FUNDAMENTAL ASPECTS OF SHOT PEENING COVERAGE CONTROL PART ONE: FORMULATION OF SINGLE AND MULTIPLE IMPACTING

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ABSTRACT

Two general geometrical/statistical methods of formulating the coverage of a substrate resulting from impacts of shots are presented [Abyaneh M Y and Fleischmann M, 'Two-dimensional transformation on substrates of infinite and finite size, J Electrochem Soc, **138**, 2486-2492, 1991]. The first geometrical representation is shown to yield the famous Avrami equation [Avrami M, J Chem Phys, **7**, 1103, 1939; **8**, 212, 1940; **9**, 177, 1941] for the coverage by <u>at least</u> a single shot, whereas the second simplifies to the equation first realized by Evans [Evans U R, Trans Faraday Soc, **41**, 365, 1945]. In this paper It is shown that the generalized form of the second approach is a much more powerful technique for the formulation of coverage and can be applied more directly to the problem of overlap of shots than the complicated form of the extended coverage developed by Avrami. The second geometrical method will be used here for formulating the extent to which regions of substrate are subjected to <u>only</u> one, two, or a greater number of impacts.

KEYWORDS

Formulation, Shot Peening, Coverage, Single Impacting, Multiple Impacting

INTRODUCTION

Shot peening is an important mechanical surface treatment that is usually very effective in improving important service performance factors such as fatigue life. During the shot peening of a component's surface an increasing proportion of that surface is said to be 'covered'. Coverage is defined as the proportion of the exposed surface that has been impacted in a given time of shot peening. Control of this coverage is an essential feature of precision shot peening. During the progress of peening we can consider a simplified model in which uniformly-sized spheres impact a flat surface creating circular impressions of constant diameter as shown in Fig.1.

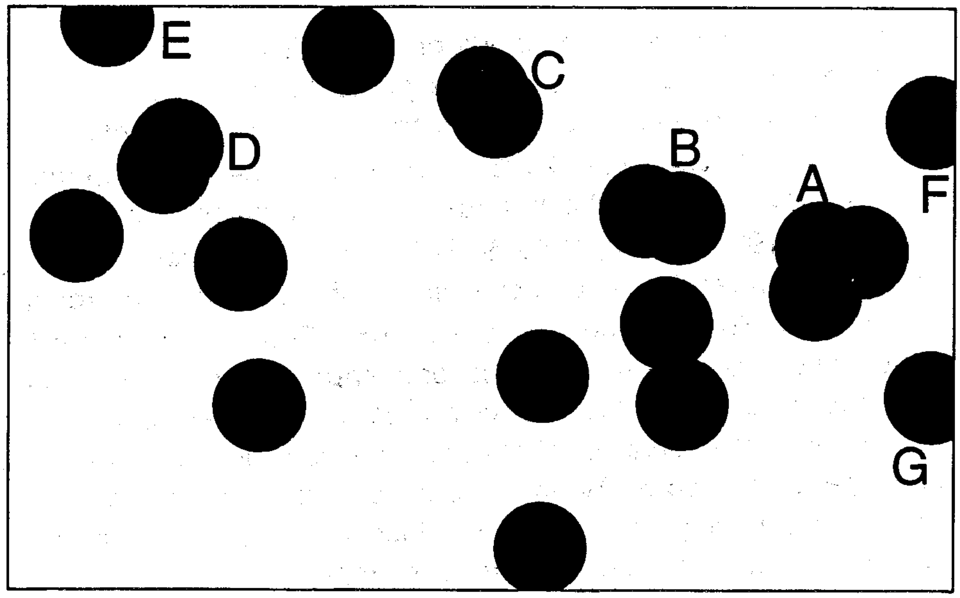

Fig.1 Simulated early stage of Shot Peening

The difficulty in formulating coverage at any given time is due to the overlap of shots on one another (see the overlap of shots at positions A, B, C and D in Fig.1). The extent of overlap increases with the progress of peening as the number of shots and consequently the probability of overlap increases with time. The well-known statistical method of taking into account the overlap of centres on one another is the Avrami postulate [2]. Another less-known but a more powerful method is that developed by Evans [3].

In this paper two general geometrical approaches to the formulation of coverage are introduced [1]. The first approach is shown to yield an equation for coverage similar to that derived by Avrami. The second one, which will be shown to be a preferred method, leads to the equation given by Evans. It must be noted that both Avrami and Evans statistical approaches have the ability to estimate only the coverage of a substrate subjected to <u>at least</u> a single impact. In addition to this, the second geometrical approach is shown to be able to predict the extent of regions subjected at any given time to <u>only</u> one, two or more impacts.

GEOMETRICAL FORMULATION OF COVERAGE

Fig.1 can be assumed to represent the situation in an early stage of peening when only a small proportion of the surface has been impacted. The box confining the area represents the observational area. Even with most of the surface being unpeened, several regions, B, C and D have received two impacts, whereas one region, A, has even received three impacts. To develop our geometrical methods let us analyse site A where three shots are seen to have overlapped on one another as shown by Fig.2.

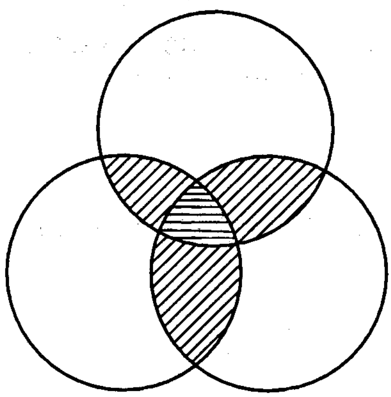

Fig.2 Region A of Fig.1 where three impacts have overlapped

The coverage, S, that is the fractional area in Fig.2 which is subjected at least to a single shot, can be calculated, geometrically, in two ways. The first one uses the concept of an extended area, S_{ext}, defined as an imaginary area which would have been peened if overlapping of impacts were not allowed. Henceforth Fig.3(a) is representing $S_{1,ext}$, that is that area which would have been impacted upon at least once if we were able to disallow overlapping of impacts. In the same way Fig.3(b) is representing $S_{2,ext}$, that is the region of overlap in Fig.2 which has at least been subjected to two impacts, disallowing again the overlap of two-impacted regions, and Fig.3(c) representing $S_{3,ext}$.

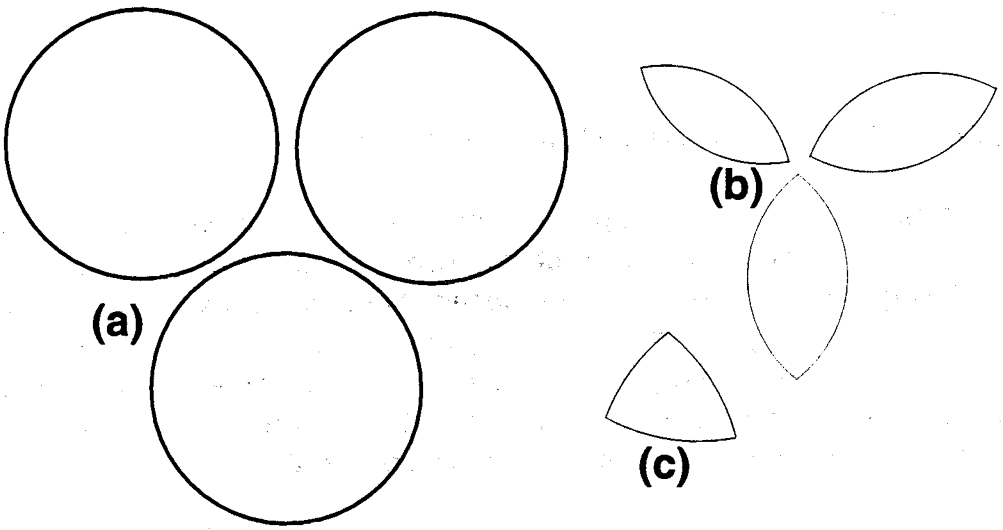

Fig.3 Extended areas subjected to at least (a) a single, (b) a double and (c) a triple impact.

The coverage, S, in Fig.2 can readily be obtained from

$$S = S_{1,ext} - S_{2,ext} + S_{3,ext} \tag{1}$$

In general one must account for the possibility of overlap of 4, 5 or any higher number of impacts. Hence a general formula relating the coverage to the extended areas can be written as

$$S = S_{1,ext} - S_{2,ext} + S_{3,ext} - \ldots\ldots + (-1)^{n+1}\, S_{n,ext} + \ldots\ldots \tag{2}$$

It can be shown [4] that

$$S_{n,ext} = \frac{(S_{1,ext})^n}{n!} \tag{3}$$

Hence, eq.(2) can be written in the form

$$S = S_{1,ext} - \frac{(S_{1,ext})^2}{2!} + \frac{(S_{1,ext})^3}{3!} - \ldots\ldots + (-1)^{n+1}\,\frac{(S_{1,ext})^n}{n!} + \ldots\ldots \tag{4}$$

The above equation remains unchanged by both adding 1 to it and subtracting 1 from it. Hence,

$$S = 1 - 1 + \sum_{n=1}^{\infty} (-1)^{n+1}\,\frac{(S_{1,ext})^n}{n!} \tag{5}$$

But $\exp(-S_{1,ext})$ can be expanded as

$$\mathrm{Exp}(-S_{1,ext}) = 1 - \sum_{n=1}^{\infty} (-1)^{n+1}\,\frac{(S_{1,ext})^n}{n!} \tag{6}$$

The coverage S is then given by

$$S = 1 - \mathrm{Exp}(-S_{1,ext}) \tag{7}$$

which is equivalent to the well-known Avrami [2] equation. A simpler way of geometrically accounting for S, Fig.2, is by adding the unhatched area, S_1, to the hatched areas, S_2 and S_3. To calculate S we must, therefore, find the coverage due to areas affected by only a single impact, S_1, by only two impacts, S_2, and by only three impacts, S_3.

FORMULATION OF COVERAGE RESULTING FROM MULTIPLE IMPACTS

As shots are arriving randomly and are independent from one another, the probability of arrival of a shot within a given time and at a given position is by its nature Poisonian. Hence the probability, p_n, that, in a given time, a given site has been covered by only n impacts, and by implication the fractional area or coverage, S_n, affected by only n impacts, is given by the Poisson equation

$$S_n = p_n = \frac{E^n e^{-E}}{n!} \tag{8}$$

where E is the expectation number. Hence the fractional coverages S_1, S_2, and S_3 are given by

$$S_1 = E\, e^{-E}, \qquad S_2 = \frac{E^2 e^{-E}}{2} \qquad \text{and } S_3 = \frac{E^3 e^{-E}}{6} \tag{9}$$

The coverage by at least one shot is then derived from Fig.2 by adding S_1, S_2, S_3, and so on together, that is from the equation

$$S = S_1 + S_2 + S_3 + \ldots\ldots + S_n + \ldots\ldots \tag{10}$$

Hence S is determined from

$$S = E\, e^{-E} + \frac{E^2 e^{-E}}{2} + \frac{E^3 e^{-E}}{6} + \ldots\ldots + \frac{E^n e^{-E}}{n!} + \ldots\ldots \tag{11}$$

If e^{-E} is factorized out we are then left with

$$S = e^{-E}\left(E + \frac{E^2}{2} + \frac{E^3}{6} + \ldots\ldots + \frac{E^n}{n!} + \ldots\ldots\right) \qquad (12)$$

or with

$$S = e^{-E}\left(e^{E} - 1\right) = 1 - e^{-E} \qquad (13)$$

Eq.(13) evidently has the same format as eq.(7) with $S_{1,ext}$ replaced by E. It has been shown [1] that $S_{1,ext}$ is related to E through the equality

$$S_{1,ext} = E\, e^{-E}\, e^{E} = E \qquad (14)$$

It is thus concluded that $S_{1,ext}$ is a more involved concept than the concept of expectation E through the multiplication factors exp(-E) times exp(E) and hence the latter concept is to be preferred. To formulate S as a function of time t, it is necessary to be able to work out the exact dependance of E on time. This is done in the next section.

EXPECTATION, E, AS A FUNCTION OF TIME

The expectation number E is defined here as the number of times a given site is to be subjected to impacts. Let us consider a given site such as point P within the boundary of the surface exposed to shots, Fig.1. Evidently point P will be subjected to impacts from all those shots the centres of which land within a radius r. Assuming that shots are fired at a uniform rate A ($m^{-2}s^{-1}$), the number of shots which are expected to land in time t within an area of circle of radius r, that is the expectation value E, is given by

$$E = \pi r^2 A t \qquad (15)$$

The time-dependence of coverage, that is the fractional area of the exposed surface which is subjected to <u>at least</u> a single impacting, is then derived by combining eqs.(13) and (15). Hence

$$S = 1 - \exp(-\pi r^2 At) \quad (16)$$

The proportion of the exposed surface that has been subjected to only n impacts as a function of time is derived by combining Eqs.(8) and (15).

$$S_n = \frac{(\pi r^2 At)^n}{n!} \exp(-\pi r^2 At) \quad (17)$$

Fig.4 compares the % area contribution of single and multiple impacts, values of n from 1 to 10, towards the total coverage, S. All plots are drawn with r=0.5 mm and A=0.293 $mm^{-2}s^{-1}$.

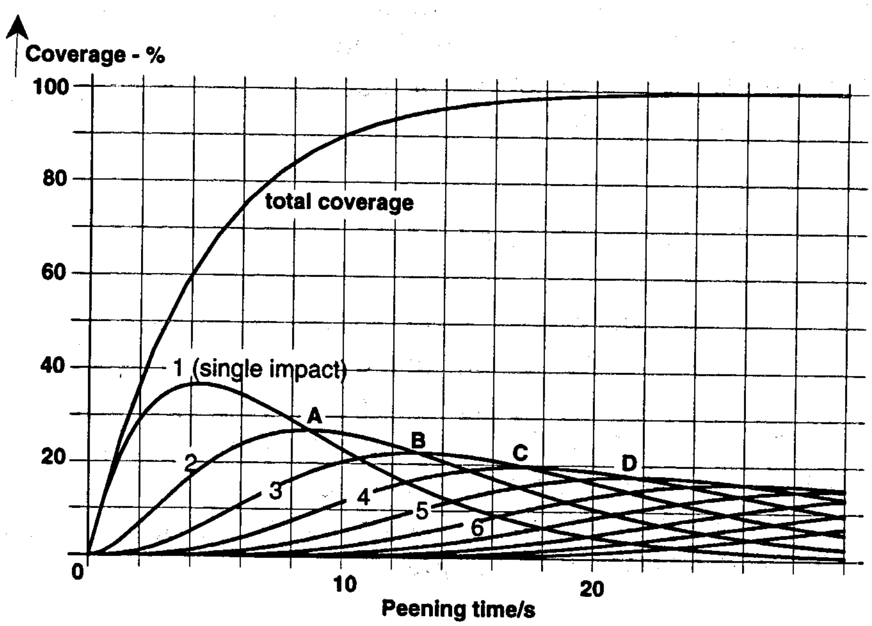

Fig.4 Contribution to the total coverage of single and multiple impacts.

The proportion of the exposed surface that has been subjected to at least n impacts as a function of time is derived by combining Eqs.(3), (14) and (15).

Hence

$$S_{n,ext} = \frac{(\pi r^2 At)^n}{n!} \tag{18}$$

Fig.5 shows how the proportion of exposed surface that has been subjected to at least n impacts changes with the number of impacts n over a range of time. The plots are all drawn with r=0.5 mm and A=0.293 $mm^{-2}s^{-1}$.

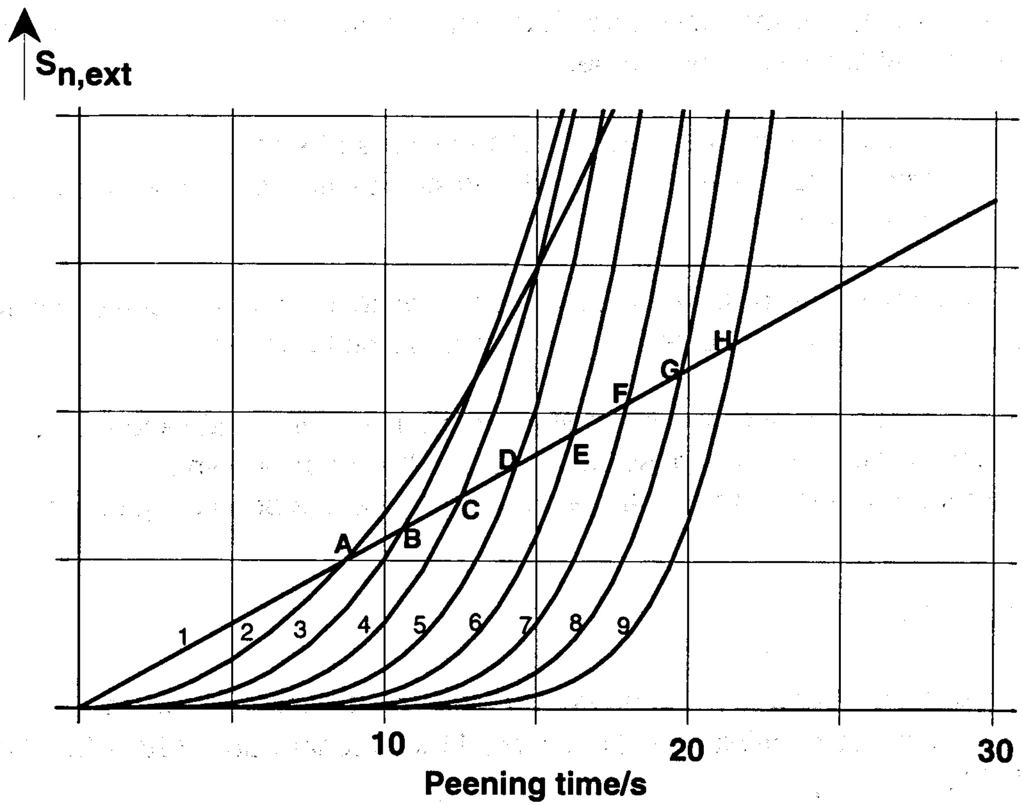

Fig.5 The variation of the proportion of exposed surface that has been subjected to at least n impacts with the number of impacts, n.

It can be seen from Fig.5 that, though up to A, $S_{1,ext}$ is large compared to all other forms of extended areas, $S_{2,ext}$ becomes the dominant area after A,

$S_{3,ext}$ after B, and so on. After H, the exposed surface which has been subjected to at least 9 impacts, $S_{9,ext}$, becomes proportionally larger than that which has been subjected to at least 1 impact, $S_{1,ext}$. This result is not intuitively obvious, but can be realized after further reflection.

CONCLUSIONS

The fractional area of regions of the exposed surface subjected to at least a single impact, i.e. the coverage, has been formulated using two proposed geo-metrical/statistical approaches.

The fractional area of those regions of the exposed surface subjected to only a single impact has been formulated and compared with those subjected to multiple impacting.

The fractional area of those regions of the exposed surface subjected to at least two or any other number of impacts has also been formulated.

In future research, once over-peening is experimentally established for a given situation, it will then be possible to relate the over-peening to a probable maximum number of impacts the exposed surface has been subjected to.

REFERENCES

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4. To be published.