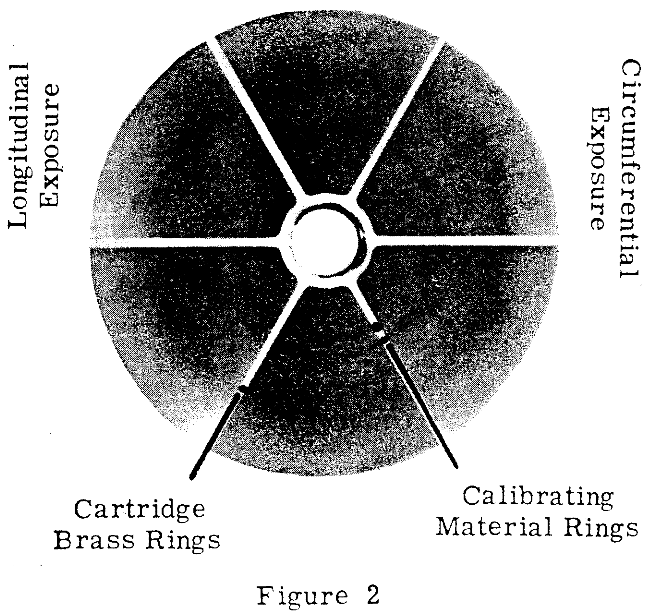


Figure 2

X-Ray Diffraction Patterns for Stress Measurements on a Cylindrical Cartridge Brass Specimen known specimen-film distance is to mount a detachable metallic spacer on the cassette. The specimen is brought up to the spacer and the spacer removed, thus providing a known specimen-film distance.

To keep variations from film shrinkage to a minimum the three photograms necessary for the evaluation of the stresses at a point on the surface of a metal are juxtaposed on a single film (see Figure 2). Then by proper precautions in film processing and drying the film shrinkage is kept constant for all three ring diameter measurements. These measurements have been made visually with a measuring scale and microphotometrically. The visual method has the advantage of rapidity which is vital when large numbers of measurements are being made. However, the microphotometric method has the advantages of giving a permanent record which may be analyzed on a more scientific basis which is largely free of the subjective errors inherent in the visual method. The photographic process may be eliminated entirely if a Geiger-Müller counter were to be used for measuring the reflection angles. This method would be very useful provided it could be made sufficiently precise for stress measurements.

The many calculations involved in the measurements are greatly facilitated by using graphs and tables relating certain of the quantities.

When chemical etching or surface preparation is necessary consideration must be given to the fact that a rough surface will give low stress values due to the release of stress in the peaks on the surface. Hence, when an etchant has been used the surface must be left in a smooth condition [4].

A final factor which affects the sharpness of the rings is the focusing action. Usually it is possible to focus both sides of the ring for a given angle of obliquity when a single ring is being photographed. However, when a calibrating material is used a compromise must be made by setting for focusing conditions between the two diffracted rings.

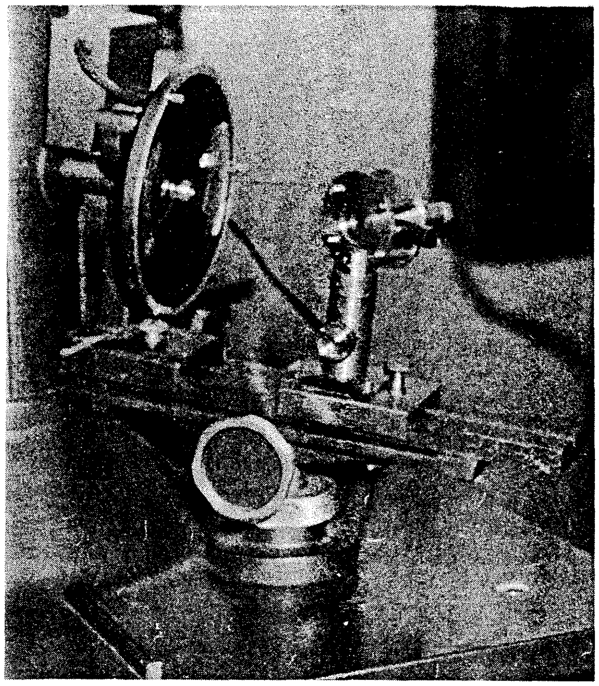


Figure 3

Specimen Holder and Cassette for X-ray Stress Measurements on Small Cylindrical Objects

DISCUSSION OF THE ELASTIC CONSTANTS FOR USE IN THE FUNDAMENTAL EQUATIONS

The two elastic constants necessary for the calculation of the x-ray stresses are Young's modulus, E, and Poisson's ratio, ν. Since the measurements are made using a particular set of atomic planes in an anisotropic crystal it is clear that the gross mechanical values of E and ν will not, in general, be applicable to the calculation of x-ray stresses. And since the methods of calculating gross mechanical moduli from single crystal moduli have not been completely successful quantitatively, it is clear that the single crystal moduli for the given set of atomic planes would not be applicable to the stress calculations.

It might be thought that the uncertainties in Young's modulus could be circumvented by keeping all results in terms of strain since the fundamental equation (5) does not contain E, but only ν. This procedure, however, would simply serve as an artifice because the introduction of equation (4), derived from equations (3), brings in the stress theory and the stress-strain relations with Young's modulus. The reported strains would not be in agreement with the mechanical strains and no logical ex-

TABLE I

Conditions for X-Ray Stress Measurements in Iron, Aluminum, and Cartridge Brass

Metal	Atomic Planes	X-Radiation	Bragg Angle	Calibrating Material
Iron and Steel	(310)	Cobalt Kα	80°38'	Gold
Aluminum and aluminum alloys	(333) (511) (420)	Copper Kα Cobalt Kα	81°22' 81° 9'	Silver Silver
Cartridge Brass	(331)	Nickel Kα	78°54'	85-15 Brass

planation of the disagreement could be made without introducing Young's modulus.

In practice it is necessary to apply known strains and measure the strains for the chosen set of atomic planes by means of the x-ray method. A linear relationship is thus established between applied and observed strains from which an empirical value of E may be calculated. The value of ν may also be calculated from these calibration experiments although it has been found for cartridge brass, at least, that the value obtained from mecha-

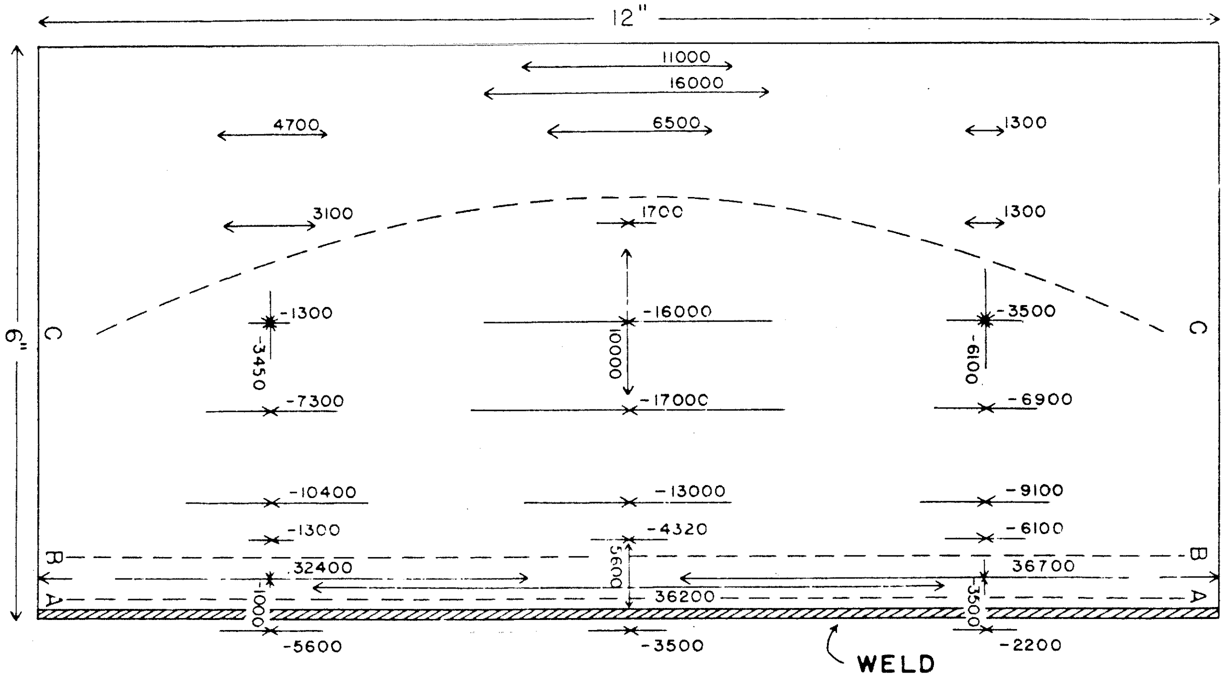


FIG. 4 SURFACE RESIDUAL STRESSES IN PLATE WITH A WELDED BEAD ON ONE EDGE (STRESSES IN POUNDS PER SQUARE INCH)

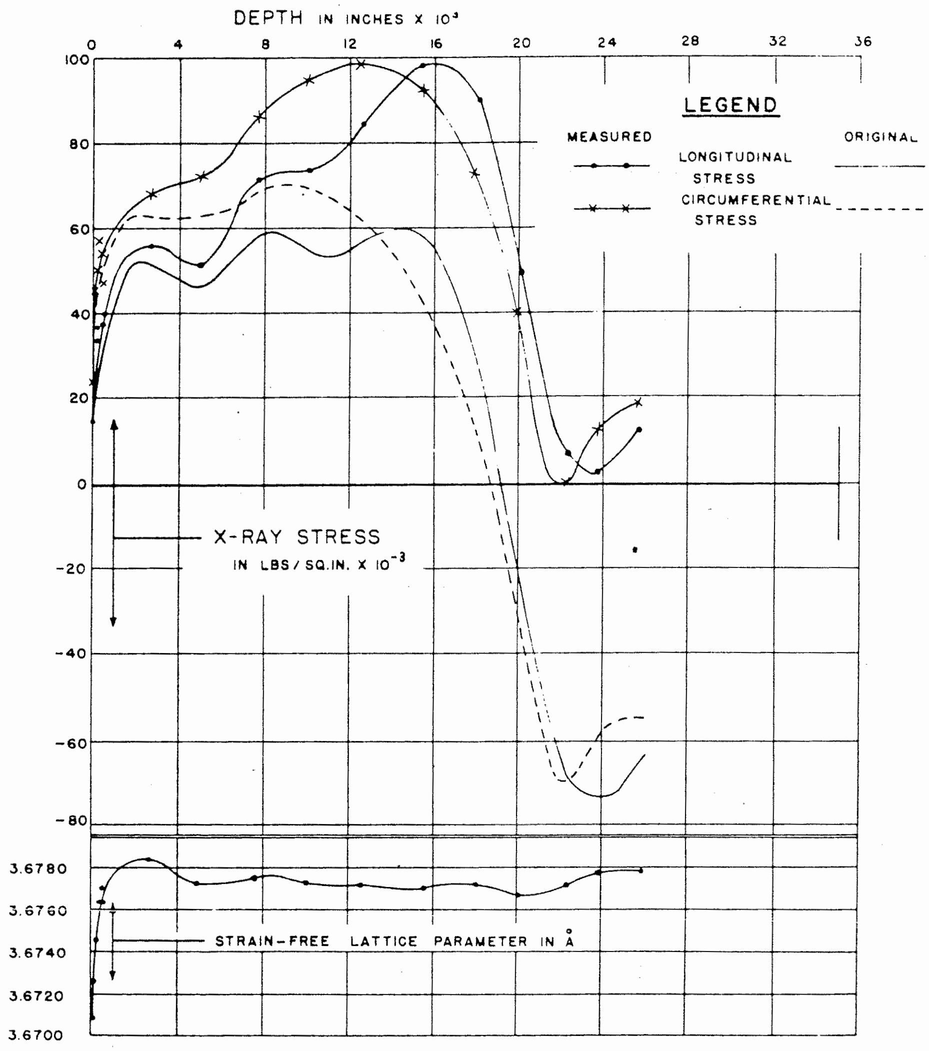


FIG. 5 STRESS DISTRIBUTION IN SUNK BRASS TUBING

nical measurements gives satisfactory results.

APPLICATIONS OF THE METHOD

The chief applications of the x-ray method of stress measurement have been to the three commercially important metals iron, aluminum and cartridge brass. The values of the critical constants and conditions for these measurements are given in Table I.

The applications to iron and steel have been very largely to welding stresses although numerous other applications have been made, for example, gun construction problems, stresses in bridges, residual stresses in connection with fatigue studies, and residual stress problems in general. A typical application of the determination of residual stresses arising from welding operations is shown in Figure 4. The example is taken from the work of Norton and Loring(5) and illustrates how the method can be applied in favorable circumstances. A sample of low carbon steel 6" x 12" x 3/8" was heat treated to remove all residual stresses and until it gave very sharp diffraction rings. Then a welded bead was made along one of the 12 inch edges and the residual stresses measured at various points on the surface of the metal. These stresses were not removed by machining off the bead, indeed did not decrease to negligible values until the bead and about 1/2 inch of the base had been removed.

The most extensive work thus far published on aluminum and aluminum alloys has been that of Frommer and Lloyd(1). Their special problem was that of making measurements on very large grained specimens and the solution was found to be oscillation of the specimen through $\pm 2^{\circ}$ accompanied, in extreme cases, by a small amount of horizontal travel transverse to the incident x-ray beam. The method was applied to such problems as the quenching stresses in forgings, types of surface stresses produced by different kinds of machining, and the stress distribution produced in thick-walled tubes by drawing and extruding. The results of the effects of various types of machining and of shot blasting on the surface stresses was particularly interesting. It was found that shot blasting and turning under conditions that produce chattering gave surface compressive stresses of the order of 40,000 psi. The milling of slots and grooves when the sample enclosed the cutting tool gave compressive stresses of about 30,000 psi. Milling and turning did not produce significantly high compressive stresses although turning a cylindrical surface with a rough cut gave compressive stresses of about 14,000 psi.

The development of the x-ray method of residual stress measurement in cartridge brass has been carried out at Lehigh University. The chief difficulty was in applying the method to the problems of the cold-worked metal. First it was necessary to demonstrate that useful measurements could be made on the diffuse type of diffraction ring shown in Figure 2. After trying a number of radiations it was found that nickel $K\alpha$ radiation with the (331) planes was the best combination for attaining measurable rings with a reasonable exposure time. After selection of the x-radiation and atomic planes, calibrations were carried out by applying known strains. This work gave a Young's modulus of 30,000,000 psi for the (331) planes. This value was viewed askance, at first as the mechanical value is only 15,000,000 psi. However, upon considering the lattice anisotropy of brass the value is found to be in the right direction although perhaps somewhat too high to fit in quantitatively with the single crystal data.

An application of the method to the stresses in a specimen of sunk brass tubing is shown in Figure 5. The sample was sunk in the dead soft condition from a diameter of 0.75 inches to 0.50 inches, the final wall thickness being 0.035 inches. The measured stresses were converted to the stress distribution originally present in the tube by means of equations (13), with the constant c equal to zero since there was no external constraint acting on the tube.

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