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The Effect of Self Stresses on High Cycle Fatigue

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ABSTRACT: It now seems to be understood that self stresses (residual stresses) increase or decrease fatigue life mainly by preventing, delaying, or accelerating the growth of cracks. The action of self stresses is currently being related to fracture mechanics. The fact that the effect of self stresses is far stronger at notches than in smooth push pull specimens, or in other smooth specimens, is gaining more and more recognition. In current publications this fact is modeled by considering not only the local surface stresses, but also the distribution of load stresses and self stresses below the surface of the specimen or part.

KEYWORDS: fatigue (materials), residual stress, shot peening, self stress, mean stress, crack arrest, strength prediction

Nomenclature

a, m	Crack length, including notch depth
c, m	Distance from neutral axis to surface
F	Function
g	Numerical factor used in fracture mechanics
I, m^4	Moment of inertia
K, MPa·$m^{1/2}$	Stress intensity factor
ΔK_{th}, MPa·$m^{1/2}$	Range of K_I below which fatigue cracks do not propagate in constant cycle testing (compressive stress is counted as zero stress)
K_t	Elastic stress concentration factor (on net section)
K_f	Fatigue notch factor. "The ratio of the fatigue strength with no stress concentration to the fatigue strength at the same number of cycles with stress concentration for the same conditions." (ASTM Definition of Terms Relating to Fatigue Testing and the Statistical Analysis of Fatigue Data [E 206]). Ratios of fatigue strengths are here taken to equal the ratios of alternating loads for equal net section
K_b	Baseline fatigue notch factor. Same as K_f at fully reversed stresses and long life (5×10^6 cycles)
M_a, N·m	Alternating bending moment
m	Coefficient of mean stress influence
R, m	Notch radius
S, MPa	Nominal stress calculated from load and net section, without considering stress concentration
S_{at}, MPa	Alternating tensile stress (compressive stress is counted as zero stress)
S_{cat}, MPa	Critical alternating tensile stress below which fatigue cracks will not propagate in constant cycle testing
S_f	Unnotched long life fatigue strength in fully reversed loading
w	Exponent in Walker's equation
ϵ	Local strain
σ	Stress (except nominal stress near notch, which is called S)
σ_y	Yield strength, monotonic
σ_y'	Yield strength, cyclic

Suffixes

a	Alternating
i	Initial
f	Final
m	Mean

Introduction

Improvement of fatigue resistance of parts which are notched, or which have crack-prone surfaces, is the main purpose of deliberately induced beneficial compressive self stress. (The term "self stress" is used by A. Cottrell to designate what many others call "residual stress." I prefer Cottrell's term for its clarity and brevity.) The observed improvement is far greater than the improvement produced by the same self stresses in smooth specimens. Conversely, deterioration of the fatigue resistance of notched parts by tensile self stresses can be far greater than expected from tests made with smooth specimens. For example, in terms of a fatigue notch factor K_f we may find by comparison of otherwise identical smooth and notched specimens at long lives, as shown later in this paper:

without self stress, $K_f = 3$;
with compressive self stress, $K_f = 1$; and
with tensile self stress, $K_f = 5$.

Any quantitative treatement of the effect of self stresses on fatigue must be able to account for the known facts that a notched part with compressive self stress may have the same long life fatigue strength as a smooth specimen tested under otherwise identical conditions, and that the effect of crack prone surfaces (for example

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decarburization or hard chrome plating or great sensitivity to small scratches) can be neutralized by compressive stresses below the surface. Four methods of quantitative analysis will be discussed; they are based on the following four criteria:

the maximum tensile stress,
the alternating tensile stress amplitude (AT),
the maximum stress times alternating stress (SWT), and
the stress intensity factor.

They all agree in explicitly or implicitly acknowledging that the fatigue resistance of notched parts may depend on a mechanism different from that which governs resistance of smooth parts, and that without tensile stresses fatigue failures will not occur.

Maximum Tensile Stress Approach

The title of an article published by Almen in 1951 [1] expresses his approach to the analysis of the long life fatigue resistance of parts. It reads "Fatigue Failures are Tensile Failures." He stated that a crack will not propagate unless there is a tensile stress present and he included self stress in determining the presence or absence of tensile stress.

While this approach is very simple and quite practical it did not find acceptance in textbooks or reviewed publications. Data on the magnitude and distribution of self stresses were not readily available; the approach contradicted the accepted octahedral shear stress criterion, and this theory did not by itself explain the long life of some parts in the presence of tensile stresses.

Alternating Tensile Stress Approach (AT)

To explain that shot-peened notched specimens can have the same fatigue strength as smooth specimens, Fuchs proposed a three-criteria method in 1963 [2] and published justification for it in 1971 [3]. The three criteria are:

(1) gross yielding,
(2) crack initiation, and
(3) crack arrest.

The criteria can be expressed in terms suitable for multiaxial states of stress [3, 4]. For the sake of simplicity the discussion is here limited to uniaxial stresses. The criterion for gross yielding then is

$$S_m/\sigma_y + S_a/\sigma_y{}' > 1$$

Nominal stress without consideration of stress concentration is S. The sum of the stress produced by the mean load and the self stress in a region near the surface is S_m. The estimation of the depth of this region will be discussed later.

The yield condition is represented by the straight lines from σ_y and $-\sigma_y$ to $\sigma_y{}'$ in the Haigh diagrams (Figs. 1, 2, and 3).

The crack initiation criterion is defined by local stresses at the notch root. They are

$$\sigma_a = K_b S_a$$
$$\sigma_m = K_b S_m$$

where K_b is the base line notch factor, equal to K_f at long life and fully reversed stresses.

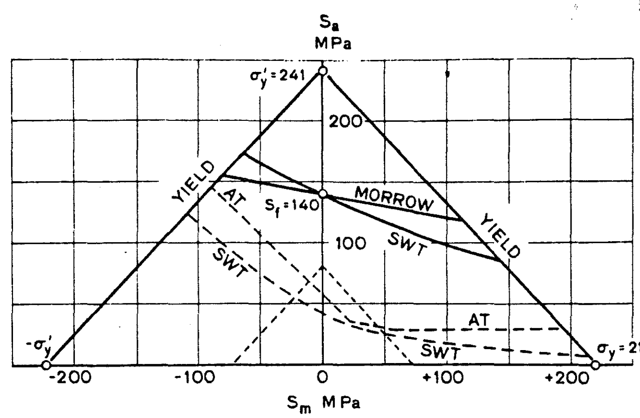

FIG. 1—*Haigh diagrams for SAE J403 (1015) steel, smooth and notched* (K_b = *3*).

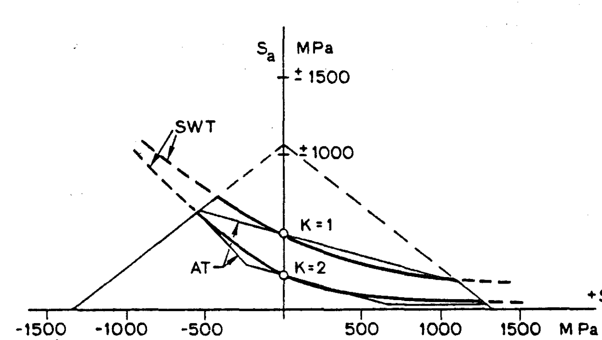

FIG. 2—*Haigh diagrams for SAE J404 (4340) steel at 409 HB, smooth and notched* (K_b = *2*).

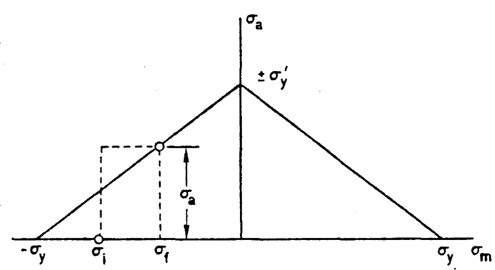

FIG. 3—*Change of initial self stress by load stress for an elastic-perfectly plastic material.*

According to Sines [5] the failure criterion is

$$\sigma_a + m\sigma_m > S_f$$

or

$$S_a + mS_m > S_f/K_b$$

where m is a number smaller than 1 and S_f is the fully reversed long life fatigue strength.

In the Haigh diagrams (Figs. 1 and 2) this condition is represented by the gently sloping lines that cross the vertical axes at S_f or at S_f/K_b.

It is worth noting that this criterion implies crack initiation at zero alternating stress if $m \times S_m > S_f/K_b$. ($S_m > 700$ MPa for the conditions of Fig. 2.) Fatigue strengths at greater mean tensile stresses are well established by numerous tests. This observation has led to the old rule of thumb that K_b should be applied only to alternating stresses, not to mean stresses. The observation of arrested (nonpropagating) cracks in sharply notched rotating bending specimens [*6*] led to the explanation of this observed effect by a crack arrest criterion.

The crack arrest criterion is

$$S_{at} < S_{cat}$$

where S_{at} is the nominal alternating tensile stress and S_{cat} is that value of S_{at} below which fatigue cracks will not propagate in constant amplitude testing. Values of S_{cat} for several materials are listed in Ref *4*. They are in the neighborhood of 50 MPa.

In the Haigh diagrams (Figs. 1 and 2) the crack arrest criterion is represented by horizontal lines in most of the area of tensile mean stress and by lines inclined at 45° in the area of compressive mean stress. These lines are shown only where they are above the crack initiation lines, in regions in which we expect to find arrested (nonpropagating) cracks.

On the tensile side the hypothesis of a critical values S_{cat} corresponds to the horizontal part of the applied stress-cycles to failure *S-N* curve which one finds for sharply notched parts and often for weldments [*7*]. On the compressive side the intersection of the crack arrest line with the yield line, at an alternating stress that is about half the yield stress, indicates the highest fatigue strength which one can expect to obtain with compressive mean stress. This has been verified by tests with compressive loading of parts without self stress [*8*], by analysis of the long life fatigue strengths of prestretched notched bars [*3*], and by analysis of the long life fatigue strengths of shot-peened parts [*2*].

Nominal instead of actual local stresses are used for the crack arrest criterion because the cracks are always arrested at some depth below the surface where the stress is less than the local stress at the surface. The self stress produced by various methods is usually higher below the surface than on the surface and it is also concentrated near notches, though less concentrated than the load stress [*9*]. These factors combine so that the approximation by the nominal stress S, including the self stress, is as good as any other unless the exact detailed distribution of stresses in depth is known.

Maximum Stress Times Alternating Stress Approach

The SWT method of Smith et al [*10*] uses the function

$$F_1 = \Delta\epsilon \times \sigma_{\max}$$

as a parameter. For its application to notched parts in high cycle fatigue we follow a suggestion by Smith [*11*] and use

$$F_2 = K_b(S_a \times S_{\max})^{1/2}$$

where

$$S_{\max} = S_m + S_a$$

The factor $S_{\max}$ implies that without tensile stress there cannot be any fatigue failures. Figure 1 shows combinations of alternating stresses S_a versus mean stresses S_m on a Haigh diagram for an expected life of five million cycles. The solid lines apply to smooth specimens. The broken lines apply to a notched specimen with a calculated fatigue notch factor $K_b = 3$.

The lines are based on data for SAE J403 (1015) (Unified Numbering System [UNS] G10150) steel taken from the *SAE Handbook* [*12*]. The curved lines SWT are drawn according to the modified Smith-Watson-Topper parameter F_2. The straight lines AT, according to the alternating tensile stress approach, show a critical tensile alternating stress of 30 MPa, a mean stress influence on crack initiation according to Morrow [*13*], and yield stresses from the *SAE Handbook* [*12*], monotonic on the abscissa, cyclic on the ordinate.

Looking first at the solid lines note the divergence between the straight line according to Morrow and the curved line SWT. Both are well documented in reviewed literature and both are correct within the tolerance band as a result of scatter and our ignorance. To expect an accuracy greater than plus or minus 5% from either rule is unrealistic and to list data of greater precision is misleading.

Looking next at the broken lines one sees that both SWT and AT show that the notched specimens have the ability to carry an alternating stress almost or quite as large as the smooth fully reversed fatigue strength *provided there is sufficient compressive mean stress (about half as high as the yield strength).* This agrees with numerous tests referred to earlier and is a requirement for any rule on the effect of mean stress on notched fatigue.

Both SWT and AT also show that tensile mean stress does much harm but that the slope of the curve for notched parts is much flatter in the tensile region than in the compressive region. This too agrees with the test data.

If the yield strength at the surface of a notch is the same as for a smooth part, which is approximately true, the local yield condition in terms of nominal stress will be shown as the broken line triangle in Fig. 1. The broken line constant life curves which extend far beyond these yield lines, and which agree with test data, confirm that *long life depends not on stresses at the surface of notches, but on the arrest of cracks at some depth below the notch root, where stresses are approximately nominal.* This is also known from direct observation [*14*] and confirmed by Fig. 4.

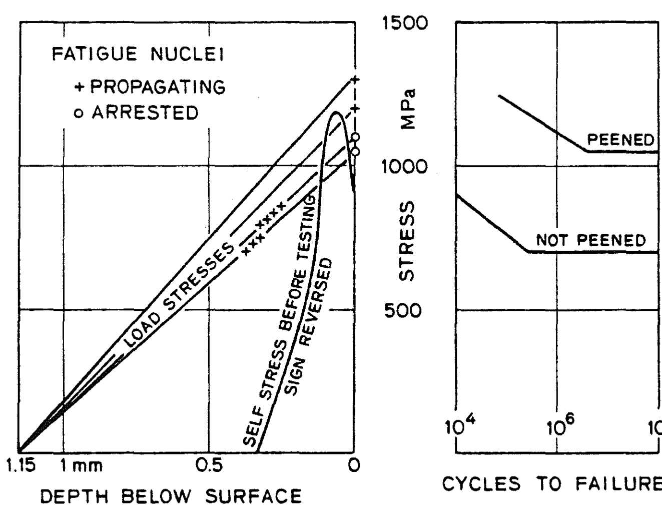

FIG. 4—*Fatigue nuclei in reversed bending. Unnotched peened steel specimens 2.3 mm thick, 830 HV, and estimated yield strength 2000 MPa* [19].

Using the diagram (Fig. 1) to obtain the fatigue notch factors for fully reversed load stresses one finds, according to SWT:

for zero self stress, 140/46.7 = 3.0;
for 110 MPa compression, 140/120 = 1.17; and
for 110 MPa tension, 140/15 = 8.

and according to AT:

for zero self stresses, 140/60 = 2.3 (cracks arrested);
for 80 MPa compression, 140/150 = 0.94; and
for 80 MPa tension, 140/30 = 4.7.

These predictions diverge in the numbers but agree qualitatively with each other and with test data. They show that one cannot expect K_f to be a constant at all values of mean stress, and that one cannot accept the often repeated statement that K_f is always less than K_t, at least if the existence of large cracks or total fracture are the failure criteria. Both predictive approaches will produce results within the tolerance band caused by scatter and our ignorance.

Figure 2, from Ref *4* show nominal stress constant life, or Haigh, diagrams according to SWT and AT for a much harder material (SAE J404 (4340) [UNS G43400] steel at 409 Brinell Hardness Number [HB]) and a notch factor of two instead of three. The general features are again the same as in Fig. 1.

Either the SWT or the AT approach gives reasonable results. The latter is more flexible in allowing for the differences in the shapes of constant life curves between notched and smooth specimens and for different slopes of the constant life line for smooth parts, and it seems more rational to the author. The former is simpler and is, through its formal similarity with Neuber's rule, in consonance with much of the current literature and therefore favored by many.

More flexibility is introduced into the alternating times maximum stress approach by Walker's rule [*15*].

$$F_3 = K_b(2S_a)^w S_{\max}{}^{1-w}$$

If we incorporate the factor two with the coefficient K_b and let $w = 0.5$, this is practically the same as the modified SWT rule given earlier as F_2.

$$F_4 = \sqrt{2}\, K_b(S_a S_{\max})^{0.5}$$

Rice et al [*16*] have found that $w = 0.4$ gives a better fit with the numerous test results that they considered. The formula

$$F_5 = K_b(S_a)^{0.4} S_{\max}{}^{0.6}$$

shows more influence of the maximum (or mean) stress than the SWT formula.

Fracture Mechanics Approach

In a paper presented in Nov. 1980 [*17*] Nelson and Socie showed that fracture mechanics can give reasonably good predictions of high cycle fatigue life. The calculation of crack propagation rates is beyond the scope of the present report, but the estimation of a threshold alternating stress, below which cracks will not propagate and failure will not occur, shall be attempted.

The general formula for stress intensity factor K (not to be confused with fatigue notch factors K_b and K_f) is

$$K = g\sigma(\pi a)^{1/2}(\mathrm{MPa}\cdot\mathrm{m}^{1/2})$$

where

σ = the stress in MPa,
g = a dimensionless numerical factor between one and two if the nominal stress is calculated from the *net* section, and
a = the depth of the notch plus the depth of a crack in metres.

The threshold stress intensity factor range ΔK_{th} is calculated from the tensile stress range (which is twice the critical alternating tensile stress S_{cat}) by the same formula which relates K to S. This gives

$$2S_{cat} = \Delta K_{\mathrm{th}}/g\sqrt{\pi a}$$

Values for ΔK_{th} are listed in Ref *4*. They range from a high of 8 $\mathrm{MPa}\cdot\mathrm{m}^{1/2}$ for a very tough pressure vessel steel to a low of 1.3 $\mathrm{MPa}\cdot\mathrm{m}^{1/2}$ for an aluminum alloy tested with a high tensile mean stress. A typical value is 3 $\mathrm{MPa}\cdot\mathrm{m}^{1/2}$.

Using $\Delta K_{\mathrm{th}} = 3\ \mathrm{MPa}\cdot\mathrm{m}^{1/2}$ and $g\cdot\pi^{1/2} = 3$ as typical values we find for incipient cracks in notches of different depth a the following values of critical alternating tensile stress amplitudes:

a in metres (inches): 0.003 (1/8), 0.005 (3/16), 0.010 (3/8), and 0.025; and
S_{cat} in MPa: 9, 7, 5, and 3.

These numbers are about 1/4 of those shown by SWT for very high tensile mean stress in Figs. 1 and 2 for the notched specimens. The new feature introduced by the fracture mechanics approach is the dependence of S_{cat} on the depth of the notch.

In design for high cycle fatigue one would use $\Delta K_{\mathrm{th}} = 0$ and $S_{cat} = 0$.

As far as the effect of self stresses is concerned the fracture mechanics approach confirms that compressive self stresses can be extremely powerful in preventing crack growth. The applied tensile stress must be greater than the compressive self stress in order to open the crack. On the other hand, if the self stress is tensile the load stress range allowable for long fatigue life becomes smaller as ΔK_{th} decreases with increasing mean stress.

This approach needs to be checked against test data. It may become very useful if suitable approximations for the numerical factor g and for the distribution of self stresses in depth can be used.

Effect of Loads on Self Stresses

When self stresses exist they can be treated as mean stresses, but they are different from stresses produced by the mean external load: the latter always persist throughout the test life; the former may disappear or be diminished and redistributed when stresses exceed the yield strength.

For an elastic-perfectly plastic material the interaction of load stresses and self stress can be visualized easily as in Fig. 3. If the self stress was σ_i initially and the alternating load stress σ_a is applied, then yielding will take place and the final self stress will be σ_f.

For work-hardening material one would have to calculate strains and obtain stresses from the stress-strain curve. The extra effort may not be worthwhile.

More importantly, the remaining self stress may have to be estimated at several depths below the surface of the notch. The alternating stress at the notch root may be large enough to wipe out any compressive self stress. At some depth below the notch root the alternating load stress will be far less concentrated and the initial self stress may be as great or greater than at the surface. Cracks will then be stopped at the depth where the sum of the self stress and load stress brings the stress intensity factor range below the threshold or brings the alternating tensile stress below its critical value, as shown in detail by Gerber [*14*].

Stress Distribution

The importance of the distribution of load stresses and of self stresses in a region below the surface of parts is very great. For the load stresses near a notch of radius R the stress gradient is given approximately by

$$d\sigma/dx = 2.5\ \sigma_{\max}/R$$

(more detailed data are quoted in Ref *4* and in Ref *18*. They show a maximum of compression at about 30% of the total depth of the compressive self stresses.)

Figure 4 shows a detailed study of the interaction of load stresses and self stresses from data by Starker et al [*19*] for shot-peened steel specimens 2.3 mm (0.09 in.) thick. Cracks started at the surface; they propagated to failure unless compressive stresses below the surface remained greater than the load stresses. Other cracks started below the surface, at a depth where there was very little or no compressive self stress, at subsurface stress values slightly higher than the unpeened fatigue limit. The fatigue limit of the peened specimens, calculated from the load, is equal to surface stresses which correspond to these subsurface stresses. It was 1.5 times the unpeened fatigue limit, or half the yield strength, as expected from Figs. 1 and 2.

In a later paper [*20*] Starker et al have shown that the increase of fatigue strength obtained by peening smooth bending specimens decreases rapidly as the size of the specimens increases. They explained this by the decrease of the stress gradient with increasing size of bending specimens. Polished push-pull specimens would show very little improvement by peening.

These data show that compressive self stresses improve the fatigue strength by arresting cracks below the surface, not by preventing their appearance at the surface. The depth at which cracks can be arrested determines the depth of the region for which self stress should be added to the stress produced by the mean load in determining S_m for use with the alternating tensile stress approach or with the SWT approach. One half the total depth of compressive self stress seems to be a reasonable estimate. Calculations which include the depth and distribution of self stresses are required only when one wants to optimize the treatment that produces the self stresses for a given application [*21*].

Conclusion

A review of four proposed methods of quantifying the effect of self stresses on long life fatigue leads to the following conclusions.

The method derived from the Smith-Watson-Topper parameter, in the form

$$K_b{}^2 S_{\max} S_a = S_f{}^2$$

where

S_f = the fully reversed smooth specimen fatigue strength,
S_a = the nominal alternating load stress amplitude,
$S_{\max}$ = the maximum stress including self stress at a small depth, and
K_b = the notch factor at long life and fully reversed stresses

seems simple and reasonably suitable for estimating stresses for a given long fatigue life.

Other methods that distinguish between resistance to crack initiation and resistance to crack propagation are available; they show the reasons for validity of $S_{\max} \times S_a$ within the limits of scatter and ignorance.

All acceptable methods show that compressive residual stresses are much more influential on notched parts and on crack-prone parts than on smooth specimens.

Approximate estimates of long life fatigue strength of notched parts exposed to constant amplitude loading are easy if one assumes "sufficient depth" of the self stresses. Detailed estimates of the depth of self stress which is sufficient for long life resistance to a given load sequence require knowledge of the distribution in depth of both self stresses and load stresses.

Self stresses have a minor effect on the resistance to the development of microscopic cracks. They have a very strong effect on the resistance to the propagation of small cracks and may arrest the growth of cracks.

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