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STRENGTH - STRESS LIFE OF HELICAL GEAR TEETH

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Any discussion on the interpretation and application of Test Data should have as its basic slogan: "Only one test can lead to a thousand misleading opinions." This slogan is particularly applicable to tests involving fatigue. Many published tests and practical experiences have definitely indicated that the service life of machine elements involves a broad scatter. The application of the data to designs depends mainly on whether a choice shall be made on minimum or average life. The factor of safety to be applied certainly must consider the statistical quantity of the data beside the usual modifications made to accommodate service variables. Many industries produce products which can be tested in large numbers to quite definitely establish life ratings. This does not necessarily mean a narrow scatter band as attested by the wide spread between minimum, average and maximum life of precision made antifriction bearings.

Unfortunately, many designers or research workers have as their problems machinery or structures of such size, cost or variability that comprehensive duplicated tests cannot be considered. Laboratory tests, even if economically feasible, would be meaningless unless compatible with field conditions. In such applications good design practice has established conservative loading limits which have resulted in service histories containing only rare field failures for comparison with any laboratory test results.

Circumstances such as these are present in the industrial gear field. The need for utmost reliability, long service life, minimum noise and uniform transmission of angular velocity has almost universally dictated the use of helical gears cut after heat treatment. Within the range of machinable hardnesses, the capacity of the industrial helical gears has been limited by the surface durability measured by the resistance to sub-surface shearing stresses, as presented by Schmitter(1)*, which cause pitting of the profiles. This is particularly true for helical gears in which the uniform tooth-to-tooth load transfer, oblique contact line and strengthening effect of the helical trace has indicated by rational analysis and field experience a load resistance against tooth fracture of three to over six times the durability capacity.

Steady improvements in the machining techniques for higher hardness steels, better surface finishing methods, more accurate generation of profiles, advanced gear technology, improved lubricants and more careful application engineering has indicated that the near future might require a more comprehensive knowledge of the strength of helical gear teeth. Accordingly, The Falk Corporation decided to embark upon a program to investigate the factors associated with the strength of helical gears. The method of approach, scope of tests and analysis of data seems appropriate to discuss since it is pertinent to the symposium presented at this meeting.

PRELIMINARY STUDIES

The literature has many papers on the strength of gear teeth. Too few of these have been adequately verified by proper tests. Those which have resulted from an analysis or correlation with actual tests were mainly

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* Numbers in parentheses pertain to references listed at the end of the paper.

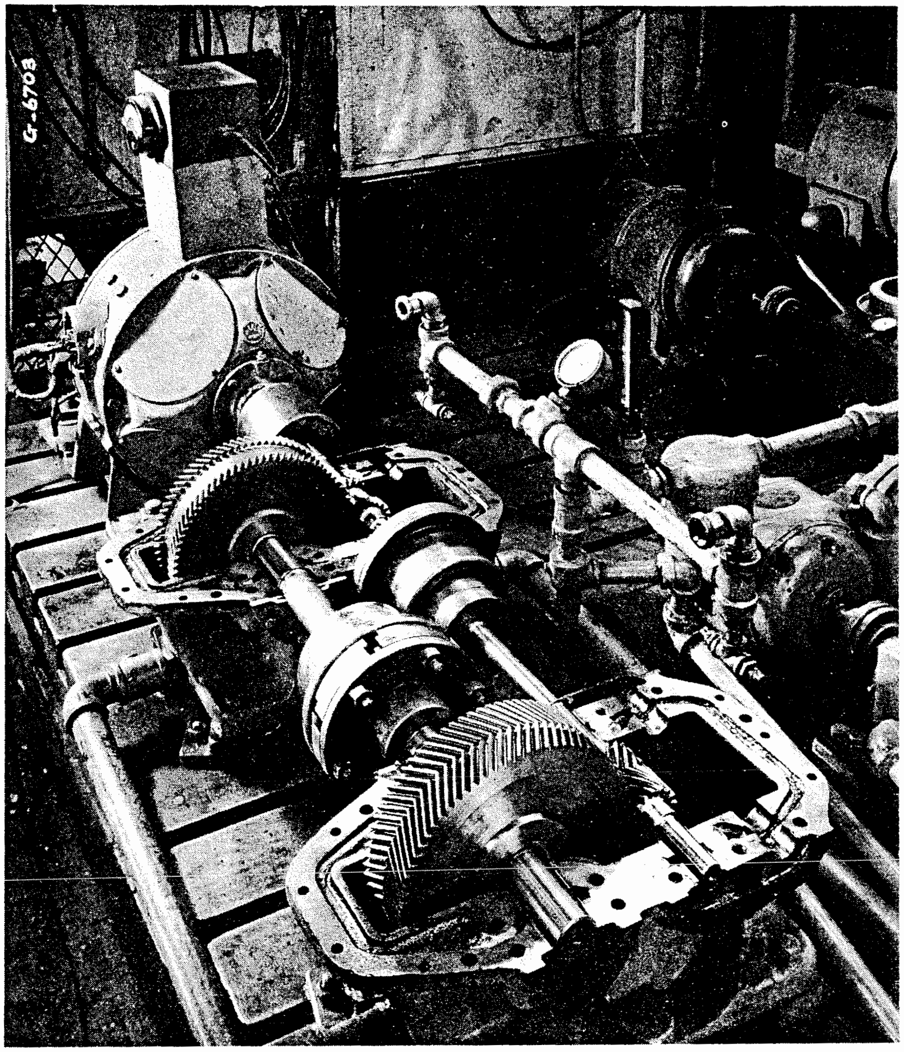

FIG. I. 6.000" CENTER x 3" FACE FALK GEAR TESTER.

reported by Almen and co-workers[2]. Unfortunately, the gears were mainly of the automotive type with expected life cycles, design proportions and materials and heat treatments not comparable to industrial gear practices. The reported results did serve notice that wide scatter dictated a comprehensive test program if the data were to have practical significance.

The usual analytical approaches were explored which, over a period of years, produced a gear tooth strength rating formula which served as a guide. Simultaneously, the knowledge of fatigue expanded and more pointedly emphasized the statistical spread or "scatter" to be expected in life cycles. Also, a growing trend seemed to prove the absence of the definite "knee" in fatigue so familiar in laboratory specimens. Add to this the definite proof of the important function played by "internal," "residual," or "locked-up" stresses and the magnitude of an adequate test program seemed immense.

However, the following test objectives were considered of first magnitude to be investigated before any detailed progress could be attained.

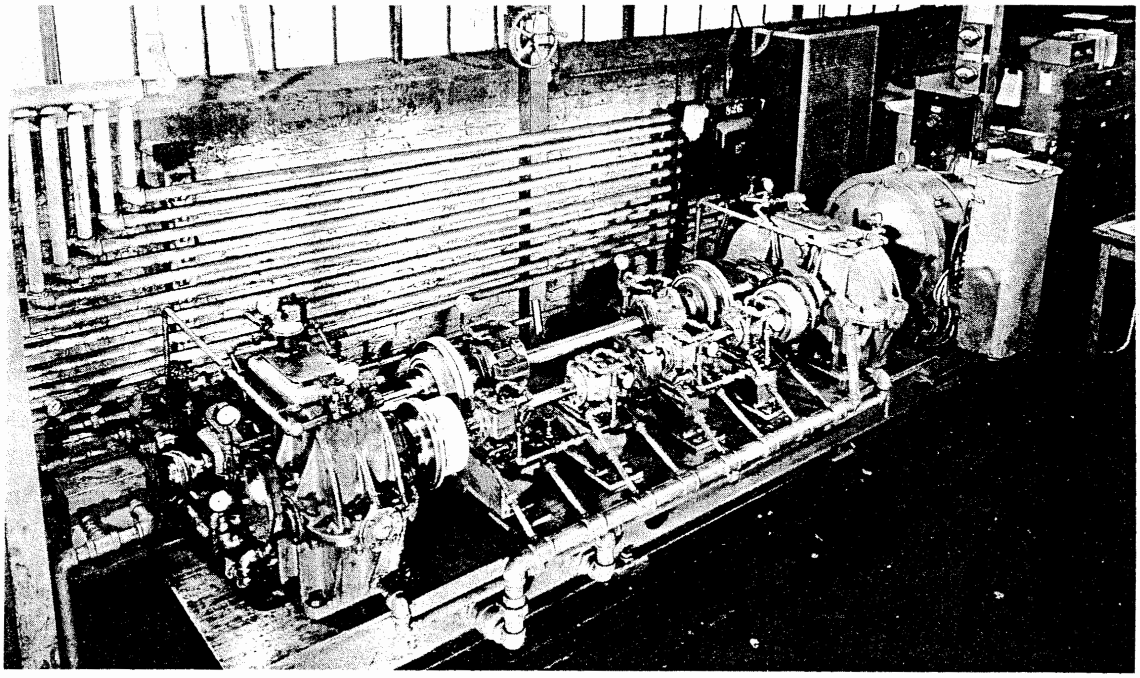

FIG. 2. 12.000" CENTER x 7.5" FACE FALK GEAR TESTER.

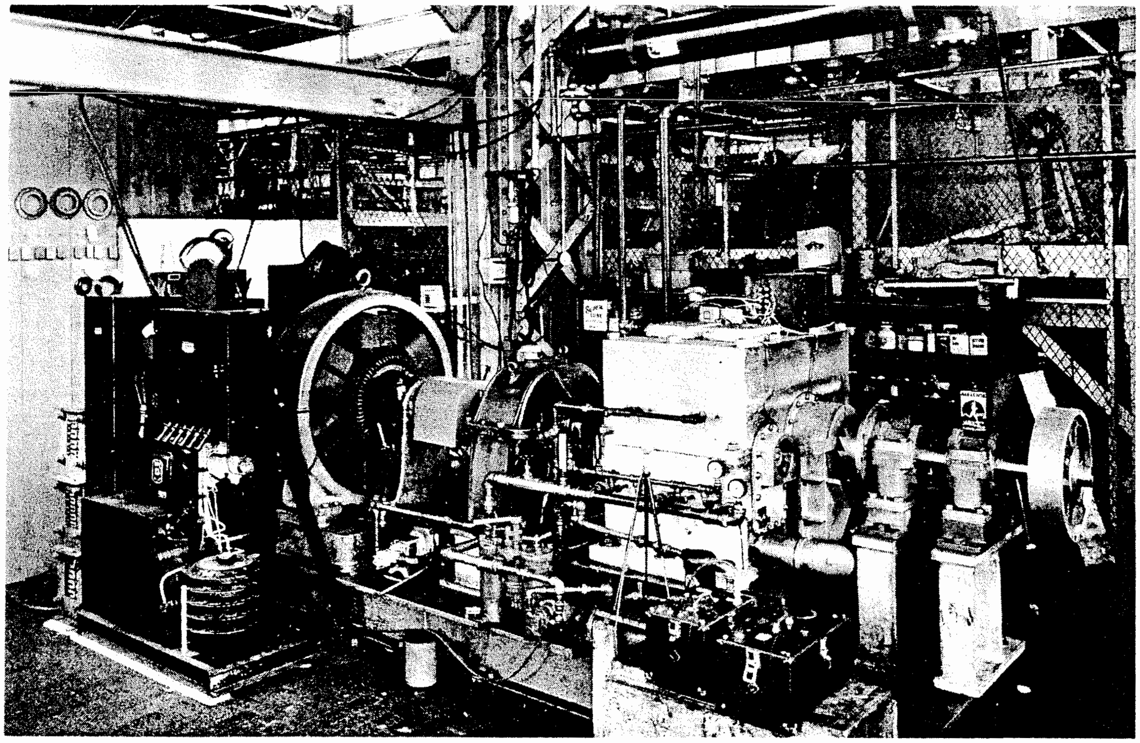

FIG. 3. 13.455" CENTER x 7.75" FACE FALK
SUPER HIGH SPEED GEAR TESTER.

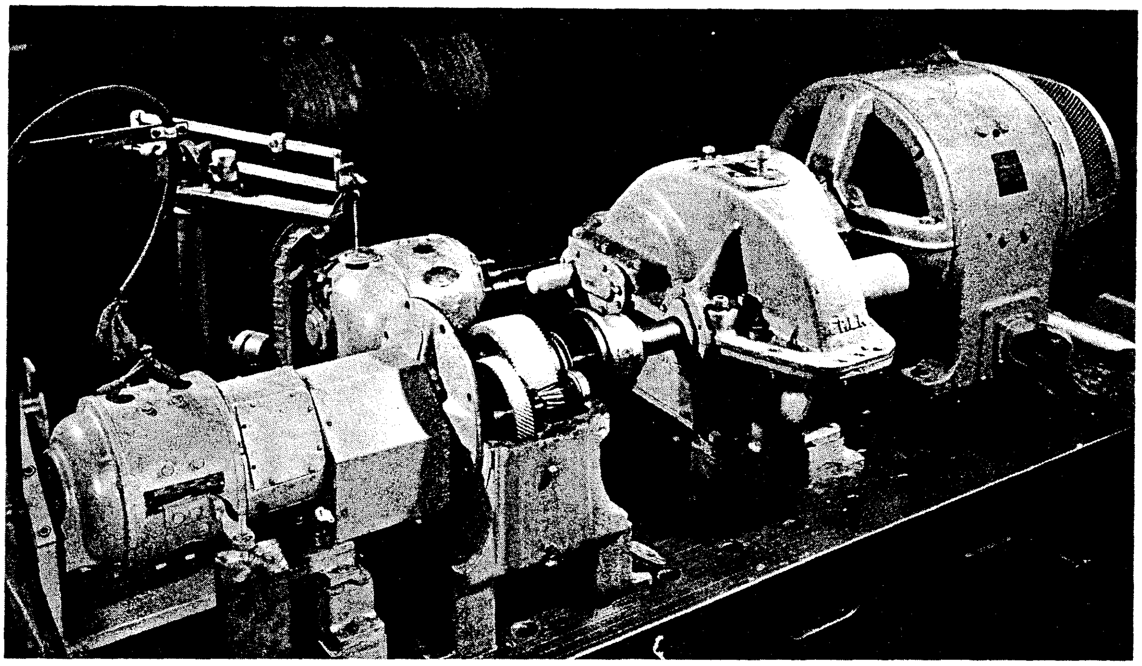

FIG. 4. 1st REDUCTION 4.688" CENTER x 1.25" FACE,
2nd REDUCTION 4.688" CENTER x 3.125" FACE,
FALK DOUBLE REDUCTION GEAR TESTER.

TEST OBJECTIVES

1. To investigate the types or modes of failure by tooth fracture.
2. To determine the statistical variation or "scatter" of a stress-life endurance curve.
3. To ascertain whether a true endurance limit or "knee" existed.
4. To determine the slope of the stress-life curve.
5. To explore the effect of variable loading.
6. To check the part played by residual stresses.
7. To provide factual data for evaluating analytically derived formulas pertinent to the strength of gear teeth.

DEVELOPMENT OF TEST EQUIPMENT

With the objectives of a test program established, the details of the test equipment required attention. Knowing the scatter expected, it was natural to think in terms of a small laboratory tester with its inherent closer control. Serious thought revealed that such small laboratory tester results would have to be compared to expected results from larger sized gears which interpretation might be difficult. An actual gear reduction unit most easily satisfies the greatly desired condition of tests conducted with the gears operating in their natural environment. To provide a large range of operating conditions, tests were made on three sizes of gear testers with checks made on a fourth. Three of the testers were loaded by interlocking the torque internally in a manner similar to the "four-square" tester used for bevel gears. Two of the machines, Fig. 1 and Fig. 2, were torqued through very resilient spring systems, the third, Fig. 3, rather stiffly through solid shafts. The fourth tester, Fig. 4, was driven by an electric dynamometer with the output connected by means of a geared system to an electrical load absorber. The type of torqueing naturally imposed a different response to the dynamic loading which varied with the degree of accuracy in the gear sets.

Using these testers, the variations in gear

TABLE I.

GEAR TEST VARIATIONS.

DIMENSION	
Center Distance	4.688 to 13.455 inches.
Gross Face Width	2.25 to 6.25
Pitch Diameter - Pinion	1.205 to 6.679
Pitch Diameter - Gear	8.17 to 23.956
GEAR DATA	
Type of Gears	Single and double helical.
Helical Angle	8° to 40°.
Pressure Angle	19° to 37° Transverse.
OPERATION DETAILS	
RPM	1,500 to 5,000 (high speed tester 32,000 RPM max.)
Pitch Line Velocities	350 to 8,750 ft./min. (high speed tester 25,000 ft./min.)
Tooth Contact cycles (pinion)	147,000 to 240,000,000
TOOTH LOADINGS	
Pressure per Inch of Face P/F	612 to 3,790
Pressure per Inch of Face per Inch of Pitch Diameter	217 to 1,873
K Factor	244 to 2,250
MATERIALS	
Steel	160 Brinell - to Carburized 60 R_c.

geometry, gear dimensions, speeds, loadings, etc., were as outlined in Table I. All gears were processed and of an accuracy suitable for industrial use at the speeds tested. It is apparent that an extremely wide range of variables of interest to gearing engineers was secured. Obviously, if these wide variables resulted in data which could be consistently correlated to the major objectives of the program practical significance and utilization was assured.

TEST PROCEDURES

There is little need to detail the test procedures which consisted principally of producing gears, thoroughly checking accuracy, loading, periodic inspections, tedious calibrations and patiently waiting for something to happen. The usual "opinions" were drawn as the test data accumulated with restrictions added as the quantity of the data increased. Each series of tests demanded re-checks and suggested new avenues of investigation. The present paper is the first presentation of the data secured to date.

ANALYSIS OF TEST RESULTS

An analysis of the test results is presented in the previously listed order of test objectives. The mass of data prevents detailed tabulations and an attempt is made to present the extremes of performance with samplings of typical test results to illustrate the scatter distribution or performance trend.

Of first interest are the types of failures resulting from the tests.

TYPES OF TOOTH FRACTURES

Fig. 5 is a typical failure originating at the root radii with the propagation of the fracture following the conventional fatigue crack appearance. Mutilation of the teeth due to broken tooth sections passing through the mesh is apparent. Profile distress is at a minimum.

Fractures occurred with a fibrous appearance as typified by Fig. 6 in which numerous teeth failed. A similarly proportioned pinion in which only a partial tooth section failed by fatigue is shown in Fig. 7. Figs. 8 and 9 show failures with a partial fracture at the center of the helix in one and complete teeth in the other.

The normally expected transverse appearance of fatigue cracks originating at the root fillet is illustrated by Fig. 10. An extreme condition resulting from radical departures in gear design is shown in Fig. 11.

A theoretical stress analysis of a gear tooth can hardly be expected to "point out" the many

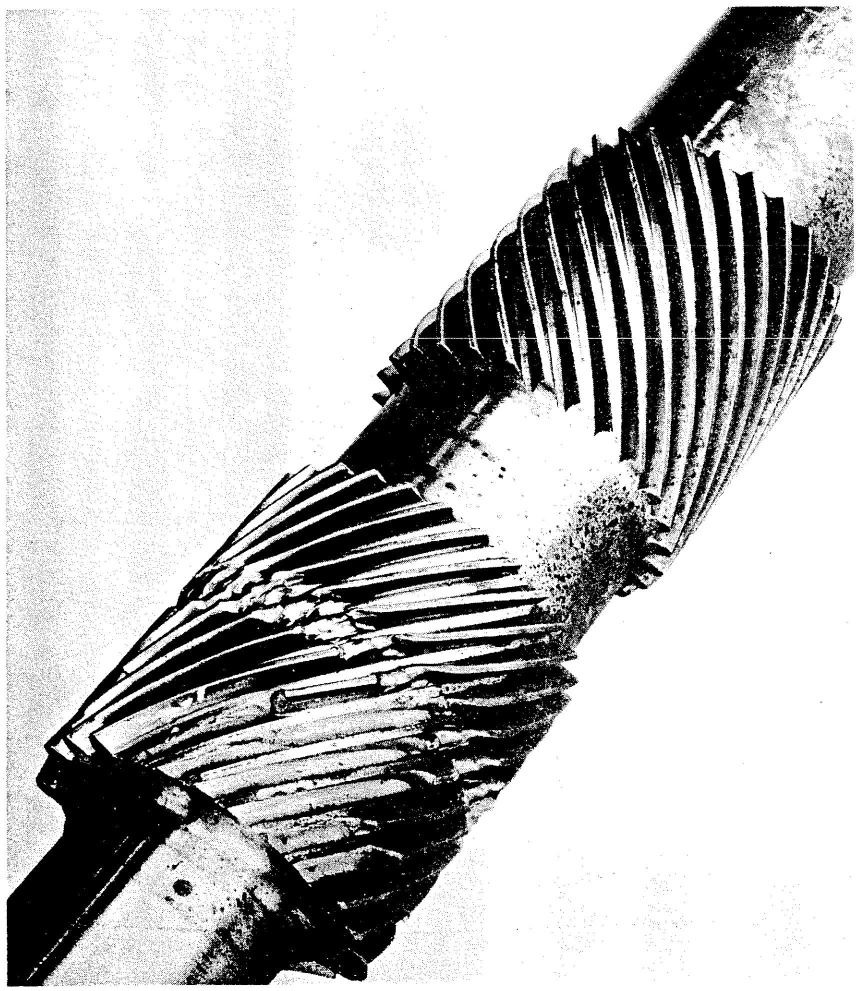

FIG. 5. TYPICAL FATIGUE FAILURE WITH MUTILATION DUE TO BROKEN TOOTH PASSING THRU MESH.

locations of failures found by these tests even though failure usually originated at the root fillet. It will be noted that severe profile mutilations have occurred on some teeth due to loading up to eight times the durability capacity. In some gears, failure originated at pits or other defects developed on the tooth surface. It is suspected that in some instances the high load deflection probably affected the involute action.

PRESENTATION OF DATA

In order to present the data on a common basis for the many different gear designs and materials tested, a rather detailed study was made to arrive at a formula for determining a relative stress criterion. All the data for each series of tests was computed on the basis of several formulas. Comparisons of the characteristics and slopes of the endurance tests were quite similar, varying only in relative placement. The stresses presented in this paper are determined by the Falk Research Formula. This formula utilizes a relative stress of 100 percent for 10,000,000 cycles of life. The 10,000,000 cycle figure was used merely because laboratory fatigue

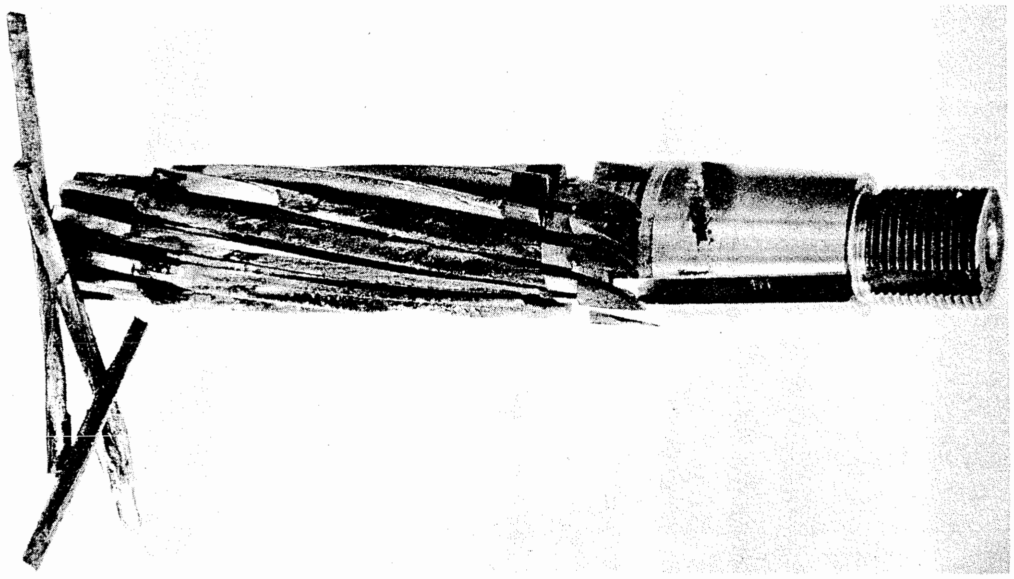

FIG. 6. COMPLETE TOOTH FRACTURE OF PINION WITH HIGH FACE WIDTH TO PITCH DIAMETER RATIO.

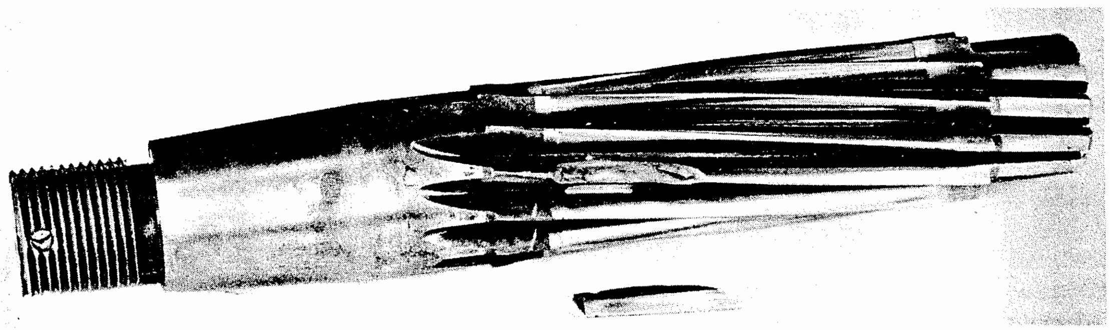

FIG. 7. FRACTURE OF PARTIAL TOOTH.

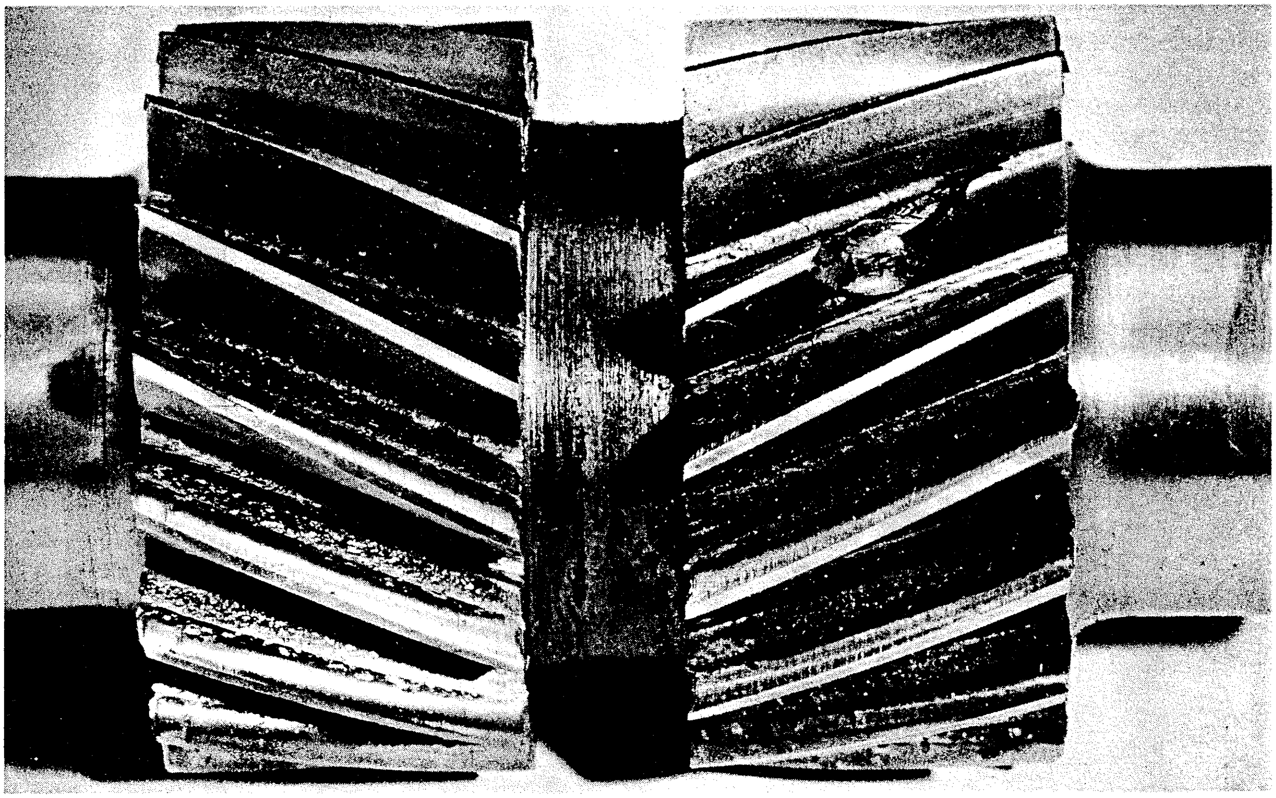

FIG. 8. PARTIAL TOOTH FRACTURE AT CENTER OF HELIX.

limits are usually established at this number of cycles. Differences in the hardness of materials were accommodated by a zero to plus endurance limit based upon the work of Lipson(3) which was presented before this society. A stress concentration factor for fillet radius was used based upon a modification of Frocht(4). The root fillet radius to root thickness ratio of well designed industrial gears is consistently within rather narrow limits such that the comparative results would not have changed greatly except in magnitude in this presentation had the stress concentration factor been neglected.

STATISTICAL VARIATION OR SCATTER

Fatigue tests are of little value unless the expectant life range or scatter is known. For gear teeth wherein the stress is importantly determined by the uniformity of load distribution across the face, the scatter is expected to be high. Since each pinion tooth contacts with each gear tooth (in a full hunting ratio combination) a gear set of 59/153 teeth has 59 x 153 or 9027 different contact conditions. Add to these diverse contacts the effects of deflections, residual stress deviations, shaft deflections, bearing diversity, etc., and a formidable number of variables must be considered.

A typical stress-life cycle endurance curve is plotted in Fig. 12 for one series of tests in the gear tester shown in Fig. 1. Obviously, stress-life tests made on one or two sets of gears can be quite misleading if formulas or opinions are derived from such limited data. The ranges for three series of tests with about the same quantity of points for each range on the testers of Fig. 1 and Fig. 4 are superimposed on Fig. 13. Of particular interest is the fair uniformity of both the location and slope of the minimum boundary which, of course, is of major interest in the practical

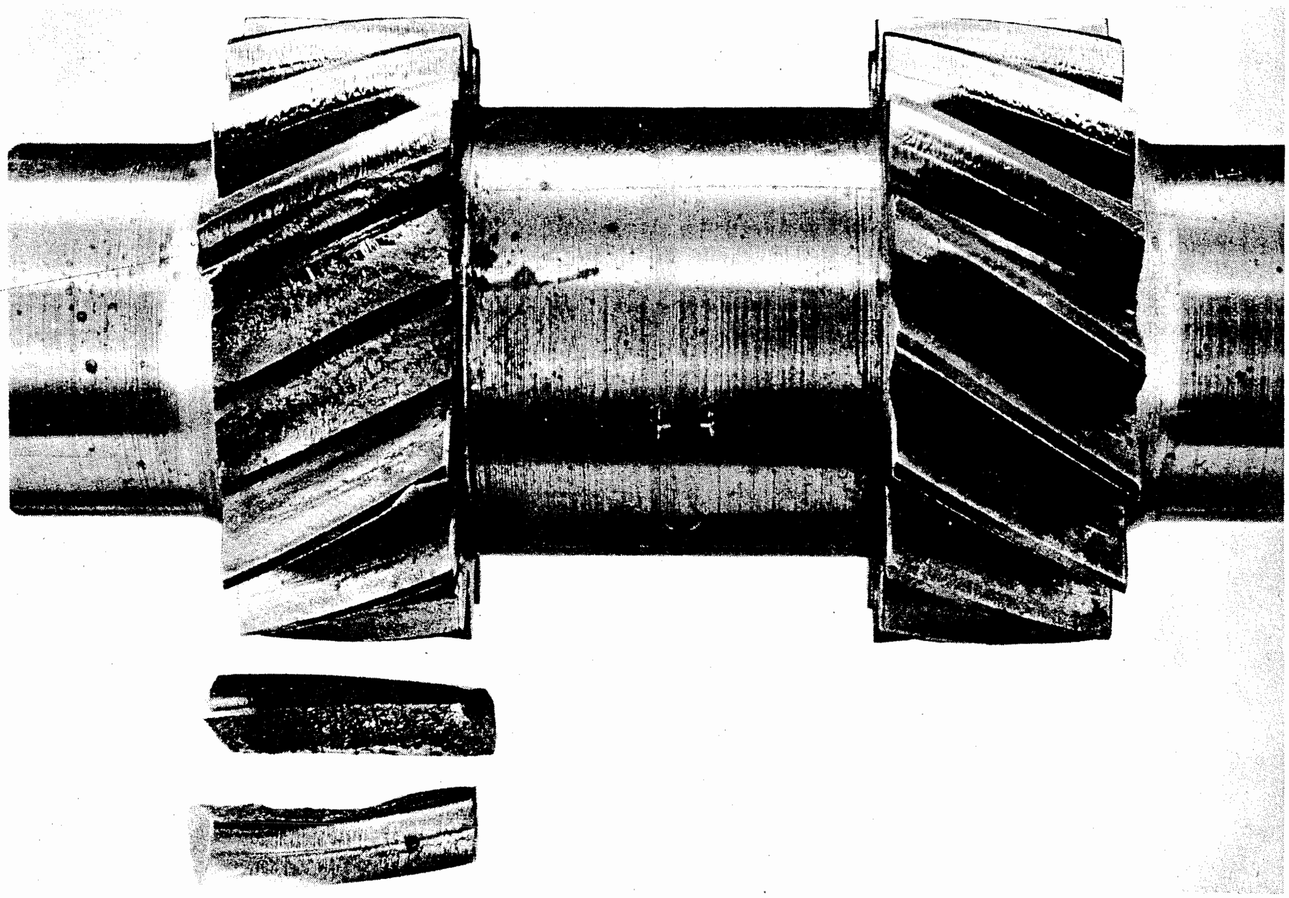

FIG. 9. COMPLETE TOOTH FRACTURES.

design of industrial gearing.

ENDURANCE LIMIT (?)

The test data definitely does not show a "knee" or true endurance limit at 10,000,000 cycles as might be expected from laboratory test specimens. A sufficient number of failures occurred within 10,000,000 to 50,000,000 cycles to definitely establish the absence of a "knee" within this region. Records of failures near 100,000,000 cycles are available. The greatest number of test cycles operated in these tests was 240,000,000. The low slope of the minimum life curve and the great increase in life with a slight decrease in stress makes the presence or absence of a "knee" over 100,000,000 cycles a theoretical point. For the present extrapolation of the curves is suggested.

SLOPE OF STRESS LIFE CURVES

The data was classified into categories representing what might be considered major groups of gear proportions and variations.

Analyzing the individual groups on the basis of the slope of the minimum boundary, the following variations were found.

$$\frac{\text{LIFE CYCLES 2}}{\text{LIFE CYCLES 1}} = \left[\frac{\text{STRESS 1}}{\text{STRESS 2}}\right]^{P}$$

GROUPS	No. 1	No. 2	No. 3	No. 4	No. 5
P VALUE	5.04	5.7	6.74	7.1	5.82

ERRATA SHEET

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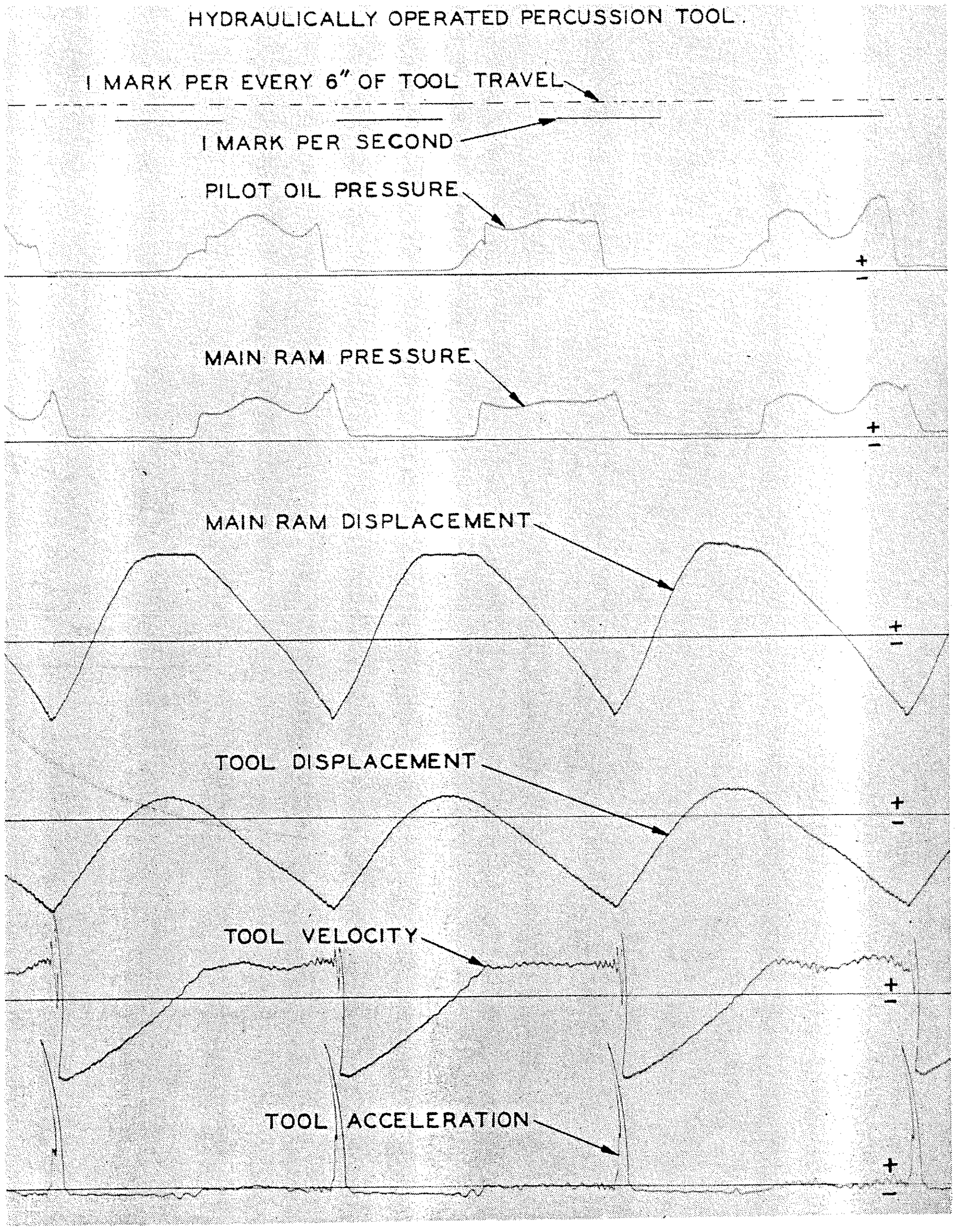

FIGURE 11. RECORD OBTAINED WITH OFFNER RECORDER.

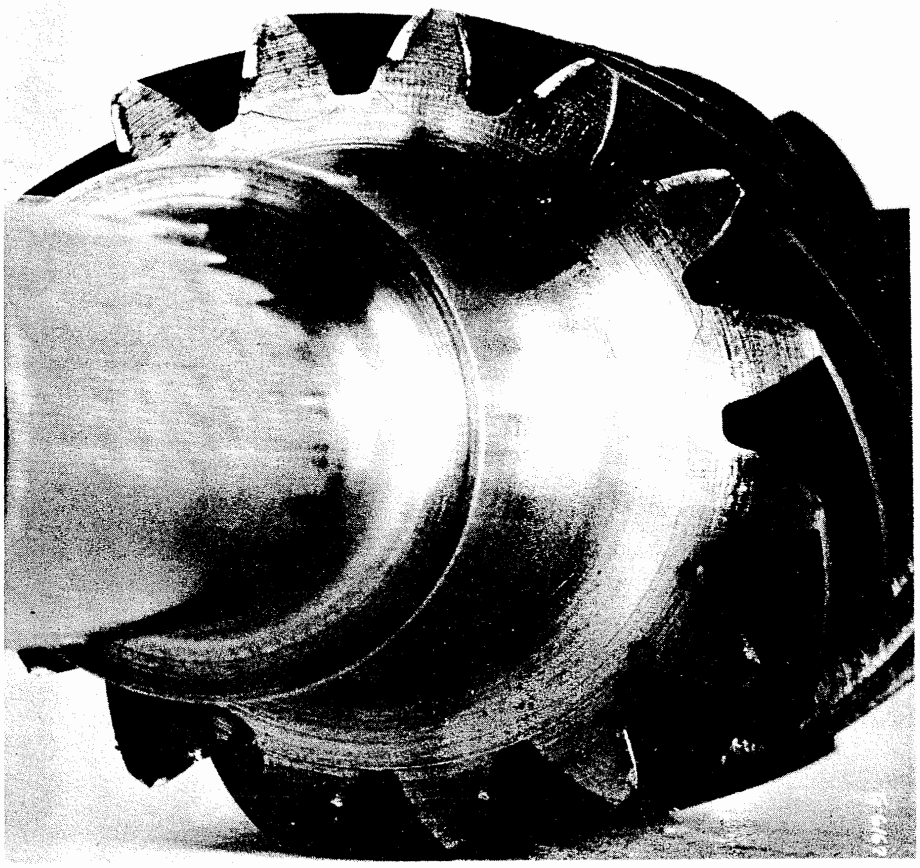

FIG. 10. USUAL FATIGUE CRACK ORIGINATING AT ROOT FILLET.

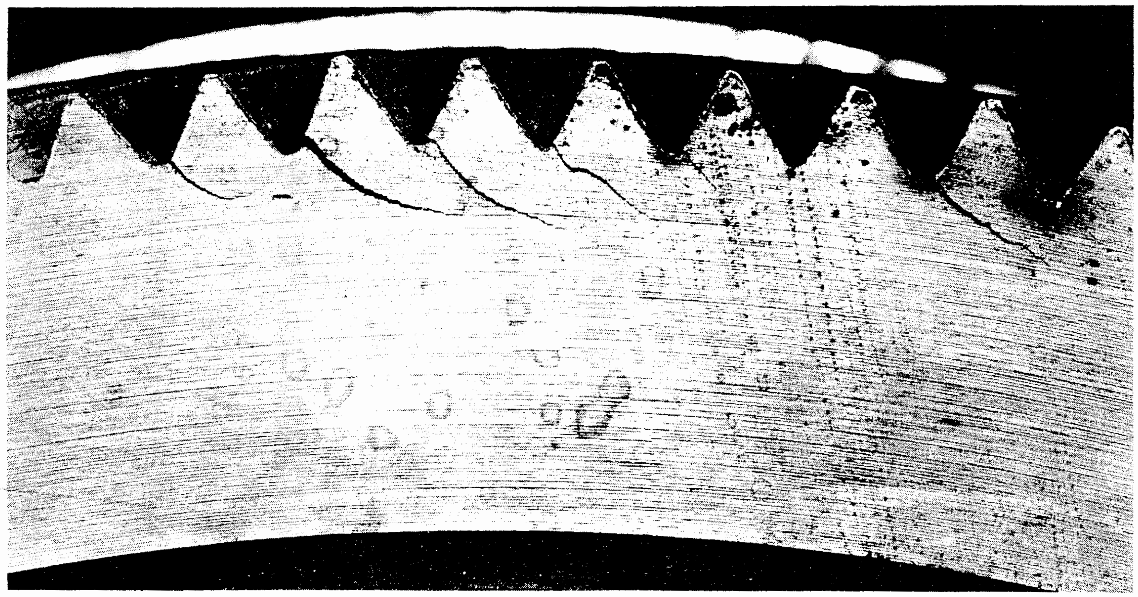

FIG. 11. UNUSUAL FATIGUE FRACTURE DEVELOPED IN UNCONVENTIONALLY DESIGNED GEAR.

An analysis of data appearing in the literature on the life of gear teeth produced "p" values mainly in the 5 to 7 range. Almen has indicated that the slope varies with the stress concentration.

The upper boundary slope "p'" was found to vary between 9 and 12.

EFFECT OF VARIABLE LOAD

The problem of life cycles at variable loadings is of particular interest to the gearing industry because many industrial applications consist of periodic operation at peak torques.

To investigate the effects of variable loading cycles, a series of tests were made in which the gears were operated for a definite number of cycles at various loads. The results are plotted in Fig. 14. The solid lines are plotted to the cycles at each load. The dash lines represent the accumulative cycles. The boundary lines represent those established by test of many sets of gears. In one instance, the first failure was at the maximum line, in another at the minimum. In neither instance did any particular shortening of life appear to result from the previous loading.

Another set of gears was operated at increasing loads to cycles for each load just short of the minimum endurance line. The results are shown in Fig. 15. Again little damage appears to have resulted from these prior loadings.

A third set of gears was operated periodically at high and low loads. Final failure occurred after a comparatively few cycles at a lower load.

It appears that the problem of damage from operation at high or low loads requires considerably more investigation as it does even

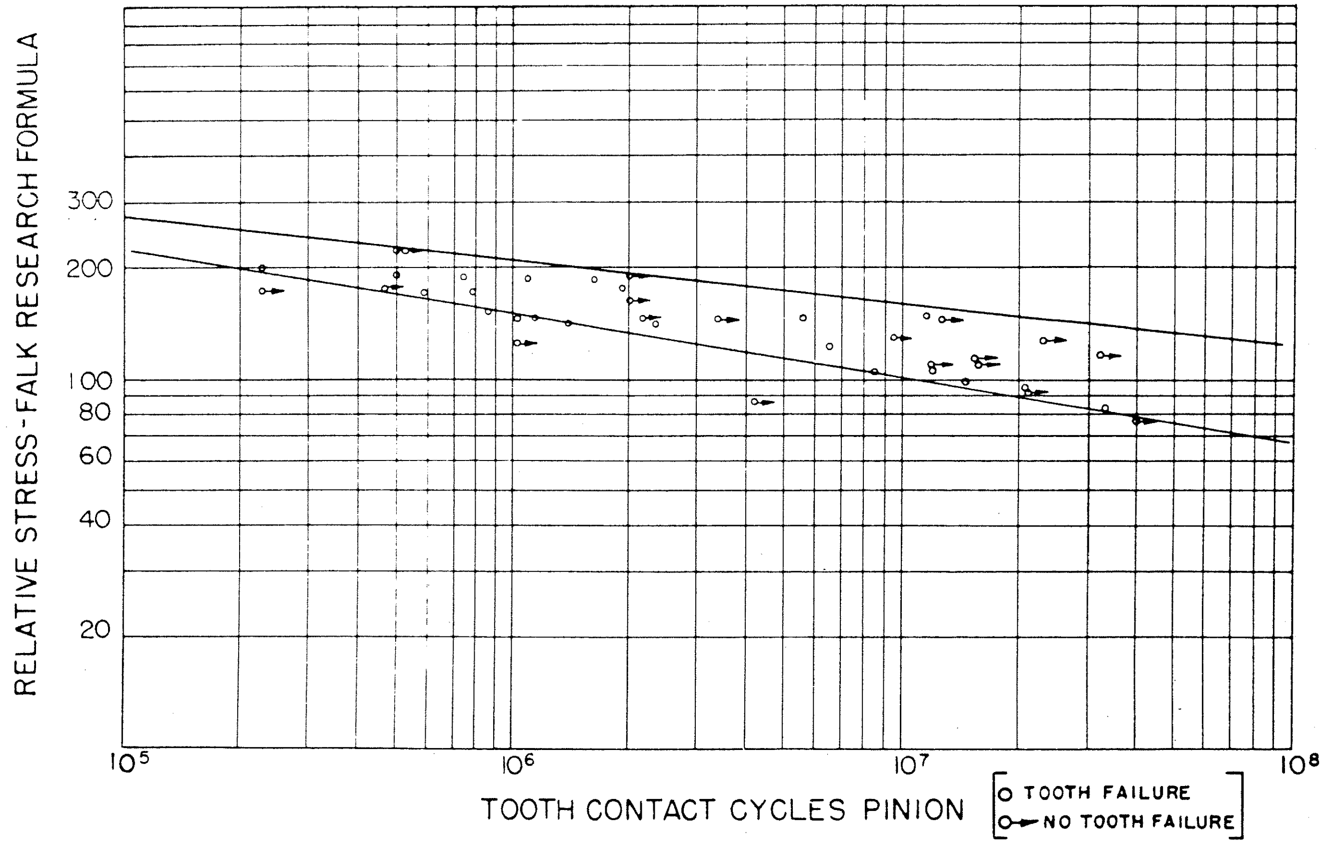

FIG. 12. TYPICAL STRESS-LIFE CYCLE VARIATION FOR GEARING.

for laboratory fatigue specimens.

RESIDUAL STRESSES

The beneficial effects of proper residual stresses has been adequately demonstrated by many practical applications. A number of tests were conducted for the purpose of ascertaining whether or not the same effects could be obtained with industrial gearing. As an example one gear was made with the teeth on one half of the circumference shot peened. The teeth un-peened failed. A number of other tests were made with surface compressive stresses introduced by various methods. Generally, an increase in life was obtained. It can be concluded that proper residual surface compressive stresses can be utilized to increase the strength capacity of industrial gearing.

Caution must be exercised when introducing surface compressive stresses to make certain that the internal tensile stresses are not increased beyond safe values. For example, surface hardening with the hardness penetration tapering from the root to fully across the tip portion can cause the upper portion of the tooth to "shell off" by fatigue. A similar failure can occur with too drastic shot peening. At present, the cost and processing difficulties seem unwarranted unless design limitations prevent the use of adequate proportions.

DEVELOPMENT OF STRENGTH FORMULA

The availability of factual tooth strength data enabled presently used strength formulas to be rigorously examined as to reliability. Few

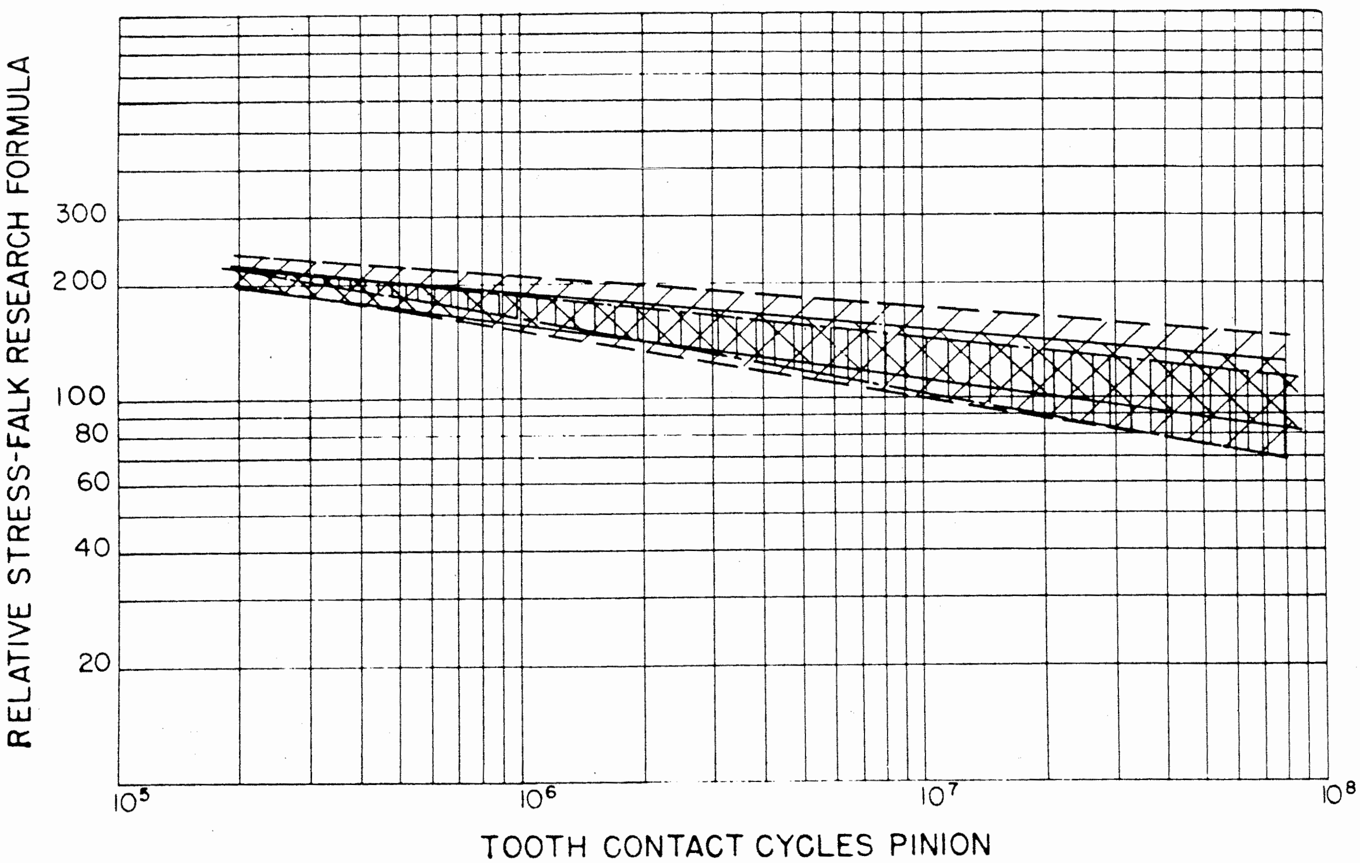

FIG. 13. SUPERIMPOSED STRESS—LIFE CYCLES RANGES.

ERRATA SHEET

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FIGURE 15. INSTRUMENTS IN FRONT PART OF MOBILE LABORATORY.

of the formulas had proper life factors since the basis was postulated upon the existence of a true endurance limit.

The fractured teeth located the apparent area of maximum stress which information, when interpreted by proper analytical methods, resulted in a formula of greater reliability and versatility.

SUMMARY OF TEST RESULTS

The results of these tests have provided the following information as applicable to the industrial type helical gear.

1. Failure due to tooth breakage of helical gears usually originates in the root fillet but occasionally is initiated by a surface pit or irregularity.
2. Considerable scatter in the stress-life curves is normal. Tests of one or a few sets of gears are rather meaningless.
3. No knee is found in the endurance curve up to 50,000,000 cycles. It appears that no knee is present below 100,000,000 cycles.
4. The life cycles vary inversely as the 5 to the 7 power of the stress.
5. It appears that cycles at low load values do not have any marked damage effect. Reliable conclusions cannot be revealed on this problem at the present.
6. The introduction of residual surface compressive stresses has possibilities for resisting tooth breakage.

APPLICATION

Having secured information on the strength of helical gear teeth, the obvious application is to test its validity under service conditions

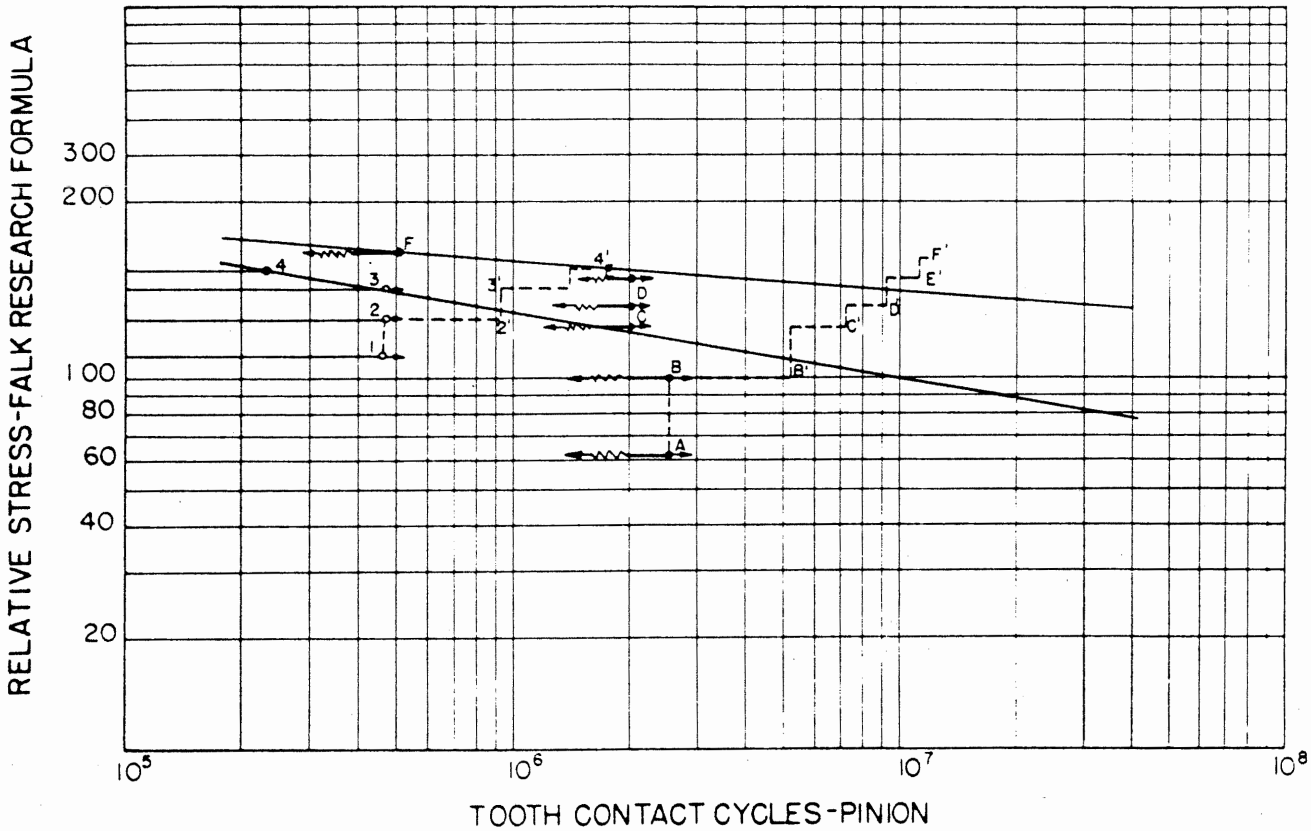

FIG. 14. EFFECT OF PERIODIC INCREASE IN LOAD ON LIFE CYCLES.

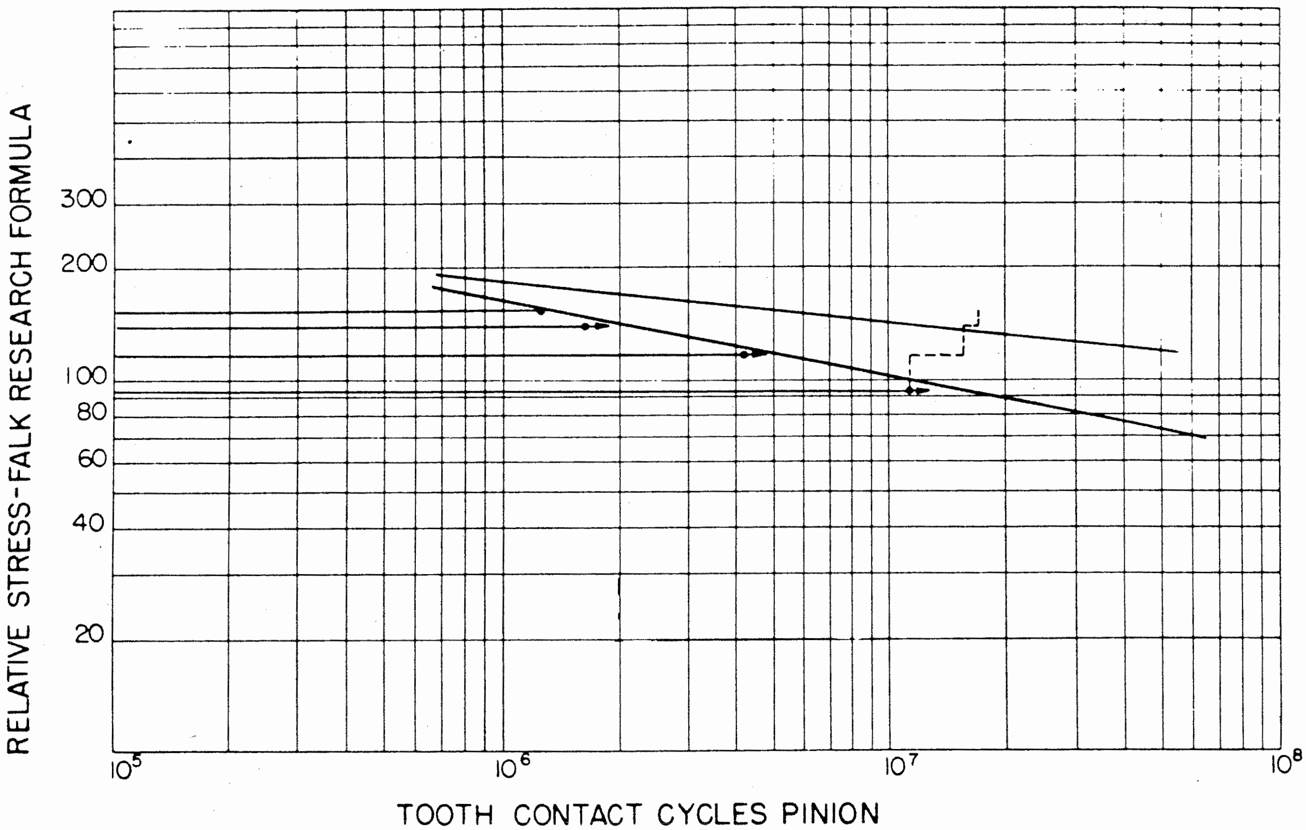

FIG. 15. EFFECT OF OPERATION AT INCREASING LOADS FOR VARIABLE CYCLES.

against designs known to have operated for long periods of time without tooth breakage. Many industrial gear sets have been in operation for over 25 years.

Since the surface durability capacity is proportionate to the square of the allowable stress at hardnesses above approximately 250 Brinell and the strength capacity is proportionate directly to the stress, a series of calculations with various hardnesses of pinion material will indicate the tensile strength or hardness level at which care must be exercised in the design to assure freedom from tooth fracture.

Such a series of calculations is plotted in Fig. 16. The horizontal lines are the durability horsepower capacity calculated on the basis of American Gear Manufacturers Association Standards with a service factor of 1. Gears so rated are considered satisfactory for uniform loads operating continuously for 10 hours per day.

The sloped lines are the cycles of life at various loads computed by The Falk Research Formula with a "P" value of 5.

The intersections of the strength and durability curves represent the minimum life at which tooth breakage could be expected. It is seen that, as the hardness (or tensile strength) increases, the intersections occur at lesser numbers of cycles.

Assuming 1750 RPM of the drive pinion, the expected minimum years of life at 10 hours per day, 365 days per year (383,300,000 cycles per year) would be as follows for the various pinions.

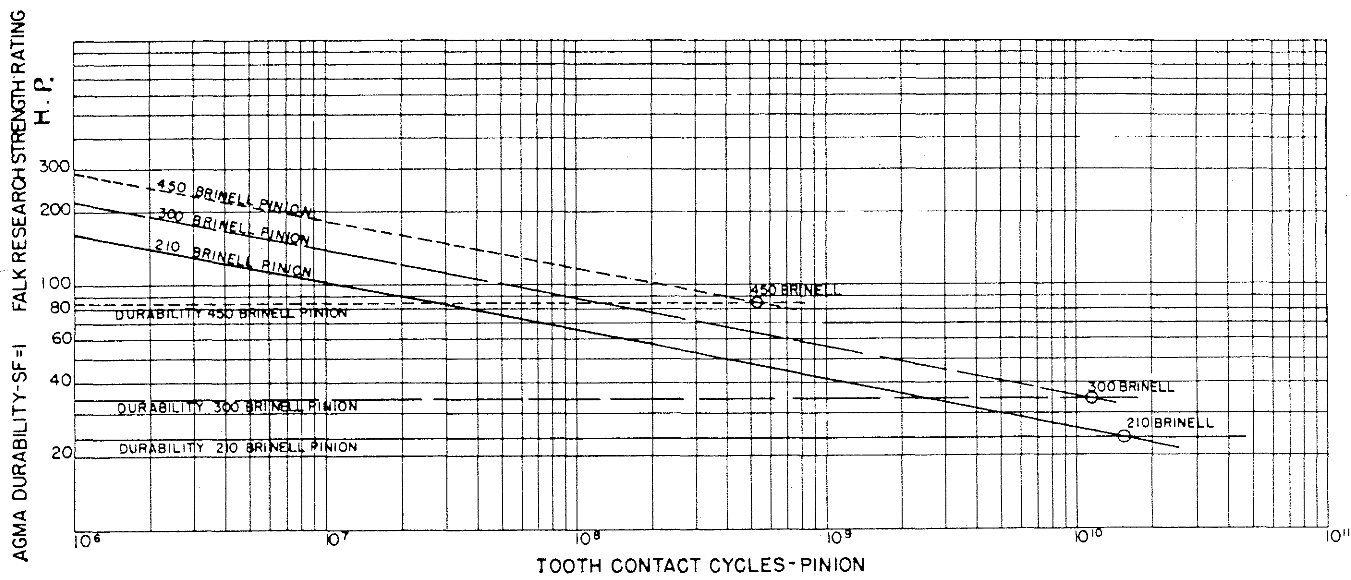

FIG. 16. CALCULATED COMPARISON OF STRENGTH AND DURABILITY LIFE BETWEEN PINION OF 210, 300 AND 450 BRINELL.

PINION HARDNESS (BRINELL)	MINIMUM STRENGTH LIFE (YEARS)
210	41
300	29
450	1.3

The author can scarcely be expected to set up a verifying test on the life at the first two hardnesses. The figures merely confirm the popular impression that the strength of industrial helical gears, designed to conventional practices, need never be seriously considered except at the higher hardness or tensile levels.

If the torque loadings were maintained at the same values for 17,500 pinion RPM, the minimum life cycles for the 210, 300 and 450 Brinell pinion would be 4.1, 2.9, and 0.13 years. Under these circumstances, the strength rating would deserve serious design investigation.

If an attempt were made to operate at twice standard rating torques, the minimum strength life cycles for the 210, 300, and 450 Brinell pinions would be 1.4 years, 11 months, and 15 days respectively. Under these overloads, severe profile distress would be experienced and add to the danger of tooth fracture. Cognizance must be given the fact that these calculations are based on minimum life whereas the average or maximum life might be 5 to 10 times longer. With higher hardnesses such as carburized gears, it is obvious that the situation is reversed and strength is the limiting factor with pitting being secondary.

In conclusion, the author predicts that further tests on the tooth strength of gearing will eventually direct improvements which will substantially increase the minimum expected life. Because of the wide spread between minimum and maximum life, it seems of primary importance to ascertain the fundamental reasons for this phenomenon.

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