# Use of a Finite Element Method for the Prediction of the Shot-Peened Residual Stress Relaxation during Fatigue

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## Introduction

The improvement of fatigue strength by surface treatment by shot peening has long been recognized. One of the reasons is the presence of compressive residual stresses [1]-[3].

Residual stresses can be relaxed by the deliberate application of mechanical and/or thermal energy. In this paper, only relaxation during a cyclic loading (fatigue) will be studied. In fact, the relaxation phenomenon depends on a complex interaction of a number of factors, such as, the stress amplitude, the number of cycles of the loading, the temperature, the state of the initial residual stress, the nature, the origin and the mechanical properties of the material.

If we want to take into account the residual stresses in a fatigue calculation, it is very important to consider the relaxation of residual stress [4]. But in the fatigue design's applications, the relaxation phenomenon is often neglected. In only a few cases, the residual stresses are systematically analysed using measurement of the residual stress state during and after fatigue testing.

So, it is necessary to develop methods of calculation for prediction of the relaxation of residual stress during fatigue. A model of relaxation using the finite element method is then presented which relates residual stress distribution in the depth plane to 1) the number of fatigue cycles, 2) the cyclic stress amplitude, 3) the ratio $R = \sigma_{min}/\sigma_{max}$, 4) the loading direction, 5) the cyclic hardening behaviours of the material, 6) the notch effect.

## Principe of the calculation method

For the inelastic analysis of structures, a simple practical approach is proposed by ZARKA and CASIER [5]. For a case of cyclic plasticity (the relaxation of the residual stress due to a local cyclic plastic strain for example), this method can't give the exact solutions, but can predict the response of a kinematic hardening material to the stabilized cycle : elastic shakedown or plastic shakedown.

The new approach of ZARKA and CASIER introduces transformed internal parameters into the calculation of the states of stress and the strains. These parameters can be calculated by the following relationship :

$$\hat{\alpha} = \alpha - \text{dév.}\,\varrho = \alpha - S + S^{el} \qquad \text{Equ. 1}$$

where

$\alpha = C.\epsilon^p$ is the tensorial internal variable connected with kinematic hardening

$C = 2/3H$ H is the cyclic hardening modulus of the material

$\epsilon^p =$ is the tensor of the plastic strains

$\varrho =$ is the tensor of the residual stresses

$S =$ is the real stress deviator tensor

$S^{el} =$ is the elastic stress deviator tensor

For the material following the Von Mises criterion, the yield condition is :

$$3/2\,(S - \alpha)^T (S - \alpha) \leqq \sigma_0^2 \qquad \text{Equ. 2}$$

$\sigma_O$ is the real yield limit

From Equ. 1 and Equ. 2 we can deduct the new plastic criterion :

$$3/2\,(S^{el} - \hat{\alpha})^T (S^{el} - \hat{\alpha}) \leqq \sigma_0^2 \qquad \text{Equ. 3}$$

This relationship (3) defines a new yield surface (or sphere) using the transformed parameters $\hat{\alpha}$ . The center of this sphere is $S^{el}$ and his radius is $\sigma_O$

In the space of the transformed parameters, the interpretation of the plastic criterion is as follows : if the state of non linearity (plasticity) of the material is defined by the tensor of the parameters $\hat{\alpha}$ , the representative point of the state of the stress is in the interior of the sphere with the center $S^{el}$ and the radius of $\sigma_O$ at all times.

When the loading condition changes with time, the position ($S^{el}$) and the size ($\sigma_0$) of the sphere changes also, but these changes depend only on the parameters calculated by the elastic calculations.

During a constant amplitude fatigue test ($F_{min} < F < F_{max}$), the representative point of the state of stress evolves between two convex ( $\mathcal{C}_{min}\,(S^{el}_{min}, \sigma_{o\,min})$ ) and ( $\mathcal{C}_{max}\,(S^{el}_{max}, \sigma_{o\,max})$ ).

According to the ZARKA and CASIER criterion, for a kinematic hardening material, the condition for elastic shakedown (stabilization of the residual stress state for example) is that the convex set $C_p = \mathcal{C}_{min} \cap \mathcal{C}_{max}$ is a non void set (Fig.**1** ).

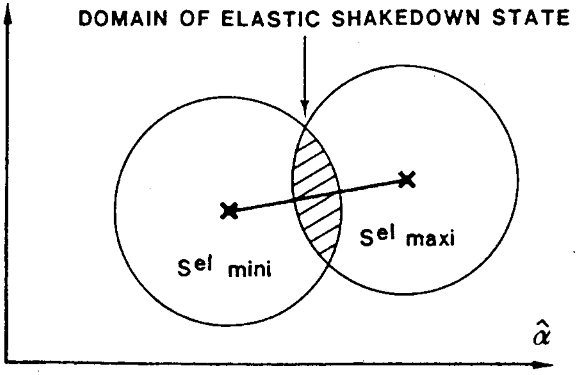


Fig.1

So the calculation of the stabilized state of the residual stress after fatigue loading consists in verifying the conditon of the elastic shakedown :

$$\sqrt{\frac{3}{2}(\Delta S^{el})^{T}(\Delta S^{el})} \leq 2\,\sigma_0 \qquad \text{Equ. 4}$$

where : $\Delta S^{el} = S^{el}\mathrm{maxi} - S^{el}\mathrm{mini}$

We can find the detail of the different steps of the calculation of the stabilized stress state in the case of elastic shakedown by an approximate method in [5] [6].

This method is very interesting. only a small number of elastic calculations are necessary to find the limiting state (elastic shakedown) using the initial residual stress field of the residual stress, the cyclic hardening characteristics of the material and the cyclic applied loading. With this approach, a software using the finite element method has been developed by the CETIM (CA.ST.OR option PC2D) for resolving the two dimensional cyclic plasticity problems. This software is only applicable in the case of elastic shakedown for a radial cyclic loading (for which the principal stresses are proportional).

## Results

For the calculation of the stabilized residual stress during fatigue, the software (PC2D) is used. We must introduce the initial plastic strains which correspond to the residual stresses. These plastic strains can be measured by the different techniques [7] - [8] or calculated by a prediction method [9]. In this paper, the residual stresses are measured by the X-ray diffraction method. The experimental results of SFM [10] and Bergstrôm [11] are used.

## 35NCD16 Grade steel [10]

Figure 2 and fig.3 show the results of the state of the residual stress after 1 load cycle for three different stress amplitudes. We can see that the relaxation of residual stress increases with the level of the stress amplitude when there is of a fully reversed fatigue loading. The direction of the applied load also influences the residual stress relaxation. In the loading (axial) direction (fig.2), the relaxation is greater important than in the tangential direction (fig.3 ).

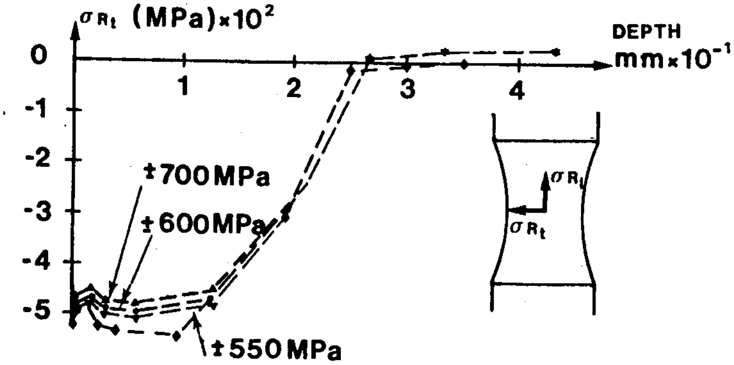


Fig.2 - Relaxation of axial residual stress after 1 cycle (influence of the stress amplitude)

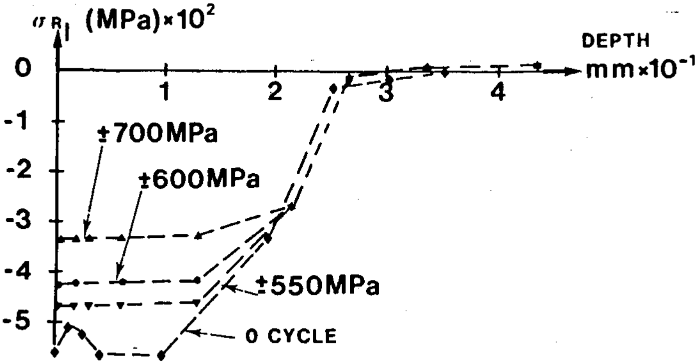


Fig. 3- Relaxation of tangential residual stress after 1 cycle (influence of the stress amplitude)

Fig.**4** show the influence of number of the fatigue cycles on the residual stress distribution. We can observe that the level of maximum compression decreases when the number of cycles increases. In this case, the material is cyclically softened. So the yield stresse decrease with the number of cycles. After each cycle, the material must a new stabilization reach situation caused by the decrease of the material cyclic properties.

The examination of fig.**5** shows that for a constant stress amplitude, the relaxation of the shot-peened residual stress decreases with the increase of the mean stress. Here the parameter R is studied.

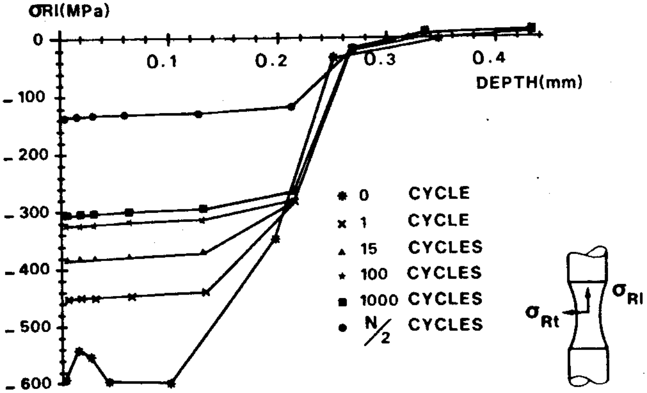


Fig.4 - Evolution of axial residual stress distribution with the number of cycles for a constant applied stress of ±600 MPa

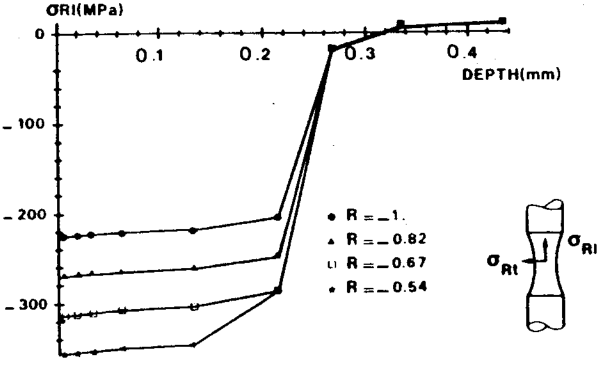


Fig.5 - Effect of the ratio R ( $\sigma$min/ $\sigma$max) on relaxation of the axial residual stress with a constant stress amplitude $\sigma_a$= 500 MPa fatigue.

The reference [12] shows the comparison between the results measured by the X-Ray diffraction method and the results calculated by the finite element method.

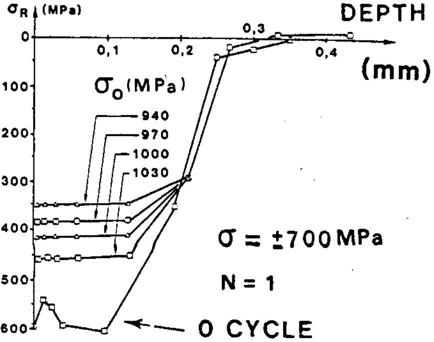
These results show that the finite element method predicts the same phenomenon as the experimental observations for the effects of the stress amplitude. For the axial stress, the calculation model over-estimates the relaxation of the residual stress using the cyclical properties of the material. For tangential stress, the model and the experimental results give the same results. If we compare the calculated results using the mechanical properties at 1 cycle and at 1000 cycles with measured results after 1000 cycles of fatigue loading, we can observe that the measured stresses are between the two calculated results. The difference between the model and the measurement can be explained by the following causes :

1- the cyclical properties of material without the shot-peening are used in the calculation. But there are two layers of material, the shot-peened layer and the non shot-peened layer. The difference of the static mechanical properties between two layers was observed [13] [14]. Probably the cyclic properties also may be different.

2- At 1000 cycles, the material is not in a stabilized state. For this state, the tensile and the compression properties can be different.

3- The initial residual stress can have some dispersions for the different specimens used in the experiment because the method of the measurement of the residual stess distribution by the X ray method is a destructive method due to an electrolytical machining of the specimen. So the measurements of the residual stress before and after fatigue testing were made on two specimens.

The first factor is the most important. The Fig.**6** shows the effects of the $\sigma_0$ on the stabilized residual stress level. The maximum residual stress (compression) level increases with the real yield stress. So it is necessary to know the real yield stress of the peened layer who is often greater than the subsurface due to the surface hardening processus by the shot peening for this type of material. For the shot peening case, a multilayer material must be considered.



AISI 4140 Grade steel [11]

Fig.**6** - The effect of the real yield stress on the level stabilized residual stress.

J. Bergstrôm have studied the relaxation of shot-peening introduced compressive stress during fatigue of notched AISI 4110 grade steel samples. Fig.**7** shows an example of the comparison between the results of the calculation and the experiments. The finite element method is used for the calculation of the real applied stress and the PC2D software is used for the determination of the stabilized residual stress state after fatigue.

Here, two layers (peened layer and the non peened layer) of material are considered.

The detail of this method is shown in [15]. The two methods of evalutation of the relaxation of residual stress show a good agreement.

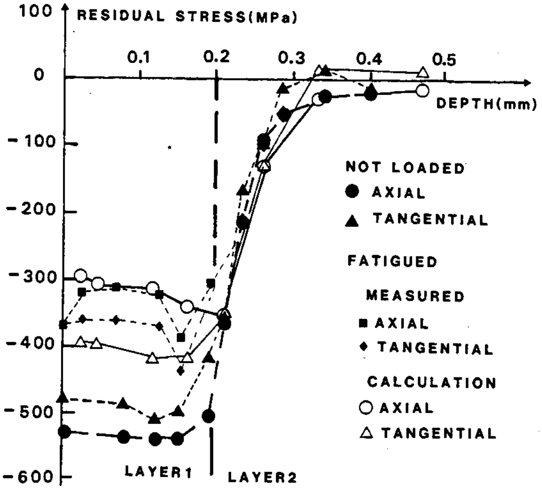


Fig.7 - Comparison of the results between the calculation and the experiment (residual stress profiles for axial and tangential directions at bottom of notch of samples shot peened under 30-35A (mm/100) Almen intensity conditions : data are shown for shot peened and shot peened plus fatigued (R = -1, nominal stress amplitude = 283 MPa, $10^6$ cycles states)

## CONCLUSION

In the work carried out, an automatic software using the finite elements method to calculate the stabilized residual stress during cyclic loading is developed.

This method is applied to shot-peened 35NCD16 grade steel and AISI 4140 grade steel. The different fatigue parameters often used in material research are studied, such as the number of cycles, the stress amplitude, the ratio R ($\sigma$min/$\sigma$ max), the notch effect .The result is satisfactory enough for the study of the residual stresses relaxation phenomenon. It is a simple method for the prediction of the stabilized multiaxial residual stress state in the depth plane. In only very few cases, residual stresses relaxation is analysed systematically due to the long experimental time of fatigue testing and the residual stress distribution measurement. This study shows a new way for understanding the mechanical relaxation of residual stress. Nevertheless, for better modelizing with this method it is necessary to introduce into the calculation the real cyclic stress strain curve corresponding to the materials present in the prestressed layer. This represents a difficulty in the case for example of shot peening. A multilayer method must be used for a better modelisation.

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