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The Stress Concentration in Keyways when Torque is Transmitted through Keys

The electroplating method of strain analysis is used to determine the peak stress at the corner of keyway

by H. Okubo, K. Hosono and K. Sakaki

ABSTRACT The stress-concentration factor for keyways is obtained by the electroplating method of strain analysis which was recently developed in Japan. The stresses in keyways are examined under most practical conditions. The shaft to be examined is set in a boss and fixed with a key so that the torque is wholly transmitted to the test piece through the key. To obtain the limit value for a shaft twisted only through a key, the fitting is devised so as to minimize the surface friction at the fitted portion. The results are compared to those reported previously by other authors in which local effects of the key are ignored.

Introduction

A number of theoretical and experimental investigations have been conducted for the determination of the stress concentration arising from torsion in a shaft containing a keyway.[1-4]

Little information, however, is available concerning the stress concentration in a shaft in which torque is applied through the keyway even though such information is of quite practical concern. In this case, the problem is of a more complicated nature and information may only be obtained experimentally, since the problem lies outside the scope of theoretical investigation.

For the determination of the stress-concentration factor of a keyway, the accurate measurement of peak stress is essential at the corner of the keyway where the radius of curvature is usually very small. Usually methods of strain analysis, such as photoelasticity or use of strain gages, are laborious for the accurate measurement of a strain of so highly a localized character.

The electroplating method of strain analysis, however, is relatively free of the technical difficulties associated with other known methods. It has proved useful in making an accurate determination of the peak stress in shafts containing grooves or holes.[5-8]

When a copper-plated specimen is submitted to cyclic stress, the surface of the plated copper changes in color owing to microflecks produced by fatigue. Microscopic examination of the surface and cross section having been color changed, electropolished, and etched, reveals the nature of grain growth in the plated copper.[9]

By means of etching, one can distinguish easily between a grown grain and the neighboring micrograins in the deposited copper, as is shown in Fig. 6. The flecking and the grain growth occur only when the magnitude of the cyclic stress exceeds a certain value proper to the plated copper for an assigned number of stress cycles. The change in constitution of the deposited copper described is affected only by the cyclic shearing stress even in a complex stress state and not by any statical stress. The new technique is based on these effects due to reversals of cyclic stress.

Recently Terada applied the technique in examining the stress concentration due to the presence of keyways.[10] In his experiments, however, the local effects due to the key were ignored. In actual application, a shaft is normally fitted into a boss with a key, and the stresses around the keyway are distributed three-dimensionally. This stress state then differs from that associated with pure torsion.

Accordingly, the experiment was carried out under those conditions which exist in most practical applications. Two different keyways were examined having various radii of curvature at the corner of the keyway. The results obtained were compared to those of Terada, in which the local effect of the key was ignored.

Experimental Procedure

When a copper-plated specimen is subjected to cyclic stress, micrograins in the plated copper gradually grow, provided that the amplitude of the cyclic shearing strain is sufficiently large. With an increase in the number of cycles, the size and the thickness of the grown grains are increased. The limiting value of shearing strain below which grain

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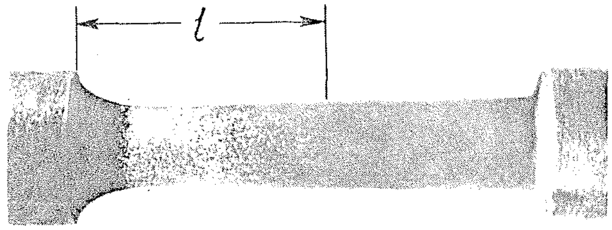


Fig. 1—Grown grains caused by cyclic strain appearing in a portion of the test piece

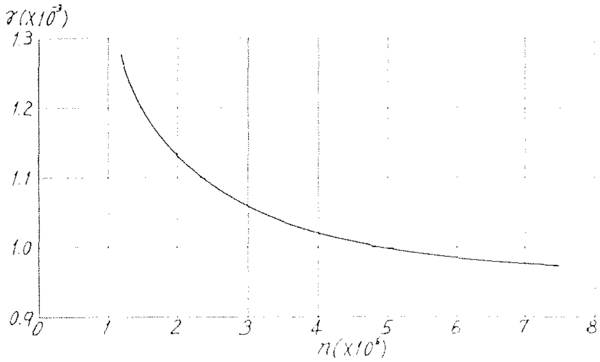


Fig. 2—Calibration curve representing the relation between γ and n

growth does not occur by fatigue is termed the "proper strain" of the plated copper and is related to a specific number of repetitions of the cyclic strain.

The value of the proper strain is keenly affected by the conditions for deposition. The conditions employed in the experiment are as follows:

Preliminary deposition: solution CuCN 23 g, NaCN 30 g, Na_2CO_3 10 g, H_2O 1000 cc; current density 0.06 amp dm²; temperature 30°C; bath voltage 0.35 v; duration of deposition 20 min.

Additional deposition: solution $CuSO_4 \cdot 5H_2O$ 250 g, H_2SO_4 80 g, H_2O 1000 cc; current density 3 amp dm²; temperature 23° C; bath voltage 0.5 v; duration of deposition 15 min.

A preliminary test was made to find the relation between the proper strain and the number of repetitions of cyclic strain. For this purpose, conical rods of fine-grained carbon steel plated with copper, varying from 8 mm to 10 mm in diameter (Fig. 1) were used. Torsional fatigue tests were made under a prescribed torque, T. For stepwise increasing repetitions of the cyclic stress, n, the corresponding diameter of the conical rod, d, is obtained by measuring l, the length of the portion of the specimen where grain growth occurred, with a microscope. The proper value of the torsional

TABLE 1 (A)—THE STRESS-CONCENTRATION FACTOR FOR GROUP A

No. of test piece	D, mm	b, mm	r, mm	ρ, (= r/D)	T, kg — m	Key end γ (× 10⁻³)	Key end α	Key end α_m	Key end β_m	Keyway γ (× 10⁻³)	Keyway α	Keyway α_m	Keyway β_m
1	10.00	2.57	0.06	0.006	0.50	1.197	3.46	3.48	3.82	1.061	3.06	3.07	3.37
2	10.00	2.57			0.49	1.173	3.46			1.043	3.07		
3	10.00	2.56			0.49	1.197	3.53			1.049	3.09		
4	10.00	2.56			0.49	1.180	3.48			1.040	3.07		
1	10.00	2.57	0.08	0.008	0.50	1.089	3.15	3.13	3.44	1.009	2.91	2.92	3.21
2	10.00	2.57			0.49	1.063	3.13			0.991	2.92		
3	10.00	2.56			0.49	1.063	3.13			0.999	2.94		
4	10.00	2.56			0.49	1.058	3.12			0.985	2.90		
5	10.00	2.56	0.12	0.012	0.60	1.147	2.76	2.76	3.03	1.082	2.60	2.59	2.85
6	10.00	2.56			0.59	1.126	2.75			1.061	2.60		
7	10.00	2.55			0.59	1.131	2.77			1.049	2.57		
8	10.01	2.55			0.59	1.121	2.75			1.053	2.58		
9	9.99	2.55	0.19	0.019	0.68	1.112	2.35	2.37	2.60	1.050	2.24	2.24	2.46
10	9.99	2.55			0.68	1.112	2.35			1.041	2.22		
11	9.98	2.55			0.68	1.121	2.36			1.056	2.23		
12	10.00	2.55			0.68	1.131	2.40			1.061	2.25		
13	9.99	2.58	0.26	0.026	0.73	1.112	2.19	2.19	2.41	1.038	2.05	2.05	2.25
14	10.00	2.57			0.73	1.104	2.18			1.038	2.05		
15	10.00	2.57			0.73	1.112	2.20			1.040	2.06		
16	10.00	2.58			0.73	1.112	2.20			1.038	2.05		
13	9.99	2.58	0.28	0.028	0.73	1.096	2.16	2.17	2.38	1.027	2.03	2.04	2.24
14	10.00	2.57			0.73	1.092	2.16			1.034	2.05		
15	10.00	2.57			0.73	1.096	2.17			1.027	2.03		
16	10.00	2.58			0.73	1.096	2.17			1.034	2.05		
17	10.00	2.58	0.40	0.040	0.81	1.121	2.00	2.00	2.20	1.044	1.86	1.86	2.04
18	10.00	2.58			0.81	1.126	2.01			1.043	1.86		
19	10.00	2.58			0.81	1.117	1.99			1.042	1.86		
20	10.01	2.58			0.81	1.117	2.00			1.034	1.85		

stress based on a circular cross section is evaluated from

$$\tau = \frac{16T}{\pi d^3} \qquad (1)$$

and the corresponding proper strain at n is

$$\gamma = \frac{\tau}{G} = \frac{16T}{\pi d^3 G} \qquad (2)$$

where G is the modulus of rigidity and equals 8.1×10^5 kg/cm² for the carbon steel used.

The γ-n curve showing the relation between the proper strain and the number of strain cycles is given in Fig. 2.

The test piece used in the experiment is a shaft of carbon steel, plated with copper and fitted into a boss with a key as shown in Fig. 3. Two different keyways were examined, the one in group A: D = 10 mm, b = 2.5 mm, t = 1 mm, the other in group B: D = 12 mm, b = 4 mm, t = 2.4 mm (DIN 6886). The major dimensions of the test pieces are given in Fig. 4, and the details of the keyways in Table 1 (A) and (B).

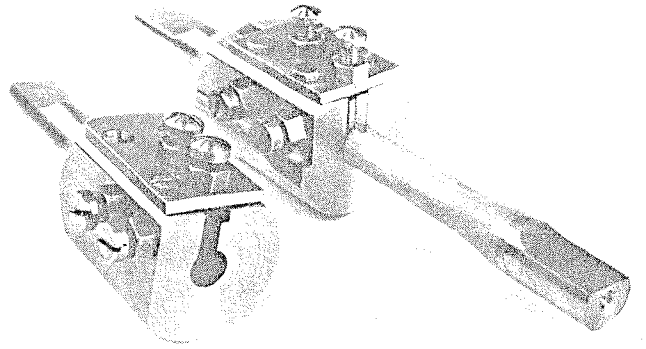


Fig. 3—Test piece fitted into the boss with key

TABLE 1 (B)—THE STRESS-CONCENTRATION FACTOR FOR GROUP B

No. of test piece	D, mm	b, mm	r, mm	ρ, (= r/D)	T, kg − m	Key end: γ (× 10^{-3})	Key end: α	Key end: α_m	Key end: β_m	Keyway: γ (× 10^{-3})	Keyway: α	Keyway: α_m	Keyway: β_m
1	11.89	4.08	0.07	0.0059	0.62	1.108	3.44	3.49	4.84	1.058	3.28	3.29	4.57
2	11.89	4.08			0.62	1.129	3.51			1.063	3.30		
3	11.88	4.07			0.62	1.141	3.53			1.058	3.28		
4	11.88	4.07			0.62	1.121	3.47			1.060	3.28		
5	11.89	4.09	0.10	0.0084	0.69	1.135	3.17	3.17	4.40	1.066	2.97	2.96	4.12
6	11.89	4.09			0.69	1.135	3.17			1.061	2.96		
7	11.89	4.08			0.69	1.131	3.16			1.061	2.96		
8	11.89	4.08			0.69	1.135	3.17			1.061	2.96		
5	11.89	4.09	0.11	0.0092	0.69	1.096	3.06	3.06	4.25	1.033	2.88	2.88	4.00
6	11.89	4.09			0.69	1.092	3.05			1.029	2.87		
7	11.89	4.08			0.69	1.092	3.05			1.031	2.88		
8	11.89	4.08			0.69	1.096	3.06			1.032	2.88		
9	11.88	4.08	0.15	0.0126	0.79	1.117	2.71	2.73	3.79	1.054	2.56	2.56	3.55
10	11.89	4.08			0.79	1.125	2.74			1.040	2.53		
11	11.89	4.08			0.79	1.124	2.74			1.049	2.56		
12	11.89	4.08			0.79	1.112	2.71			1.053	2.57		
9	11.88	4.08	0.17	0.0143	0.79	1.078	2.62	2.63	3.65	1.005	2.45	2.45	3.40
10	11.89	4.08			0.79	1.082	2.64			1.005	2.45		
11	11.89	4.08			0.79	1.078	2.63						
12	11.89	4.08			0.79	1.078	2.63						
13	11.89	4.08	0.25	0.0210	0.94	1.141	2.34	2.32	3.22	1.049	2.15	2.14	2.98
14	11.89	4.08			0.94	1.121	2.30			1.047	2.14		
15	11.89	4.08			0.94	1.126	2.31			1.046	2.14		
13	11.89	4.08	0.27	0.0227	0.94	1.104	2.26	2.26	3.14	1.023	2.09	2.10	2.92
14	11.89	4.08			0.94	1.104	2.26			1.033	2.12		
15	11.89	4.08			0.94	1.108	2.27			1.027	2.10		
16	11.88	4.07	0.40	0.0336	1.07	1.131	2.03	2.01	2.79	1.042	1.87	1.89	2.62
17	11.88	4.07			1.07	1.112	2.00			1.053	1.89		
18	11.89	4.08			1.07	1.117	2.01			1.051	1.89		
19	11.89	4.08			1.07	1.104	1.99			1.051	1.89		
16	11.88	4.07	0.41	0.0345	1.07	1.131	2.03	2.01	2.79	1.042	1.87	1.89	2.62
17	11.88	4.07			1.07	1.112	2.00			1.053	1.89		
18	11.89	4.08			1.07	1.117	2.01			1.051	1.89		
19	11.89	4.08			1.07	1.104	1.99			1.051	1.89		

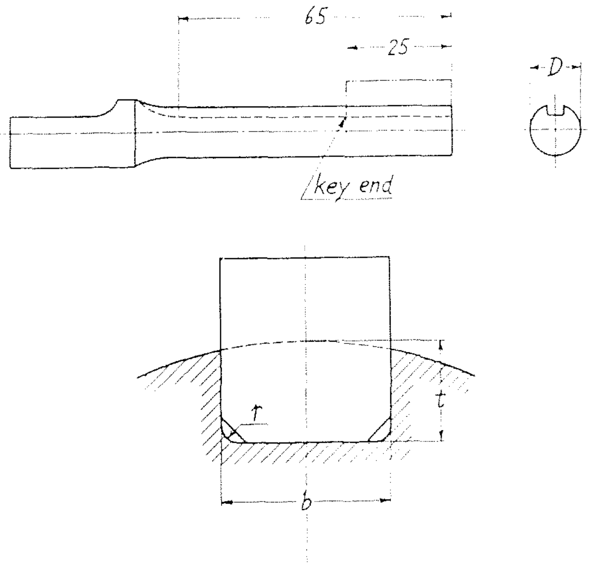
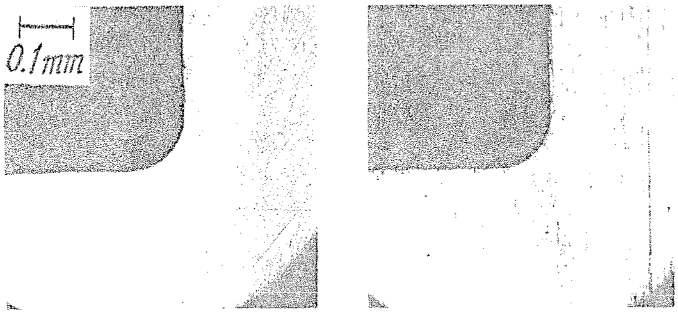


Fig. 4—Dimensions of test piece



(a) (b)

Fig. 5—Radius of curvature at the corner of keyway. D = 11.9 mm, r = 0.10 mm. (a) Before cutting: (b) after cutting

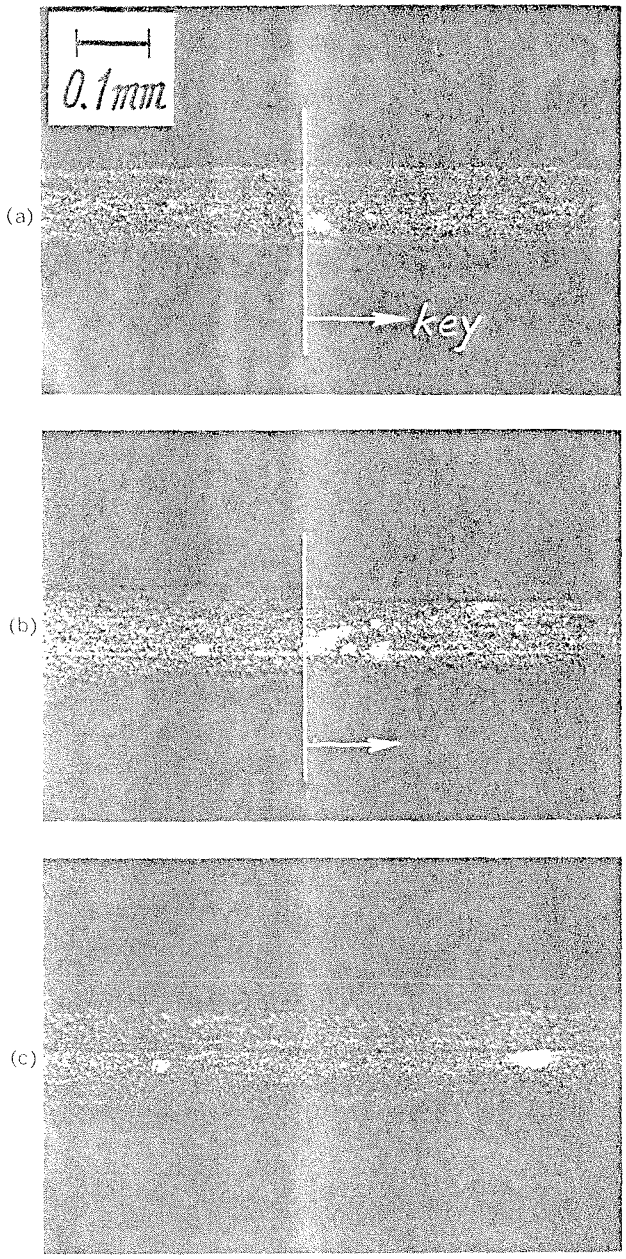


Fig. 6—Initial grown grains at key end and keyway. (a) $n = 2.0 \times 10^6$, key end; (b) $n = 2.95 \times 10^6$, key end: (c) $n = 2.95 \times 10^6$, keyway distant from the key end

Precautions were taken in machining to get keyways of accurate dimensions, especially with respect to the radius of curvature at the corner, r. To eliminate the error in cutting arising from the abrasion of the cutter, the radii of curvature at corners of the finishing cutter were carefully examined by cutting a keyway in a shaft of epoxy resin before and after the finishing cut of the keyways of the test pieces, and by measuring r precisely with a microscope. Little discrepancy was observed between the two values of r, as is shown in Fig. 5. But, in some instances, the radii of curvature at the adjacent corners of a keyway are somewhat different as shown in Table 1. For the test, however, it is not necessary to have a keyway with equal values of r. Moreover, this requirement results in machining difficulties in fabrication of the cutter.

A torsional fatigue test was made using test pieces thus prepared. The number of cycles when the micrograins in the plated copper begin to grow at the corner of the keyway is determined for a prescribed torque, T. In determining a value of n, the testing machine was stopped repeatedly near the presumed number of cycles, namely for every increase of $n = 0.1 \sim 0.15 \times 10^6$. The specimen was then examined for the appearance of grown grains using a microscope.

The results show that the stress at the corner of the keyway is concentrated at the key end and the initial grown grains always appear there. The occurring of grain growth begins perceptively later at the corner of the keyway most distant from the key end. Micrographs of grown grains showing the stress concentration at the key end are given in Fig. 6.

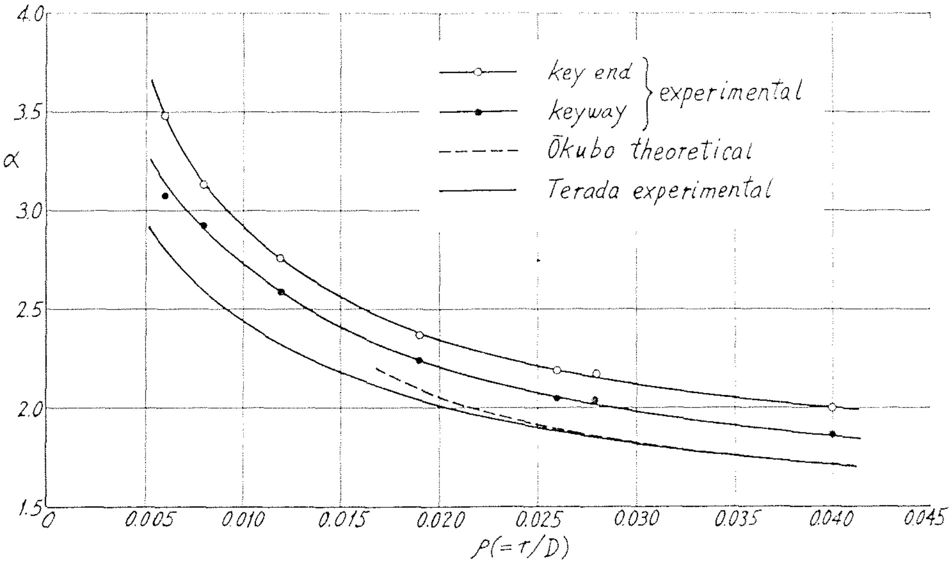


Fig. 7—Stress-concentration factor at the key end and the keyway distant from the key end, group A

When grown grains begin to appear at the key end after n cycles, the maximum shearing strain of the keyway attains the proper strain and the value of γ can be obtained from the calibration curve given in Fig. 2. Then the stress-concentration factor of the keyway is calculated from

$$\beta = \frac{\pi D^3 G \gamma}{16T} \tag{3}$$

where β is the stress-concentration factor based on the nominal stress of a shaft having the same diameter with no keyway but subjected to the same torque.

The stress-concentration factor may also be represented by the ratio of the maximum stresses in two shafts with and without a keyway, both twisted in the same angle. Since the torsional rigidity of the shaft with a keyway is less than that of the shaft without a keyway, the former shaft is submitted to a reduced torque in the ratio of the torsional rigidities of the two shafts. Denoting the ratio of the torsional rigidities by k, the stress-concentration factor described above becomes

$$\alpha = k\beta \tag{4}$$

The values of k obtained by statical torsion test are 0.91 for group A and 0.72 for group B.

Using the experimental results obtained, α and β are evaluated from eqs (4) and (3), respectively. They are given in Table 1, where α_m and β_m denote the mean values of α and β for the same r, respectively.

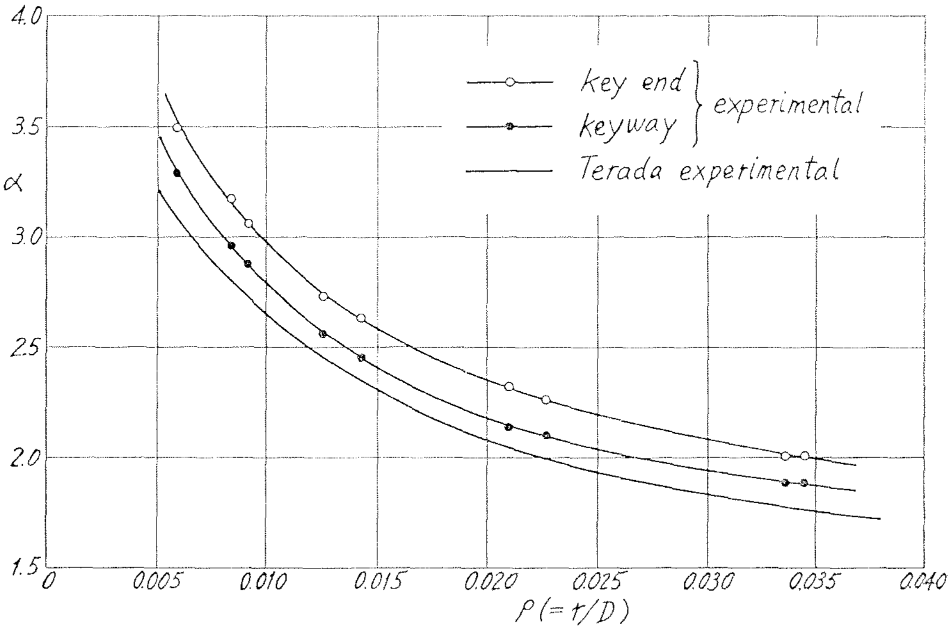


Fig. 8—Stress-concentration factor at the key end and the keyway distant from the key end, group B

TABLE 2—VALUES OF α/α_0

ρ (= r/D)		0.006	0.010	0.015	0.020	0.025	0.030	0.035	0.040
Group A	Key end	1.24	1.19	1.18	1.16	1.17	1.16	1.17	1.16
	Keyway	1.12	1.11	1.11	1.09	1.09	1.09	1.09	1.08
Group B	Key end	1.14	1.12	1.12	1.13	1.13	1.14	1.14	
	Keyway	1.07	1.05	1.04	1.05	1.05	1.06	1.07	

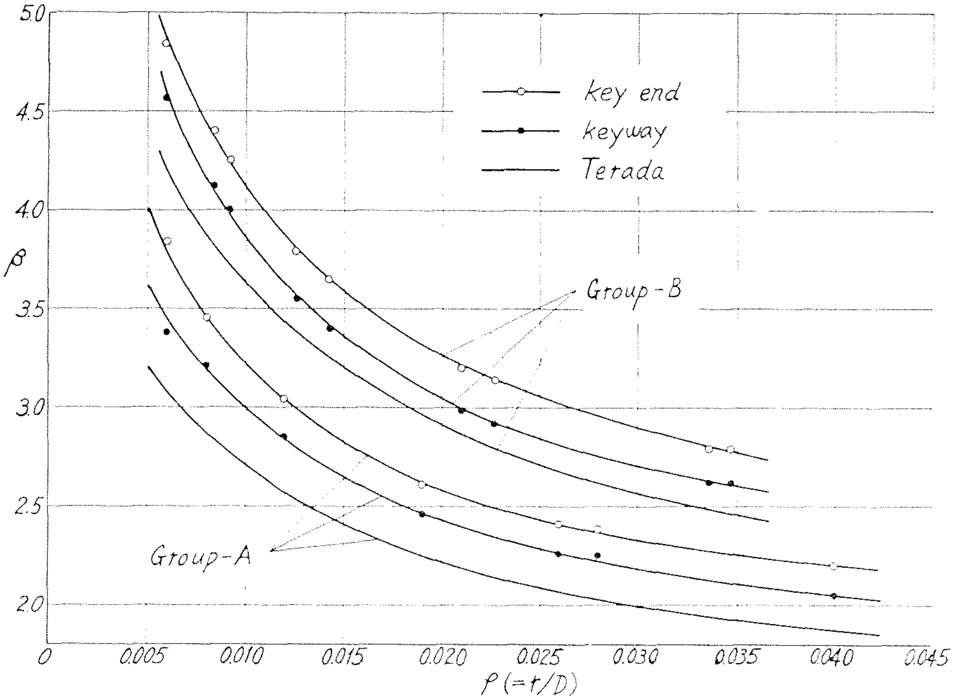


Fig. 9 — Stress-concentration factor β converted from α

Results and Conclusions

From the experimental results given in Table 1, the curves showing the relation between the stress-concentration factor and the radius of curvature at the corner of the keyway are obtained. They are shown in Figs. 7, 8 and 9. In the illustrations, some theoretical and experimental results of previous authors for the case of pure torsion are also shown for comparison.

As is shown in the figures, the effect of the key on the stress in the keyway is perceptible particularly at the key end. The ratio of α to α_0 is obtained from the curves in Figs. 7 and 8, where α_0 denotes the stress-concentration factor of the keyway for pure torsion. It is given in Table 2. For group A, the stress at the key end is 16 ~ 24 percent larger than that for pure torsion and 12 ~ 14 percent for group B, as is shown in the table. It is seen from Table 2 that the stress in the keyway at the point most distant from the key end is also affected by the key and is slightly larger than the stress for pure torsion.

The remarkable forte of the technique described is that the stress concentration occurring even in a minute area can be detected effectively. Recently the technique was applied with success to detect the peak points in an alpha-brass rod of 0.04-mm grain size.[11] The photoelastic analysis may be applied to the problem herein treated. The least thickness of a slice effectively used, however, is usually 2 ~ 3 mm and the peak stress can only be determined by extrapolation.

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