An X-ray Diffraction Study of the Residual Stress-Strain Distributions in Shot-peened Two-phase Brass

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ABSTRACT

X-ray residual stress determination and line-broadening analysis were used to study the deformation distributions existing in the surface layers of a shot-peened two-phase brass specimen. The results indicate inhomogeneous partitioning of the plastic strains between the α and β grains, as well as between the surface layers and the bulk. The residual microstresses, which occur because of the differential deformation between the α and β grains, have a maximum in the β grains at the material surface, while the residual macrostresses, which arise because of the differential plastic deformation between the surface and the bulk, exhibit a maximum deeper in the material. A qualitative model (describing these residual stress profiles in terms of the plastic strains caused by the peening operation) is presented.

1. INTRODUCTION

Shot-peening is an industrially important manufacturing process in which the surface of materials is plastically deformed by a stream of high velocity shot [1, 2]. This process usually introduces beneficial compressive residual stresses in the surface layers since the bulk of the material, which suffers no plastic deformation, elastically constrains the surface layers. The compressive residual stresses in the surface may significantly increase the service life of the treated part if the appropriate peening parameters that control the plastic deformation, and hence the residual stress distribution, are employed during the operation [1-4]. These parameters control the magnitude and location of the maximum compressive residual stress with depth in the plastically deformed surface layers [1, 2]. Experiments have shown that the maximum compressive stress usually occurs at the surface of the workpiece if the material is soft. This is so since the maximum elongation occurs at the surface and decreases with increasing depth into the material [2]. For hard specimens peened at high (shot) intensities, however, or for soft materials peened at very high intensities, the position of the maximum compressive residual stresses moves into the material [2]. In these cases it has been suggested that the maximum plastic elongation, and thus the maximum residual stress, occurs below the surface since the Hertzian pressure distribution existing under the shot at the moment of impact predicts a maximum shear stress at a point below the surface [2]. These analyses assume a homogeneous single-phase workpiece and, similarly, the residual stress values reported have been determined from the matrix phase only, even when a multiphase material was examined. Such treatment, however, may not be appropriate for multiphase materials, where the yield points of the phases are different. In such cases there may be two sources of residual stress: macrostresses exerted by the deformation-free regions on the surface layers, and microstresses set up because of the unequal partitioning of plastic deformation between the phases [5-7]. These stresses will also change with depth (since the total plastic deformation is a function of depth) and will exhibit a maximum where peening causes the maximum differential plastic deformation between the phases. To date, no experimental study of the variations in these different stress components with depth has been made. Such an analysis will also yield information on the distribution of...
the plastic deformation between the surface layers and the bulk as well as about the partitioning of plastic deformation between the different phases and may be used to check the models that describe the plastic deformation distribution with depth in peened materials.

In this paper, X-ray techniques sensitive to elastic and plastic deformation are utilized in an examination of the deformation distributions existing in a two-phase brass alloy peened with high intensity steel shot. Interplanar spacing \( d \) versus \( \sin^2 \psi \) data (where \( \psi \) is the tilt of the specimen from the normal focusing condition on a diffractometer and is also defined in Fig. 1) measured for both phases is analyzed to yield the residual macrostress-strain and microstress-strain distributions in the respective phases [6]. As the specimen was electropolished to expose new depths to the X-ray beam after each set of measurements (from both the \( \alpha \) and the \( \beta \) phases), the analysis also yields the variation in the macrostresses and average residual microstresses in both phases as functions of depth. In addition, data obtained from line-broadening experiments, variation in peak breadth at half the maximum intensity etc. is used to determine qualitatively the variation in plastic deformation with depth. These results are then utilized to form a new model which qualitatively relates the formation of residual stresses in soft materials peened with (high intensity) hard shot to the distribution of plastic strains with depth.

2. THEORY

2.1. X-ray stress determination

The coordinate systems used in the following discussion are shown in Fig. 1. The steel pellets used in shot peening impinged normal to the surface, causing plastic flow along the normal direction \( P_3 \), and transverse directions \( P_1 \) and \( P_2 \). The plastic strains in the transverse directions cause a permanent elongation of the surface layers along these directions, which is constrained by the undeformed bulk material. Such constraint can be represented as an equivalent force along the specimen boundary and causes compressive residual macrostresses \( \sigma_{11}^m \) and \( \sigma_{22}^m \) in the surface, which are balanced by tensile residual macrostresses in the bulk. Furthermore, in a two-phase material where the phases \( \alpha \) and \( \beta \) have different yield points, the plastic strains \( \epsilon_{ij}^p \) will be different in the grains of different phases, causing mutual constraint between these volumes [6, 7]. The forces due to this mutual constraint cause a microstress field to be set up between the grains of the different phases. These stresses will be balanced between the \( \alpha \) and \( \beta \) grains and will not exist in the bulk layers which have suffered no plastic deformation [6].

For such a stress state* the average lattice parameter measured by X-rays from a given phase along the direction \( L_{\phi \psi} \) (Fig. 1) can be written as [6, 8]

\[
\frac{(d_{\phi \psi})_i - (d_0)_i}{(d_0)_i} = \left( \frac{1 + \nu}{E} \right) \left\{ (\sigma_{11}^m + \langle \sigma_{11}^\text{pm} \rangle)_i \cos^2 \phi + \right.
\]

\[
+ (\sigma_{22}^m + \langle \sigma_{22}^\text{pm} \rangle)_i \sin^2 \phi - \left( \sigma_{33}^\text{pm} \right)_i \sin^2 \psi + \left( \frac{1 + \nu}{E} \right) (\sigma_{33}^\text{pm})_i - \right.
\]

\[
- \left( \frac{\nu}{E} \right) \left\{ (\sigma_{11}^m + \langle \sigma_{11}^\text{pm} \rangle)_i + \sigma_{22}^m + \right.
\]

\[
+ (\sigma_{22}^\text{pm})_i + (\sigma_{33}^\text{pm})_i \right\} \right. \right]
\]

*The residual stress state occurring in response to the deformation caused by shot peening has been discussed extensively in the literature. The microstress components \( \langle \sigma_{13}^\text{pm} \rangle \) and \( \langle \sigma_{23}^\text{pm} \rangle \) of the stress tensor are zero since the plastic strains \( \epsilon_{13}^p \) and \( \epsilon_{23}^p \) are zero. The term \( \sigma_{33}^m \) is zero by definition to satisfy the equations of force equilibrium at the surface.
Here \( E_i \) and \( \nu_i \) are the elastic constants of the particular phase, \((d_0)_i\) is the unstressed plane spacing for this phase and \((\sigma_{ij}^{pm})_i\) are the average microstress components, which are also termed residual pseudo-macrostresses. Thus, the slope \( m \) of an experimentally determined \( d \) versus \( \sin^2 \psi \) plot is [6]

\[
(m_{\phi=0})_i = \left( \frac{1 + \nu}{E_i} \right) \{\sigma_{11}^m + (\sigma_{11}^{pm})_i\} - \{\sigma_{33}^{pm}\}_i \ (d_0)_i
\]

for \( \phi = 0^\circ \), and

\[
(m_{\phi=90})_i = \left( \frac{1 + \nu}{E_i} \right) \{\sigma_{22}^m + (\sigma_{22}^{pm})_i\} - \{\sigma_{33}^{pm}\}_i \ (d_0)_i
\]

for \( \phi = 90^\circ \). The intercept \( I \) of the lines for both values is invariant and equal to [6, 8, 9]

\[
I = \frac{(1 + \nu)(\sigma_{33}^{pm})_i - \{\sigma_{11}^m\} + \{\sigma_{22}^{pm}\}_i + \{\sigma_{33}^{pm}\}_i}{(d_0)_i}
\]

Thus eqns. (2a) and (2b) may be solved simultaneously to obtain the terms \(\sigma_{11}^m + (\sigma_{11}^{pm})_i\), \(\sigma_{22}^m + (\sigma_{22}^{pm})_i\), and \(\sigma_{33}^{pm}\)_i existing in a given phase. This procedure, however, requires a very precise value of \((d_0)_i\) [9, 10] that is accurate to the fifth decimal place. Less accurate values will cause a large error in the intercept term (2c) and thus large errors in the final stress values [9, 10].

If \( d \) versus \( \sin^2 \psi \) data from both phases are available, it is possible to separate the macrostress components from the pseudo-macrostress terms for the normal stresses along \( P_1 \) and \( P_2 \). For each phase, eqns. (2) yield the stress values

\[
(\sigma_{11}^m)_\alpha = \sigma_{11}^m + (\sigma_{11}^{pm})_\alpha
\]

\[
(\sigma_{11}^m)_\beta = \sigma_{11}^m + (\sigma_{11}^{pm})_\beta
\]

It can be shown that \((\sigma_{11}^{pm})_\alpha\) and \((\sigma_{11}^{pm})_\beta\) obey the equilibrium equation [3, 4]

\[
(1 - f) (\sigma_{11}^{pm})_\alpha + f (\sigma_{11}^{pm})_\beta = 0
\]

where \( f \) is the volume fraction of \( \beta \). Thus, from eqns. (3) and (4) the macrostress \(\sigma_{11}^m\) and the pseudo-macrostress terms \((\sigma_{11}^{pm})_\alpha\) and \((\sigma_{11}^{pm})_\beta\) can be determined.

Determination of the macrostresses \(\sigma_{11}^m\) and \(\sigma_{22}^m\) is possible with this scheme even when exact values of \((d_0)_i\) for either phase are not available [3]. For both phases, eqn. (2a) can be rewritten as

\[
(m_{\phi=0})_i = \left( \frac{1 + \nu}{E_i} \right) \{\sigma_{11}^m + (\sigma_{11}^{pm})_i\} - \{\sigma_{33}^{pm}\}_i \ (d_0)_i
\]

The total pseudo-macrostress term \((\sigma_{11}^{pm})_i - (\sigma_{33}^{pm})_i\) in eqn. (5) must also obey eqn. (4):

\[
(1 - f) (\sigma_{11}^{pm})_\alpha + f (\sigma_{11}^{pm})_\beta = 0
\]

Thus, \(\sigma_{11}^m\) and the total pseudo-macrostress \((\sigma_{11}^{pm})_i - (\sigma_{33}^{pm})_i\) for both phases can be obtained from eqns. (5) and (6). Similarly, eqns. (2b) and (6) may be used to determine \(\sigma_{22}^m\).

2.2. Line-broadening analysis

In the above procedure the peak positions of the X-ray lines were used to determine the elastic deformation existing in the irradiated volume after plastic flow. Plastic deformation also affects the shape of the X-ray lines and causes line broadening [11]. There are two major contributions to line broadening: (1) the particle size effect, which may be due to a change in the size of the coherently diffracting domains during plastic deformation (due to the change in the spacing between dislocations, the formation of small-angle boundaries etc.) and/or a change in the density of planar stacking faults and (2) the distortion effect, which is caused by the local microstrains that vary from point to point in the diffracting volume.

The most general and quantitative method of treating X-ray line broadening is that described by Warren and Averbach [12] and Warren [11]. In this procedure the X-ray peak is expressed as a Fourier series, the cosine coefficients of which are related to the microstructural changes described above. In order to obtain the true broadening effects due to deformation, the instrumental broadening is removed by analyzing the corresponding peaks of both the deformed sample and an annealed sample and applying the correction given by Stokes [13]. The corrected cosine coefficients of the X-ray peak can then be expressed as

\[
\ln A_L = \ln A_L^s - \frac{2\pi^2h_0^2L^2(\epsilon L)^2}{a_0^2}
\]

where \( A_L \) is the X-ray peak intensity, \( A_L^s \) is the corrected X-ray peak intensity, \( h_0 \) is the X-ray wavelength, \( L \) is the thickness of the sample, \( \epsilon \) is the mean X-ray strain, and \( a_0 \) is the lattice constant of the unstrained material.
where $A_L$ is the Stokes corrected cosine coefficient of the X-ray peak, $A_L^s$ is the particle size coefficient, $h_o^2 = h^2 + k^2 + l^2$, $a_0$ is the true lattice parameter and $\langle \varepsilon_L^2 \rangle$ is the mean squared strain normal to the reflecting planes, averaged over all the diffracting regions of length $L$ (normal to the surface). Thus, if the corrected cosine coefficients of at least two orders of a peak can be obtained, in a plot of ln $A_L$ versus $h_o^2$ the slope $M$ of the line for each $L$ is given by [11, 12]

$$M = \frac{2\pi^2 L^2 \langle \varepsilon_L^2 \rangle}{a_0^2}$$  

from which the variance of the microstrains $\langle \varepsilon_L^2 \rangle$ in the measurement volume of length $L$ can be obtained. The intercepts of the ln $A_L$ versus $h_o^2$ lines at $h_o^2 = 0$ give the particle size coefficients (ln $A_L^s$) for that $L$. The average coherently diffracting length $D_{\text{eff}}$ normal to the diffracting planes can then be obtained from [11, 12]

$$\left( \frac{\partial A_L^s}{\partial L} \right)_{L \to 0} = \frac{1}{D_{\text{eff}}(hkl)}$$

2.3. Determination of microtwin and stacking fault densities

These parameters are also indicative of the plastic deformation suffered by the irradiated layers and were determined only for the $\alpha$ phase, since Rothman and Cohen [14] have reported negligible densities for both types of fault for $\beta$ brass.

The twin-fault probabilities were determined using a modified version of the procedure described by Cohen and Wagner [15]:

$$2\theta_{200} - \text{CG}(2\theta_{200}) = \Delta\{\text{CG}(200)\} = 14.6\beta_{\text{MT}} \tan \theta_{200}$$

where $\text{CG}(2\theta_{200})$ is the center of gravity of the 200 peak and $2\theta_{200}$ is the peak position. The term $\beta_{\text{MT}}$ is the twin-fault probability.

The stacking fault density $\alpha_{\text{SF}}$ can be determined using a modified form of the equation given by Mikkola and Cohen [16] for the peak shift, for a given $hkl$ peak, between an annealed sample and a deformed sample. This formula relates the peak shift to the various parameters that cause it:

$$(2\theta)_{\text{def}} - (2\theta)_{\text{ann}} = J_1 \alpha_{\text{SF}} \tan \theta + J_2 \alpha_{\text{SF}} \varepsilon_1 \tan \theta + J_5 \sigma_1 \tan \theta + J_6 \frac{\Delta a}{a} \tan \theta$$

where the $J$ values are constants for a given $hkl$ and are given in ref. 16, $\varepsilon_1$ is the spacing change between layers at the stacking fault, $\sigma_1$ is the first stress invariant ($\sigma_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$) and $\Delta a/a$ is the change in lattice parameter.

Since, with Cr $K\alpha$ radiation, only three peaks could be obtained, eqn. (11) was rewritten as

$$\{\Delta(2\theta)\}_{hkl} = J_1 \alpha_{\text{SF}} \tan \theta + J_2 \alpha_{\text{SF}} \varepsilon_1 \tan \theta + \frac{360}{\pi} \delta$$

where $\delta = K_e (\sigma_1 + \Delta a/a)$ and $K_e = \text{constant}$ for all $hkl$. Thus, the parameters $\alpha_{\text{SF}}, \varepsilon_1$ and $\delta$ for a given sample can then be determined from eqn.(12), when the peak positions of three reflections are available from the deformed sample and an annealed sample.

3. EXPERIMENTAL PROCEDURE

3.1. Specimen preparation

The starting material used in this study is commercially available Muntz Metal (60-40 brass), which was supplied in the form of a cold-rolled plate. The nominal composition of the alloy is 60 wt.% Cu and 40 wt.% Zn, with the permissible copper content between 59 wt.% and 63 wt.% and containing 0.30 wt.% Pb and 0.07 wt.% Fe maximum as allowable impurities. Standard fatigue specimens (Fig. 2) were machined from the plate and then annealed for 6 h at 498 K to relieve the residual stresses due to rolling and machining. After annealing, the residual stress was less than 15 MPa at the surface. The microstructure at this point was heavily textured and consisted of two phases $\alpha$ and $\beta$. The average grain size was 40 $\mu$m. The volume fraction of $\beta$, obtained from repeated applications of a square grid on the photograph and counting the number of points falling within the grains, is 0.22 (this corresponds to a copper content of 59.6 wt.% from the equilibrium phase diagram).
The annealed specimens were then shot peened on all four sides of the gauge section (the shaded area in Fig. 2) to an Almen A arc height of 0.46 mm with M1460 steel shot. Surface topography after peening was examined by scanning electron microscopy. No cracking was observed and the deformation features were uniformly distributed, except near the edges where some bulging (flow of material out of the surface) was observed. The average depth of oscillations in the surface, as measured by a calibrated optical metallographic microscope, was 6 μm. The standard sample to be used in the line-broadening analysis to correct for the instrumental broadening was annealed after peening at 713 K for 2 h and then oven cooled to ensure complete recrystallization.

Fig. 2. Nominal specimen dimensions (in millimeters).

### 3.2. Electropolishing procedure

Residual stresses and line profiles were measured at the shot-peened surface, and then the specimen surface was electropolished to expose deeper layers to the X-ray beam. All the specimen, except the region in the gauge section where electropolishing was desired (1.8 cm²), was masked (with Microstop) from the polishing solution as this minimized polishing time and stress relaxation due to material removal. The following electropolishing conditions were selected so that preferential etching of the phases was minimized [17]: solution composition, 60 vol.% of phosphoric acid and 40 vol.% of H₂O; cathode material, stainless steel; current density, 2.2 A cm⁻²; voltage, 2–4 V (variable); bath temperature, 298 K.

The bath was agitated with an electromagnetic stirrer and the specimen was agitated by hand. After removal from the bath, the specimen was rinsed in distilled water and then alcohol. Then the thickness removed was measured with a micrometer, after which the X-ray measurements were performed.

### 3.3. Line-broadening experiments

In this set of experiments a General Electric diffractometer equipped with a scintillation counter and a pulse height analyzer set for 90% acceptance of iron radiation was used. The pertinent instrument parameters are shown in Table 1. Alignment of the center of the diffracting volume over the center of the diffractometer was accomplished through the use of the Nelson–Riley analysis [18] for the peak positions of the 111, 220 and 311 peaks from the α phase. In all cases this displacement was less than 0.07 mm.

Two orders of the 110 peak (110 and 220) from the β phase and of the 111 peak from the α phase (111 and 222) were recorded for line-broadening analysis. Intensity versus 2θ

<table>
<thead>
<tr>
<th>Instrumental parameters for the line-broadening studies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tube</strong></td>
</tr>
<tr>
<td>Operating voltage and current</td>
</tr>
<tr>
<td>40 kV, 8 mA</td>
</tr>
<tr>
<td>Incident beam</td>
</tr>
<tr>
<td>3.6° horizontal divergence, vertical and horizontal beam limiter. No soller slits</td>
</tr>
<tr>
<td>Beam size</td>
</tr>
<tr>
<td>1.5 cm × 0.4 cm on the specimen</td>
</tr>
<tr>
<td>Diffracted beam</td>
</tr>
<tr>
<td>0.2 receiving slit with Kβ filter</td>
</tr>
</tbody>
</table>
TABLE 2
Measurement range, observed intensities and measurement parameters for the line-broadening studies

<table>
<thead>
<tr>
<th>$2\theta$ range (deg)</th>
<th>$2\theta$ step size (deg)</th>
<th>Counting time (s)</th>
<th>hkl</th>
<th>Peak intensity (counts s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>51-58</td>
<td>1</td>
<td>100-200</td>
<td>111</td>
<td>742</td>
</tr>
<tr>
<td>58-66</td>
<td>2</td>
<td>100-200</td>
<td>110</td>
<td>286</td>
</tr>
<tr>
<td>122-140</td>
<td>2</td>
<td>400</td>
<td>220</td>
<td>226</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average background</td>
<td>200</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3. The 111-110 doublet measured from a shot-peened 60-40 brass sample. The 111 peak is from the $\alpha$ phase; the 110 peak is from the $\beta$ phase. (Iron radiation was used in the measurement.)

3.4. Microtwin and stacking fault density determination
The peak positions of the 111, 200 and 220 peaks, measured on the Picker diffractometer with the same instrumental parameters described below for the residual stress measurements, were used to obtain $\alpha_{SF}$, $\alpha_{SF}\epsilon_{i}$ and $\delta$. All these peaks were determined by fitting a five-point parabola to the top 15% of the peak, with alignment of the sample over the diffractometer center closer than 0.003 mm. The counting times were chosen so that the standard deviation due to counting statistics was (for any peak) less than 0.01° ($2\theta$).

The 200 peak profiles for the microtwin density determination were measured on the General Electric diffractometer. The instrument parameters for this case are also shown in Tables 1 and 2.

3.5. Residual stress measurements
These measurements were performed on a minicomputer-controlled Picker diffractometer equipped with a chromium tube and a
TABLE 3
Instrumental parameters for the residual stress measurements

<table>
<thead>
<tr>
<th>Tube</th>
<th>Operating voltage and current</th>
<th>Incident beam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cr 50 kV, 20 mA</td>
<td>1° divergent slit for the α phase and 2° divergent slit for the β phase.</td>
</tr>
<tr>
<td>Beam size</td>
<td>Vertical and horizontal beam limiter; no soller slits</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.173 cm x 0.254 cm for the α phase; 0.173 cm x 0.508 cm for the β phase.</td>
<td></td>
</tr>
<tr>
<td>Diffracted beam</td>
<td>Receiving slits of 0.2° and 0.4° were used for the α phase and the β phase respectively. The slit box also carried the Kβ filter</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 4
Typical measurement parameters for the residual stress determination

<table>
<thead>
<tr>
<th>hkl</th>
<th>2θ (deg)</th>
<th>Number of points fitted</th>
<th>Counting time per point (s)</th>
<th>Maximum intensity (counts s⁻¹)</th>
<th>Background intensity (counts s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>220</td>
<td>122</td>
<td>7</td>
<td>15</td>
<td>637</td>
<td>340</td>
</tr>
<tr>
<td>211</td>
<td>143</td>
<td>11</td>
<td>45</td>
<td>880</td>
<td>636</td>
</tr>
</tbody>
</table>

scintillation counter with associated electronics. The instrument parameters for this case are shown in Table 3. A General Electric quarter-circle goniometer was used as a sample holder in all measurements. The φ rotation of the quarter-circle and the θ rotation of the diffractometer were used respectively to define the φ and θ tilts required for stress analysis. The intensities across a peak were corrected on-line for background, absorption, Lorentz polarization and detector dead-time factors [18]. The code also determined the location of a diffraction peak and its associated interplanar spacing d₂φ by fitting a parabola to the top 15% of the intensity. The peak positions obtained were also corrected for the overlapping of the Kα₁-Kα₂ doublet [8, 10]. (This correction was important only for measurements from annealed samples which had sharp peaks. The shot-peened specimens generally had peaks too broad to be affected by this correction.) The reflections employed from each phase, the number of points used in parabola fitting and other pertinent measurement parameters are given in Table 4. The elastic constants (1 + ν)/E and ν/E were calculated from single-crystal compliances in the Kröner limit [7, 8] and are given in Table 5 for both phases.

The peak positions were measured to a statistical accuracy of less than 0.02° (2θ), resulting in a statistical error of less than 15 MPa in the stress determined from the slope [18]. The calculated errors due to geometric parameters (such as sample displacement, horizontal divergence etc.) were less than 5 MPa. Similarly, all components of the

TABLE 5
X-ray bulk elastic constants in various limits for the α and β phases calculated from the single-crystal elastic constants

<table>
<thead>
<tr>
<th>Phase</th>
<th>hkl</th>
<th>ν/E x 10⁻⁵ (MPa⁻¹)</th>
<th>(1 + ν)/E x 10⁻⁵ (MPa⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voigt limit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant strain</td>
<td>α</td>
<td>220</td>
<td>0.240</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>211</td>
<td>0.218</td>
</tr>
<tr>
<td>Reuss limit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>constant stress</td>
<td>α</td>
<td>220</td>
<td>0.306</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>211</td>
<td>0.486</td>
</tr>
<tr>
<td>Neerfeld limit</td>
<td></td>
<td>average of Voigt and Reuss values</td>
<td></td>
</tr>
<tr>
<td></td>
<td>α</td>
<td>220</td>
<td>0.273</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>211</td>
<td>0.352</td>
</tr>
<tr>
<td>Kroner limit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>α</td>
<td>220</td>
<td>0.262</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>211</td>
<td>0.283</td>
</tr>
</tbody>
</table>

According to ref. 20 we have, for the α phase, C₁₁ = 0.144 x 10⁶ MPa, C₁₂ = 0.107 x 10⁶ MPa and C₄₄ = 0.71 x 10⁶ MPa and, for the β phase, C₁₁ = 0.128 x 10⁶ MPa, C₁₂ = 0.109 x 10⁶ MPa and C₄₄ = 0.82 x 10⁵ MPa.
residual stresses tensor determined from the $d$ versus $\sin^2\psi$ data obtained from a thin layer of chromium powder dusted on the specimen surface were less than 5 MPa [9, 21]. As the values should be zero, this is a good check of all procedures.

The stress results reported in the following are not corrected for layer removal and depth of penetration, since these terms (calculated according to the standard procedures described in ref. 4) are very small (less than 2 MPa) for the gradients observed. However, it must be noted that these procedures are derived assuming boundary conditions and equilibrium equations applicable to only macrostresses and hence may not be appropriate for stress profiles containing average microstress (pseudo-macrostress) components.

4. RESULTS AND ANALYSIS

4.1. Residual stress analysis

The $d$ versus $\sin^2\psi$ plot from both phases of the as-received brass sample exhibited oscillations, indicating an inhomogeneous surface strain distribution. Since the variation in relative intensity* versus $\sin^2\psi$ was also oscillatory for both cases, both phases have texture.

After this material is annealed and then shot peened, the oscillations in both $d$ versus $\sin^2\psi$ and relative intensity $versus \sin^2\psi$ disappear for both phases, indicating a homogeneous average strain distribution and a surface with randomly oriented grains. This transformation is due to the highly random plastic deformation distribution associated with the peening operation. However, as the surface is removed by electropolishing, the plastic deformation decreases and, at a depth of 110 $\mu$m from the original surface, oscillations in $d$ versus $\sin^2\psi$ appear, together with a non-linear relative intensity variation. Equation (1), which implicitly assumes a homogeneous strain distribution in its derivation, cannot account for these oscillations and thus is inapplicable to such data. Thus at this depth the data analysis was terminated.

The $d$ versus $\sin^2\psi$ plots for both phases between 0 and 90 $\mu$m either were linear within experimental error or exhibited slight curvature, with no oscillations in relative intensity. Such curvature may be caused by a gradient of $\sigma_{ij}$ in $-P_3$. For these cases the $d_{\phi\psi}$ points at large $\psi$ angles ($\psi$ greater than 18.43°) were used in the regression analysis, since these points would be less affected by a stress gradient [21, 22].

The slopes and intercepts of the $d$ versus $\sin^2\psi$ plots obtained from linear least-squares analysis are shown in Table 6 for two directions, $\phi = 0^\circ$ and $\phi = 90^\circ$, for both phases. It can be seen that the intercepts of $d$ versus $\sin^2\psi$ for both directions are the same within experimental error ($\Delta d < 0.00015 \text{Å}$), at each depth for any phase. This is predicted by eqn. (2c) and shows the good agreement of the data with the theory.

In Fig. 4 the total residual stresses at each depth for the respective phases, obtained from eqns. (2a) and (2b) are shown. It can be seen that the total stresses ($\sigma_{ij}^{PM}$ and ($\sigma_{ij}^{PM}$) are different at the surface; however, they gradually approach each other and become equal for both $\phi$ directions*. This profile indicates a variation in plastic deformation with depth, since the residual stresses are due to the elastic response of the undeformed layers to the permanent elongation of the deformed layers [6].

These total stresses can be separated into their macrostress components $\sigma_{ij}^{m}$ and total average microstress components ($\sigma_{ij}^{pm} - \langle \sigma_{33}^{PM} \rangle$) by utilizing eqns. (5) and (6). The results are shown in Fig. 5(a) and Fig. 5(b) for $\phi = 0^\circ$ and $\phi = 90^\circ$ respectively. It can be seen that (i) the average microstress (pseudomacrostress) component in the matrix is negligible and (ii) the average microstress component in the $\beta$ phase is larger for the $\phi = 90^\circ$ direction (however, for both $\phi$ values, the average microstress components decrease to zero at approximately 90 $\mu$m).

In order to obtain the $\langle \sigma_{33}^{pm} \rangle$ value at each depth, accurate values are needed for $(d_0)_n$.

---

*The relative intensity at a given $\psi$ tilt is defined as the peak intensity, expressed as a fraction of the maximum intensity encountered over the total $\psi$ range used in that experiment. In this procedure, all intensities are corrected for Lorentz polarization and absorption.

*The broken lines in Figs. 4 and 5 indicate the expected (qualitative) behavior of the residual stress after 90 $\mu$m. Such behavior has been extensively reported for data obtained from a single phase in shot-peened materials [1, 20, 23]. In the present study, as noted before, analysis after this depth is not possible because of the re-emergence of texture.
TABLE 6
Slopes and intercepts of the $d$ versus $\sin^2\psi$ plots measured at various depths for $\phi = 0^\circ$ and $\phi = 90^\circ$ from both phases of the shot-peened material

| Depth ($\mu$m) | $\phi = 0^\circ$ | | $\phi = 90^\circ$ | |
|----------------|------------------|------------------|------------------|
|                | $\alpha$ phase   | $\beta$ phase    | $\alpha$ phase   | $\beta$ phase    |
|                | Slope            | Intercept (Å)    | Slope            | Intercept (Å)    |
| 0              | -0.00198         | 1.30572          | -0.00102         | 1.20446          |
| 10             | -0.00217         | 1.30903          | -0.00158         | 1.20500          |
| 20             | -0.00201         | 1.30957          | -0.00199         | 1.20532          |
| 30             | -0.00335         | 1.30964          | -0.00238         | 1.20529          |
| 40             | -0.00396         | 1.30984          | -0.00282         | 1.20614          |
| 50             | -0.00402         | 1.31013          | -0.00355         | 1.20632          |
| 60             | -0.00355         | 1.31018          | -0.00364         | 1.20643          |
| 70             | -0.00439         | 1.31045          | -0.00468         | 1.20747          |
| 90             | -0.00458         | 1.31044          | -0.00434         | 1.20665          |
|                |                  |                  |                  |                  |
| Annealed sample| 0.00012          | 1.30469          | 0.00020          | 1.20426          |

Fig. 4. Variation with depth of total residual stress $\sigma$ in the $\alpha$ phase ($\bigcirc$) and the $\beta$ phase ($\triangle$) in a shot-peened 60-40 brass specimen: (a) $\phi = 0^\circ$; (b) $\phi = 90^\circ$. 
Fig. 5. Separated macrostress components $\sigma_{\text{macro}} (\times)$ and average microstress (pseudo-macrostress) components $\sigma_{\alpha}^{\text{pm}} (+)$ and $\sigma_{\beta}^{\text{pm}} (\ominus)$ as functions of depth $z$: (a) $\phi = 0^\circ$; (b) $\phi = 90^\circ$.

and $(d_0)$. Such values were not available for this study, since the current techniques of $d_0$ determination (such as annealing) also cause variation in other parameters (such as stacking fault and microtwin density) that affect $d_0$, yielding large errors in the stress analysis [9]. However, since the penetration depth of X-rays in brass is very small for Cr Kα radiation (90% of the observed intensity is from a layer 3.5 μm thick for both phases) and, in previous studies, negligible $\sigma_{33}$ values were reported even for the cases where deeper penetration distances could be achieved [24], the gradient in $\langle \sigma_{33}^{\text{pm}} \rangle$ may be assumed to be negligible in both phases. Thus, the average microstresses (pseudo-macrostresses) contributing to the total stress determined by X-rays are only the stresses $\langle \sigma_{11}^{\text{pm}} \rangle$ and $\langle \sigma_{22}^{\text{pm}} \rangle$ in the surface plane, and Fig. 5(a) and Fig. 5(b) represent the total separated stress profiles for $\phi = 0^\circ$ and $\phi = 90^\circ$ respectively.

4.2. Line-broadening studies

In Fig. 6 the peak breadths at half the maximum intensity for 211 peaks from the $\beta$ phase and for 220 peaks from the $\alpha$ phase are shown as functions of depth. It can be seen that the peak breadths for both phases decrease with increasing depth into the material. This indicates qualitatively that the damage suffered by the material is maximum
Fig. 6. Peak breadth at half the maximum peak intensity PBHMI as a function of depth \( z \) for 211 (from \( \beta \) phase) (\( \Delta \)) and 220 (from \( \alpha \) phase) reflections (\( \times \)).

**TABLE 7**

Effective coherent diffracting column length \( \langle D \rangle_{\text{eff}} \) and root-mean-square strain \( \langle \varepsilon_L^2 \rangle^{1/2} \) \( (L = 50 \text{ Å}) \) with depth in the respective phases of shot-peened 60-40 brass

<table>
<thead>
<tr>
<th>Depth (µm)</th>
<th>( \langle D \rangle_{\text{eff}} ) (Å)</th>
<th>( \langle \varepsilon_L^2 \rangle^{1/2} \times 10^{-3} )</th>
<th>( \langle D \rangle_{\text{eff}} ) (Å)</th>
<th>( \langle \varepsilon_L^2 \rangle^{1/2} \times 10^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>110</td>
<td>4.6</td>
<td>71</td>
<td>2.5</td>
</tr>
<tr>
<td>13</td>
<td>87</td>
<td>3.7</td>
<td>78</td>
<td>3.7</td>
</tr>
<tr>
<td>40</td>
<td>248</td>
<td>4.5</td>
<td>103</td>
<td>2.6</td>
</tr>
<tr>
<td>70</td>
<td>245</td>
<td>4.9</td>
<td>125</td>
<td>3.2</td>
</tr>
<tr>
<td>90</td>
<td>270</td>
<td>4.6</td>
<td>128</td>
<td>4.6</td>
</tr>
<tr>
<td>110</td>
<td>500</td>
<td>4.7</td>
<td>83</td>
<td>2.6</td>
</tr>
</tbody>
</table>

at the surface and decreases with increasing distance from the surface. The variation with depth of the breadth from the hard \( \beta \) phase is less in the near-surface regions, indicating more uniform deformation in these layers, whereas the softer \( \alpha \) phase exhibits a more or less continuous decrease from the deformation at the surface. A similar trend can be observed from the results of the complete Fourier analysis (eqns. (7)-(9)) listed in Table 7. The domain size from the \( \alpha \) phase increases steadily with depth, while the domain size of the \( \beta \) phase is approximately constant in the layers analyzed.

### 4.3. Microtwin and stacking fault density analysis

The results of the analysis for the microtwin density are shown in Table 8*. It can be seen that \( \beta_{\text{mT}} \) is negligible at all depths. (As a basis for comparison, Wagner [25] has reported that \( \beta_{\text{mT}} = 0.066 \) for 65-35 \( \alpha \)-brass filings.)

The stacking fault density, however, does demonstrate variation with depth, as shown in Fig. 7. It can be seen that \( \alpha_{\text{SF}} \) is low at the surface, increases to a stable level immediately below the surface and decreases slowly after 100 µm.

*According to Cohen and Wagner [15], using a single peak increases the instrumental error associated with this analysis. To check this effect, \( \beta_{\text{mT}} \) at a depth of 30 µm was determined on the General Electric and Picker diffractometers using different radiations and slit systems. Both results are shown in Table 8. Both values are negligible, and the difference between them gives an idea of the error associated with this analysis.
TABLE 8
Microtwin probability distribution with depth in the \( \alpha \) phase of shot-peened two-phase 60-40 brass

<table>
<thead>
<tr>
<th>Depth (( \mu )m)</th>
<th>( {\Delta (CG)}_Fe )</th>
<th>( (2\theta_{200})_Fe )</th>
<th>( \beta_{mT} \times 10^{-3} )</th>
<th>( {\Delta (CG)}_Cr )</th>
<th>( \beta \times 10^{-2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.005</td>
<td>62.76</td>
<td>0.560</td>
<td>-0.017</td>
<td>1.48</td>
</tr>
<tr>
<td>30</td>
<td>-0.006</td>
<td>62.66</td>
<td>0.875</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>-0.017</td>
<td>62.54</td>
<td>1.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>-0.010</td>
<td>62.54</td>
<td>1.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>-0.046</td>
<td>62.64</td>
<td>5.17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7. Variation in stacking fault density with depth \( z \) in the \( \alpha \) phase of shot-peened 60-40 brass.

5. DISCUSSION

5.1. Pseudo-macrostress distribution

From Fig. 5 it can be seen that the average microstresses due to inhomogeneous partitioning of plastic deformation between the phases are negligible in the \( \alpha \) phase at all depths. This result is in good agreement with previous studies, where stresses due to shot peening were measured with mechanical methods and X-ray methods (in the matrix only) and were found to agree within experimental error [23]; mechanical methods would measure only the macrostresses and thus differ from the X-ray values by the pseudo-macrostress components which are small in this study.

The average microstresses in the \( \beta \) grains are tensile. This indicates that the \( \alpha \) grains (i.e. the soft matrix) elongate more under shot peening than the \( \beta \) particles (the hard phase) do. Thus, after the peening operation, \( \alpha \) grains will be in compression and \( \beta \) grains in tension as a result of the mutual constraints from each phase. From Fig. 5 it can also be seen that the average microstresses have their maximum at the original surface where the plastic deformation is a maximum. They decay with depth and the pseudo-macrostress tensor is negligible approximately 60 \( \mu \)m into the material.

5.2. Distribution of plastic strains and macrostresses with depth

Since residual stresses form as a reaction to the inhomogeneous distribution of plastic strains, let us consider first the plastic strain distribution. In Table 9 the various parameters related to the amount of plastic deformation and their variation with depth are shown. All properties except microtwin density indicate decreasing plastic deformation with increasing depth. The low stacking fault
TABLE 9
Various parameters related to the amount of plastic deformation and their variation with depth for shot-peened 60-40 brass

<table>
<thead>
<tr>
<th>Property</th>
<th>Behavior with depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-breadth of the (220)₀ peak</td>
<td>Decreases continuously</td>
</tr>
<tr>
<td>Half-breadth of the (211)₂ peak</td>
<td>Decreases continuously</td>
</tr>
<tr>
<td>Domain size from the α phase</td>
<td>Increases continuously</td>
</tr>
<tr>
<td>$d$ versus $\sin^2\psi$ for the (220)₀ peak</td>
<td>Linear for $z &lt; 90 \mu m$; oscillatory for $z &gt; 90 \mu m$</td>
</tr>
<tr>
<td>Relative intensity versus $\sin^2\psi$ for the (220)₀ peak</td>
<td>Linear for $z &lt; 90 \mu m$; oscillatory for $z &gt; 90 \mu m$</td>
</tr>
<tr>
<td>$d$ versus $\sin^2\psi$ for the (211)₂ peak</td>
<td>Slightly curved or linear for $z &lt; 90 \mu m$; oscillatory for $z &gt; 90 \mu m$</td>
</tr>
<tr>
<td>Relative intensity versus $\sin^2\psi$ for the (211)₂ peak</td>
<td>Slightly curved or linear for $z &lt; 90 \mu m$; oscillatory for $z &gt; 90 \mu m$</td>
</tr>
<tr>
<td>Microtwin density in α</td>
<td>Negligible in the depths investigated</td>
</tr>
<tr>
<td>Stacking fault density in α</td>
<td>Initial increase, constant over 30–90 μm and then decreases steadily</td>
</tr>
</tbody>
</table>

Fig. 8. (a) Qualitative depiction of plastic deformation $\epsilon_{12}^p$ and $\epsilon_{22}^p$ in surface layers of shot-peened 60-40 brass; (b) the residual stress $\sigma$ profile expected from such a plastic deformation distribution with depth $z$.

Density in the α phase at the surface may be due to dynamic recovery caused by the high energy supplied by the peening process since a similar change was observed by Adler and Otte [26] for wire-drawn 70–30 brass at the high plastic deformation levels. Thus, the plastic deformation distribution can be qualitatively depicted as shown in Fig. 8(a).

The residual stress profile that such a plastic deformation distribution will cause is shown in Fig. 8(b). Such a profile is expected since the layer at the surface with the maximum deformation must be constrained the most by the undeformed layers. However, our experimentally determined residual stress profiles (Fig. 5) do not agree with the stress distribution shown in Fig. 8(b).

Previous models explaining stress distributions similar to those shown in Fig. 8 assume that the plastic deformation distribution is proportional to the Hertzian pressure generated by the shot at the instant of impact [2]. This stress profile is shown schematically in Fig. 9. It is assumed that the maximum...
plastic deformation occurs at the point of maximum (Hertzian) shear stress and thus, since layers below the surface elongate more, the maximum residual stress is observed below the surface. In this case, however, other parameters which are proportional to plastic deformation would be expected to exhibit a maximum at the same depth as the residual stresses, and this is contrary to Table 9. It is thus concluded that the current models cannot explain the formation of residual stresses in soft materials peened with hard shot at high peening intensities.

An alternative explanation is as follows. If the layers near the surface slip with respect to each other because of the high intensity of the peening operation, i.e. there is some unconstrained flow $\delta_i$ which decreases with depth, the residual stresses will exhibit a maximum value below the surface, with the maximum total plastic deformation occurring at the surface. The depth at which the residual stress is a maximum is the depth at which maximum constrained plastic flow occurs.

This explanation is shown schematically in Fig. 10(a), with the resultant plastic deformation and residual stress profiles shown in Fig. 10(b) and Fig. 10(c) respectively. The partial relaxation of the compressive residual stresses by such unconstrained plastic flow may also be important in other cases where the maximum residual stress occurs below the surface.
and should be checked using procedures similar to those described in this study.

6. CONCLUSIONS

The stress state in both phases of a shot-peened two-phase brass alloy was examined with the following conclusions.

(1) The matrix contains mainly macrostresses $\sigma_{11}$ and $\sigma_{22}$ in the region penetrated by chromium radiation.

(2) The second-phase particles contain macrostresses and significant average microstresses (pseudo-macrostresses).

(3) Differential deformation of $\alpha$ and $\beta$ phases (and the microstress field set up because of this relative deformation) is a maximum at the surface and decays to zero at 60 µm below the surface.

(4) Various other X-ray measurements, such as line-broadening and stacking fault distributions, also indicate that the deformation is a maximum at the surface and decays gradually with depth.

(5) The coexistence of the particular residual stress and plastic strain distributions discussed above implies the presence of unconstrained plastic flow in the surface layers, which results in the partial relaxation of the residual stresses at the surface and causes the stress profile to exhibit a maximum deeper into the material.

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