Effect of Residual Stress on Fatigue Fracture of Case-Hardened Steels—An Analytical Model


ABSTRACT: The influence of residual stress at and near the surface on fatigue life was modeled by utilizing linear elastic fracture mechanics to quantify the stress intensity change due to the residual stress. The initiation and the propagation stages of the fatigue cracking process were distinguished using the concept of the threshold stress intensity amplitude; with a constant load amplitude, the propagation stage commences from the crack length at which the maximum fatigue stress intensity is equal to the threshold value. The influence of residual stress is to increase or decrease the crack length which corresponds to the initiation stage, thereby controlling the fatigue crack initiation life.

KEY WORDS: fatigue, residual stress, crack initiation, threshold fatigue stress intensity, linear elastic fracture mechanics

Fatigue accounts for a significant portion of the failures of case-hardened components. It is known that in bend testing of unnotched steel specimens near the fatigue limit, most of the total fatigue life corresponds to the crack initiation stage [1]. Residual stress, ever present in a hardened case, has a marked influence on fatigue limit; the greater the compressive stress at and near the case-hardened surface, the higher the fatigue limit [2]. Therefore a prerequisite for compositions and heat treatments for case-hardened steels is that they produce compressive residual stress in the case.

Application of fracture mechanics in understanding the influence of residual stress on fatigue fracture has been limited to relatively large initial crack lengths and simple linear stress profiles [3,4]. Little effort has been devoted to analyzing the influence of residual stress on crack initiation. In the present paper, a fracture mechanics approach is presented by which the influence of residual stress on stress intensity is quantified. The importance of residual stress in affecting the crack initiation life is demonstrated, and, in the process, a definition of the term

1 Senior Research Engineer, Metallurgy Department, General Motors Research Laboratories, Warren, Mich. 48090; formerly at Climax Molybdenum Company of Michigan, a subsidiary of AMAX Inc., Ann Arbor, Mich. 48105.
2 Research Supervisor and Manager of Research, respectively, Climax Molybdenum Company of Michigan, a subsidiary of AMAX Inc., Ann Arbor, Mich. 48105.
3 The italic numbers in brackets refer to the list of references appended to this paper.
"crack initiation" is obtained by utilizing the concept of threshold stress intensity amplitude.

Influence of Residual Stress on Stress Intensity

Extension by fatigue of a crack having a finite length is determined by stress intensity amplitude. Therefore the influence of residual stress on fatigue can be quantified by analyzing the change in stress intensity due to residual stress. In this paper, the residual stress is assumed to act perpendicular to the surfaces formed by a crack, as was done previously [3,4]. It is further assumed that only the residual stress acting across the crack face can influence the stress intensity; residual stress present in the unbroken ligament is assumed to have no effect on the stress intensity. For an infinite body, it has been shown that a stress acting in the ligament contributes zero stress intensity at the crack tip [5]. Similarly, for a semi-infinite solid with a point force acting on the back face in the ligament side, the resulting stress intensity is zero [5].

The stress intensity caused by a residual stress field may be predicted by utilizing either the weight function given by Bückner [6] or the stress intensity formula for a point force acting on crack face [5]. The latter method was chosen in the present study; a residual stress field is represented as a contiguous group of point forces acting normal on the crack face. The stress intensity caused by a force acting on a point in the crack face of a single-edge-notched specimen is given [5] as

\[ K_I = \frac{2P}{\sqrt{\pi a}} F\left(\frac{c}{a}, \frac{a}{W}\right) \]  

where

\[ F\left(\frac{c}{a}, \frac{a}{W}\right) = \frac{3.52 \left(1 - \frac{c}{a}\right)}{\left(1 - \frac{a}{W}\right)^{3/2}} - \frac{4.35 - 5.28 \frac{c}{a}}{\left(1 - \frac{a}{W}\right)^{1/2}} \]

\[ + \left\{ \frac{1.30 - 0.30 \left(\frac{c}{a}\right)^{3/2}}{\sqrt{1 - \left(\frac{c}{a}\right)^2}} + 0.83 - 1.76 \frac{c}{a} \right\} \left\{ 1 - \left(1 - \frac{c}{a}\right) \frac{a}{W} \right\} \]

where \( P \), \( W \), \( a \), and \( c \) denote load, specimen width (thickness), crack length, and distance from the specimen face on the crack side to the position of the point force, respectively (Fig. 1). The load \( P \) has a dimension of force per unit length (in the breadth direction). The equation may be used to calculate stress intensity changes due to residual stresses of any profile by replacing \( P \) with \( \sigma_r(c) \times dc \) and integrating over the length of the crack, where \( \sigma_r(c) \) signifies the residual stress field expressed as a function of \( c \). Therefore
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$$K_I = \frac{2P}{\pi a} F\left(\frac{c}{a}, \frac{a}{W}\right)$$

$$F\left(\frac{c}{a}, \frac{a}{W}\right) = \frac{3.52 \left(1 - \frac{c}{a}\right)}{\left(1 - \frac{a}{W}\right)^{3/2}} - \frac{4.35 - 5.28 \frac{c}{a}}{\left(1 - \frac{a}{W}\right)^{1/2}} + \frac{1.30 - 0.30 \left(\frac{c}{a}\right)}{\sqrt{1 - \left(\frac{c}{a}\right)^2}} + 0.83 - 1.76 \frac{c}{a} \left| 1 - \left(1 - \frac{c}{a}\right) \frac{a}{W} \right|$$

FIG. 1 — Stress intensity formula for a single-edge-notched specimen with a point force acting on the crack face [5].

$$K_I = \int_0^a 2\sigma_r \frac{dc}{\sqrt{\pi a}} F\left(\frac{c}{a}, \frac{a}{W}\right) = \int_0^1 2 \sigma_r \frac{dc}{\pi a} F\left(\frac{c}{a}, \frac{a}{W}\right) d\left(\frac{c}{a}\right)$$

Since Eq 1 contains a term which has $1 - (c/a)^2$ in the denominator, the value of the function is infinite if $c = a$. However, this singularity can be eliminated, and the integral in Eq 2 converges, by expressing $c/a$ as sine or cosine of a new variable. In most cases the integration is best carried out numerically with a computer.

In the following sections, the foregoing computing procedure was applied to analyze the influence of surface residual stress on crack initiation and the influence on fatigue crack propagation of the position of the residual stress peak along the face of the crack.

Crack Initiation and Surface Residual Stress

Application of fracture mechanics to fatigue fracture has been limited only to the propagation of a pre-existing crack, and efforts made in analyzing the crack initiation stage have been relatively meager. Crack initiation is usually reserved for the mechanism by which a crack forms to some finite length, but it could also be defined as that portion of the fatigue process not involved in crack propagation. A crack of a finite length may be present, but if the fatigue stress intensity amplitude is less than the threshold stress intensity amplitude, defined as the
value below which no crack propagation occurs, then the system may be considered to be in the crack initiation stage. In this study, crack initiation is defined as that portion of the fatigue process where the stress intensity amplitude is less than the threshold stress intensity amplitude.

Consider a fatigue crack propagation process that obeys Paris’s relationship, 
\[ \frac{da}{dN} = A (\Delta K)^m \]. Furthermore, limit the consideration to bend testing of steels for which \( \Delta K_t \), the stress intensity amplitude, may be replaced with \( K_{\text{max}} \). Then, the influence of compressive residual stress is to decrease \( K_{\text{max}} \) by the amount predicted from Eq 2. It is known that a threshold value of stress intensity amplitude, \( \Delta K_{\text{th}} \) or \( K_{\text{th}} \), exists, below which a crack cannot propagate. Threshold values for steels are about 3 to 8 MPa√m (3 to 7 ksi√in.) [7]. In the present study, \( \Delta K_{\text{th}} \) will be assumed to have a value of 4.4 MPa√m (4.0 ksi√in.).

The residual stress profile used to study the influence of surface stress on crack initiation is shown in the upper left corner of Fig. 2; the maximum stress is at the very surface and decreases linearly to zero at a depth of 0.25 mm (0.010 in.). The specimen was assigned to have a 7.0 mm (0.276 in.) thickness, a breadth of 20 mm (0.787 in.), a span-to-thickness ratio of 8, and to be tested in three-point bending in an unnotched condition. Two values were assigned for the surface residual stress, -138 and -276 MPa (-20 and -40 ksi). The stress intensity due to residual stress is plotted as a function of crack length in Fig. 2. Also shown is the maximum stress intensity for various crack lengths caused by an externally applied fatigue load with a constant load-amplitude of 454 kgf (1000 lbf). The stress intensity values for both the externally applied load and the compressive residual stress are plotted as positive values. Therefore the effective or net stress intensity is calculated by subtracting the residual stress contribution from the stress intensity due to the external loading. Unless the applied stress intensity overcomes the stress intensity due to the compressive residual stress, no crack propagation is expected to occur. In fact, the crack will not propagate until the net stress intensity exceeds the threshold stress intensity.

It is observed in Fig. 2 that only at crack lengths greater than about 0.089 mm (0.0035 in.) does the net stress intensity exceed the threshold stress intensity (\( \Delta K_{\text{th}} \)) of 4.4 MPa√m (4 ksi√in.) when the surface compressive residual stress is 138 MPa (20 ksi). If the surface residual stress is -276 MPa (-40 ksi), this critical crack length is 0.178 mm (0.007 in.). Without any residual stress, the crack length only has to reach 0.036 mm (0.0014 in.) for the stress intensity to exceed \( \Delta K_{\text{th}} \). The actual mechanism by which flaws or cracks reach these specific critical lengths pertains to the process of crack initiation and is not discussed in the present paper.

Figure 3 is the same as Fig. 2 except that an externally applied load of 907 kgf (2000 lbf) was also considered and the surface residual stress values ranged from -138 to -689 MPa (-20 to -100 ksi) in increments of 138 MPa (20 ksi). The 907 kgf (2000 lbf) external force corresponds to an outer fiber stress of 779 MPa (113 ksi), which closely approximates typical endurance limits for case-hardened steels [8]. For the combination of a 454 kgf (1000 lbf) external force and surface...
compressive stresses of 414, 552, and 689 MPa (60, 80, and 100 ksi), the critical crack lengths corresponding to the initiation stage are 0.23, 0.25, and 0.30 mm (0.009, 0.010, and 0.012 in.), respectively. If the externally applied load is increased, the critical crack length for each residual stress condition decreases. For example, critical crack length for the 907 kgf (2000 lbf) external load and \(-689\) MPa \((-100\) ksi\) surface residual stress is 0.15 mm (0.006 in.) compared with 0.30 mm (0.012 in.) for the 454 kgf (1000 lbf) load.

**Influence of the Position of Residual Stress Peak on Fatigue Crack Propagation**

The discussion made in conjunction with stress profiles shown in Figs. 2 and 3 demonstrates the importance of the surface residual stress on fatigue, but does not consider stress peaks occurring further away from the surface, as often observed in actual carburized cases. Therefore it is of interest to find out which part of the stress field (for example, the part far away from the crack tip or that part very near the crack tip) gives rise to a greater change in stress intensity. To analyze this problem, the contribution to stress intensity by residual stress with a pyramidal profile was computed for various peak positions. The base length of the pyramid profile was fixed at 0.254 mm (0.010 in.). The peak compressive residual stress could be assigned any value, but for this specific analysis was fixed at \(-345\) MPa \((-50\) ksi\). It is seen in Fig. 4 that as the residual stress peak is
shifted away from the surface, its contribution to the stress intensity is also shifted. The maximum contribution to stress intensity always occurs when cracks are slightly beyond the peak location of the residual stress profile. Figure 5 is a plot of the maximum stress intensity due to the residual stress peak analyzed in Fig. 4 as the peak is shifted away from the surface. The values remain fairly constant except for a slight increase when the profile and crack are close to the surface. Figure 4 shows that cracks must propagate into the compressive residual stress field before any effect of the residual stress is realized. Also included in Fig. 4 is the stress intensity curve for a constant applied load of 454 kgf (1000 lbf). Because the stress intensity due to the external force increases with crack length, the further the residual stress field is from the surface, the less will be the relative contribution from the residual stress to the net stress intensity.

**Fatigue Crack Propagation in Carburized Cases**

The stress intensity concept and analytical method explained so far can be used to analyze the fatigue fracture process in a typical residual stress field found in
carburized cases. The assumed typical residual stress field and its contribution to stress intensity as a function of crack length are shown in Fig. 6. The residual stress field has a maximum compressive stress of 345 MPa (50 ksi) at a 1.0 mm
FIG. 6—Stress intensity change due to a residual stress field typical in carburized steels.

(0.40 in.) depth. The surface residual stress was assumed to be $-138$ MPa ($-20$ ksi), and the same value persists to a 0.25 mm (0.010 in.) depth. Also included in Fig. 6 is a stress intensity curve for an external force of 725 kgf (1600 lbf), which corresponds to a 625 MPa (90 ksi) outer fiber stress. Analogous to the treatment in Figs. 2 and 3, it is the net stress intensity between the applied stress intensity and the residual stress intensity that affects fatigue crack propagation. Only if this net value exceeds the threshold intensity will fatigue propagation occur. Figure 6 shows that crack 0.25 mm (0.010 in.) in length must exist for $\Delta K$ to exceed $\Delta K_{th}$ under an applied load of 725 kgf (1600 lbf). Once the crack length exceeds this critical value, $a_{cr}$, the crack will extend to a depth of 1.0 mm (0.039 in.) by the propagation mechanism. At this location, the net stress intensity falls below the threshold value and continues to decrease as the crack length increases. If the applied load never exceeds 725 kgf (1600 lbf), the crack will arrest at this location.
Discussion

The application of fracture mechanics to the fatigue fracture of case-hardened steels with compressive residual stresses in the case has led to a quantitative definition of crack initiation. Crack initiation of case-hardened steels is defined as the process by which a crack of critical length, $a_{cr}$, is formed. This critical length not only depends on the applied load, but also on the residual stress profile in the case. For applied loads of 454 and 907 kgf (1000 and 2000 lbf) and a residual stress field with a 138 MPa (20 ksi) compressive residual stress at surface, which linearly decreases to zero at a 0.25 mm (0.010 in.) depth (Fig. 3), the critical crack lengths that correspond to the crack initiation stage are 0.08 and 0.005 mm (0.003 and 0.0002 in.), respectively.

For the residual stress profiles shown in Figs. 2 and 3, one can determine the combination of applied load, surface compressive residual stress, and critical crack length necessary for fatigue crack propagation in a carburized case. For any combination which results in a net stress intensity amplitude exceeding $\Delta K_{th}$, assumed to be about 4.4 MPa $\sqrt{m}$ (4 ksi $\sqrt{in.}$), crack propagation will occur.

Oxidized grain boundaries in carburized cases have long been believed to contribute to reduced fatigue lives. The depth of surface oxidation can easily exceed the critical length of crack initiation, especially if the surface compressive residual stress is 138 MPa (20 ksi) or less. A typical depth of surface oxidation is 0.013 mm (0.0005 in.). The fatigue load required to propagate a 0.013-mm (0.0005-in.)-long crack in the compressive residual stress field at the surface of 138 MPa (20 ksi) is 860 kgf (1900 lbf). The outer fiber stress corresponding to this load is 738 MPa (107 ksi), which incidentally is typical of value of fatigue limit for a case-hardened steel. Fatigue limits for carburized steels have been reported to be in the range of 689 to 827 MPa (100 to 120 ksi) [8]. A steel with less surface oxidation should have a higher fatigue limit, provided the compressive residual stress at the surface remains constant. Likewise, a steel having a high surface compressive residual stress at the surface can tolerate a greater depth of surface oxidation. Unfortunately, steels having excessive oxidation also tend to form nonmartensitic microstructures at the surface, which in turn decreases compressive residual stresses [9].

Shot-peening treatments have been reported to produce surface compressive residual stresses in excess of 690 MPa (100 ksi) [8]. The effect of such treatments can easily compensate for the detrimental influences of surface oxidation and other surface flaws. In fact, a fatigue load of 1588 kgf (3500 lbf), corresponding to an outer fiber stress of 1363 MPa (198 ksi), would be required to propagate a crack in a shot-peened specimen with a 550 MPa (80 ksi) surface compressive stress and a surface crack or flaw 0.013 mm (0.0005 in.) in length.

Figure 6 illustrates the stress intensity of cracks extending into a compressive residual stress profile that approximates the shape typical in carburized cases. For the conditions depicted, the net stress intensity amplitude decreases as the crack propagates from a length of 0.76 mm (0.03 in.) to a depth of 1.0 to 1.3 mm (0.04 to 0.05 in.). If the applied fatigue load is low enough, the crack will actually arrest and not propagate again until the applied load is increased.
At the present time, there are no definitions of the initiation and propagation stages in a fatigue fracture process for which there is universal agreement. Socie et al have proposed a definition for crack initiation: a fatigue crack is initiated when the fatigue damage due to propagation mechanisms exceeds that due to crack initiation or strain cycle fatigue mechanisms [10]. Low-cycle fatigue concepts and linear elastic fracture mechanics were used to describe crack initiation and propagation stages, respectively. However, for high-carbon, high-hardness, low-toughness materials such as carburized case, the usefulness of the concept of a low-cycle strain-controlled test is doubtful. Such materials do not exhibit an appreciable degree of plasticity in a tension test, and in the fracture toughness test, the plastic zone at the crack tip may be as small as one micron in radius, many orders of magnitude smaller than typical plastic zone sizes found in tough materials. Therefore, defining the initiation and propagation stages of fatigue fracture in terms of the threshold stress intensity amplitude seems to be a reasonable approach to the fatigue fracture of case-hardened steels. In a sense, Socie’s and the present authors’ definitions of crack initiation may be considered as complementary to each other by providing a means for treating crack initiation in low-strength high-toughness materials and high-strength low-toughness materials, respectively.

The influence of tensile residual stress was not analyzed in the present study. For materials which obey the Paris fatigue equation, crack propagation rate is not affected by tensile residual stress; this is because only the mean stress intensity is raised and not the stress intensity amplitude. It is known, however, that $\Delta K_{th}$ can decrease by as much as a factor of 2.5 compared with the residual stress-free condition [7]. If such is the case, tensile residual stress is expected to have an effect on the crack initiation life.

It is clearly a difficult task to develop a theory that can describe the fatigue fracture process quantitatively. The authors present this paper as a demonstration of the usefulness of linear elastic fracture mechanics and empirically derived fatigue laws in explaining the influence of residual stress on crack initiation in high-strength low-toughness materials.

Summary

The crack initiation stage of fatigue of carburized steels was defined as the process of formation of a crack whose length is such that the fatigue stress intensity amplitude is equal to the threshold stress intensity amplitude. The process of fatigue crack extension that follows the initiation stage was defined as the propagation stage. The critical initiation crack length depends not only on applied load but also on residual stress. Examples of the effect of various residual stress profiles on the fatigue stress intensity amplitude were given. It was shown that a compressive residual stress at the surface has a greater influence on the critical crack length than a similar residual stress profile existing further inside the case. An analysis of a residual stress profile typical of a carburized case showed
that even after a crack begins to propagate, the stress intensity amplitude may
decrease, and at relatively low stress levels this can cause the crack to be arrested.

Carburized cases having surface cracks corresponding in length to the depth of
grain boundary oxidation were analyzed. It was found that applied stresses
required for crack propagation were similar in magnitude to measured fatigue
limits of carburized steels. Steels having greater compressive residual stresses at
the surface require greater applied loads for the crack to extend by the propagation
mechanism. The influence of surface oxidation on fatigue depends on the depth
of the oxidized boundaries as well as the residual stress at the surface. For
example, shot-peened specimens having a high compressive residual stress
should have a high tolerance for surface oxidation. The high fatigue limit of
shot-peened specimens can be explained by the present model, according to
which a compressive residual stress field near the surface increases the critical
crack size required for propagation.

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