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When Logic Takes Over Intuition

This is simply a story of events from the production shop at Electronics, Inc. It is a story about false intuition and where logic finally triumphed. Yes, some of you peening experts must be thinking "greenhorns", but some of you might be caught off guard, as I was.

HERE IT GOES: Electronics, Inc. has a significant variety of media types in inventory which are used to test valves, sensors and Almen strips. Recently we reviewed the inventory list, and for some of the containers we had actual weight-in-stock. We looked for a way to get an approximate weight simply by measuring the fill-level of the container. Intuition struck and the need for a bulk-density table based on media type was on the table. Intuition said, smaller media equals higher bulk-density!

No problem we thought—let's call the experts at Ervin Industries! Michael Konecny and Rick Payne with Ervin promptly fed us the answers: 7.14 – 7.69 g/ccm (~7.5 g/cm3) for the shot-material density and 286.82 lb/cu-ft for the shot-media bulk-density—the handwritten note said: "currently 280 shot". A bit frustrated, having now only one data point, the 280 shot, I decided to make a simple test, weighing some different sizes of cast-steel shot in EI's inventory. The results baffled me as the given volume for all shots had approximately the same weight! How could this be? Smaller media has smaller voids around itself, while larger shot has bigger voids. I was convinced, therefore, the larger shot would have a smaller bulk density over the smaller shot. So, I consulted "Uncle Google"! (See QR codes at the end of the article.)

Here is what I learned quickly: While smaller media has smaller voids and larger shot has bigger voids, for smaller shot there are many more of these voids, while for larger shot there a fewer of them. This balanced things out, simply by geometric volume ratio.

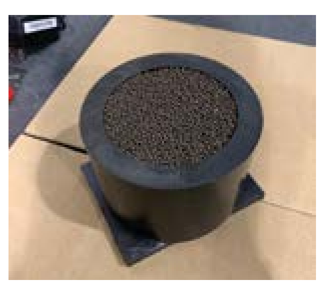
One reference[i] gave a nice table with packing-density equations based on different mathematical models and packing processes (see Table One). Another reference[ii] pointed out the "wall effect" where the bulk-density is affected by the ratio of the wall-area to the sample volume and the size of the spheres. Higher wall-area to sample volume results in lower bulk density. This phenomenon was apparent on my first, crude weight test with a tall, narrow lab measuring cylinder. The larger media sizes weighed noticeably less.

Table One
Packing Density Equations

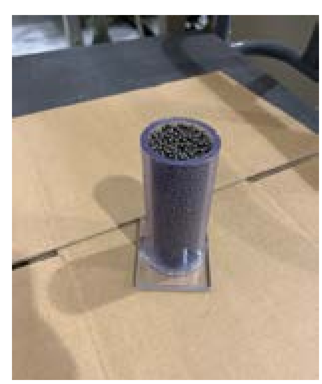
packing	analytic η	η	reference
loosest possible	--	0.0555	Gardner (1966)
tetrahedral lattice	$\frac{\pi\sqrt{3}}{16}$	0.3401	Hilbert and Cohn-Vossen (1999, pp. 48-50)
cubic lattice	$\frac{\pi}{6}$	0.5236	
hexagonal lattice	$\frac{\pi}{3\sqrt{3}}$	0.6046	
random	--	0.6400	Jaeger and Nagel (1992)
face-centered cubic close packing	$\frac{\pi}{3\sqrt{2}}$	0.7405	Steinhaus (1999, p. 202), Wells (1986, p. 29; 1991, p. 237)
body-centered cubic close packing	$\frac{\pi\sqrt{3}}{8}$	0.6801	
hexagonal close packing	$\frac{\pi}{3\sqrt{2}}$	0.7405	Steinhaus (1999, p. 202), Wells (1986, p. 29; 1991, p. 237)

A third reference[iii] got so much into the packing theory with differential, integral and matrix math, I did not take the effort to absorb all the details of the article. Instead, we decided to make two additional tests with six (6) cast-steel shot sizes and two (2) different sample volumes:

- Test #2a the media was "as-poured" into a 20 cu-in volume cylinder with a diameter/length ratio of 1.06 (~1.0) [-]
- Test #2b the media was "as-shaken" (not stirred! - 007 ☺) into a 20 cu-in volume cylinder with a diameter/length ratio of 1.06 (~1.0) [-]
- Test #3a the media was "as-poured" into a 5.22 cu-in volume cylinder with a diameter/length ratio of 0.37 [-]
- Test #3b the media was "as-shaken" into a 5.22 cu-in volume cylinder with a diameter/length ratio of 0.37 [-]



20 cu-in volume



5.22 cu-in volume

These are the results of these two tests.

20 cu-inch Volume Test #2

El-Test #2a	Diameter		3.000 inch
As Poured	Length		2.830 inch
	Volume		20.00409 cu-in
Media	Weight [g]	Weight [lbs]	Bulk Density [lbs/cu-in]
S-930	1430	3.14600	0.1573
S-660	1425	3.13500	0.1567
S-230	1470	3.23400	0.1617
S-170	1455	3.20100	0.1600
S-110	1455	3.20100	0.1600
S-70	1445	3.17900	0.1589
		Avg	**0.1591 lbs/cu-in**
			274.93 lbs/cu-ft
		Packing Density	**63.0%**

El-Test #2b	Diameter		3.000 inch
As Shaken	Length		2.830 inch
	Volume		20.00409 cu-in
Media	Weight [g]	Weight [lbs]	Bulk Density [lbs/cu-in]
S-930	1515	3.33300	0.1666
S-660	1505	3.31100	0.1655
S-230	1550	3.41000	0.1705
S-170	1520	3.34400	0.1672
S-110	1520	3.34400	0.1672
S-70	1520	3.34400	0.1672
		Avg	**0.1673 lbs/cu-in**
			289.18 lbs/cu-ft
		Packing Density	**66.2%**

5.22 cu-inch Volume Test #3

El-Test #3a	Diameter		1.350 inch
As Poured	Length		3.645 inch
	Volume		5.21741 cu-in
Media	Weight [g]	Weight [lbs]	Bulk Density [lbs/cu-in]
S-930	367.86	0.80929	0.1551
S-660	371.28	0.81682	0.1566
S-230	383.24	0.84313	0.1616
S-170	378.16	0.83195	0.1595
S-110	376.14	0.82751	0.1586
S-70	374.52	0.82394	0.1579
		Avg	**0.1582 lbs/cu-in**
			273.38 lbs/cu-ft
		Packing Density	**62.7%**

El-Test #3b	Diameter		1.350 inch
As Shaken	Length		3.645 inch
	Volume		5.21741 cu-in
Media	Weight [g]	Weight [lbs]	Bulk Density [lbs/cu-in]
S-930	389.17	0.85617	0.1641
S-660	389.41	0.85670	0.1642
S-230	400.76	0.88167	0.1690
S-170	395.74	0.87063	0.1669
S-110	397.79	0.87514	0.1677
S-70	395.77	0.87069	0.1669
		Avg	**0.1665 lbs/cu-in**
			287.65 lbs/cu-ft
		Packing Density	**66.1%**

Here we draw the conclusion for the two tests combined:

- **Bulk-density "as poured"** 273.4 to 274.9 lb/cu-ft
  → **avg. 274.2 lb/cu-ft**
  Packing density 62.7 to 63.0%
  (see Table 1: ≈ "random" - Jaeger & Nagel 1992)

- **Bulk-density "shaken"** 287.7 to 289.2 lb/cu-ft
  → **avg. 288.5 lb/cu-ft**
  Packing density 66.1 to 66.2%
  (see Table 1: ≈ "body-centered" - cubic close packing)

The numeric results are very consistent between the two tests.

In both the "as-poured" tests, the wall-effect manifests itself by the slightly lower weights at coarser media. In the "as-shaken", only the test 3b with the lower diameter/length ratio volume indicates some wall-effect.

Comparing these numbers with the value given by Ervin Industries shows the consistency of data:

"as-poured" = 274.2 lb/cu-ft → "ERVIN" = 286.82 lb/cu-ft
→ "as-shaken" = 288.5 lb/cu-ft

Bottom line, our friends at Ervin Industry, in their wisdom, gave us the correct, single number—yes, there is only one bulk density for shot-peen media, independent of the different sizes.

While these tests were made with spherical, cast-steel shot media only, the single value feature of different size bulk-density translates to other media types: i.e., cut-wire, ceramic, glass, etc. However, the single, numeric value will naturally change as the particular material density of different media types varies greatly.

The days are not lost—we learned something!

Google Search References

[i] *https://mathworld.wolfram.com/SpherePacking.html*

[ii] L. Burtseva, B. Valdez Salas, F. Werner, V. Petranovskii - **Packing of monosized spheres in a cylindrical container: models and approaches**; Revista Mexicana de F'ısica 61 (2015) 20–27 – Wall Effect paragraph 4.2

[iii] H. Cuhn (Microsoft) – **Conceptual Breakthrough in Sphere Packing**; Feb2017_rnoti-p102