

## $\psi$ -SPLITTING IN RESIDUAL STRESS TEST FOR CURVATURE

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### ABSTRACT

" $\psi$ -splitting" phenomenon, generally due to shear stresses, can be produced in the residual stress measurement of machining parts with tangential components. But the same phenomenon is observed when the residual stress of shot peened notch part is measured. X-ray diffraction geometry of curve surface is examined in this paper in order to explore the mechanism of  $\psi$ -splitting by curvature. The theoretic analysis shows, when the sample displacement is involved, the linear relationship between  $2\theta$  and  $\sin^2\psi$  is destroyed and  $\psi$ -splitting can be observed for curvature, due to X-ray beam shining curvature in different incident angles. This effect increases with sample axial displacement and the ratio of beam size to curve radius. The influences of the " $\psi$ -splitting" in curvature can be mostly eliminated by taking the average value of  $+\psi$  and  $-\psi$  direction tests, so that the satisfactory values of curvature residual stresses can be got. The experiments for the residual stress test of shot peened notch confirm the theoretic analysis. Finally the comparison between the  $\psi$ -splitting by curvature and that by tangential stresses is made.

### KEYWORDS

$\psi$ -Splitting, Residual Stress, Notch, X-Ray Diffraction

## INTRODUCTION

The "  $\psi$ -splitting " phenomenon in X-ray residual stress testing had been reported over ten years, which is generally induced by machining with high tangential components such as grinding or honing and surface wear[1]. The same phenomenon had been observed when the residual axial stresses of shot peened notch parts were tested, but no such appearance for the smooth specimen treated by the same machining and shot peening processes. This effect strongly interrupted the residual stress testing, so that many problems are brought about.

This paper will examine the mechanism of the  $\psi$  - splitting in terms of X-ray curvature diffraction geometry, and discuss some associate problems.

### $\psi$ -SPLITTING FOR CURVATURE

In order to measure the residual axial stress of notch root with small radius, the special equipment and attachment are used. The round bar specimens with 1 mm radius circumferential notch were shot peened after quenched and high temperature tempered. Experiments were carried out on Dmax-3A diffractometer. Cr-K $\alpha$  radiation, ( 211 ) peak of  $\alpha$ -Fe, step scanning with 40 seconds fixed time per step, parabolic method for peak locating are employed. The radius of X-ray collimator is 0.25 mm. The test data are shown in Fig. 1.

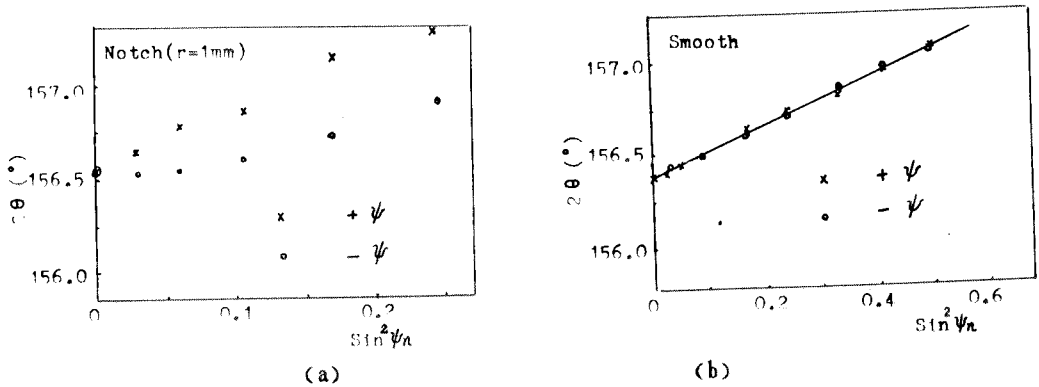


Fig.1  $2\theta$ - $\sin^2\psi_n$  relationships of shot peened specimens  
 (a) Notched (r=1mm) (b) Smooth

The different  $2\theta$ - $\sin^2\psi_n$  curves for notched specimen by  $+\psi$  and  $-\psi$  tests are illustrated, i.e.  $\psi$ -splitting (Fig. 1a). For comparison, the smooth specimen treated by the same heat treating, machining and shot peening processes was tested under the same experiment conditions, no such phenomenon can be found ( Fig.1b ).

### DIFFRACTION GEOMETRY OF CURVATURE

The  $\psi$ -diffractometer ( side-inclination method ) is used. The diffraction geometry of curvature as shown in Fig.2. Assume the curvature as a semi-circular notch ( the similar result can be got for convex surface ), the notch root center O locates at the center of  $\psi$ -rotation, a beam element covers an area  $dS$  on the notch surface with a angle  $\psi$  to the surface normal, but its nominal tilt angle is  $\psi_n$ , and its diffraction angle is  $2\theta_*$ . According to the idea of central gravity method, the average value of  $2\theta_*$  in the whole irradiation area is  $2\theta$ .

$$2\theta = \frac{\int \int_S 2\theta_\psi dI_\psi}{\int \int_S dI_\psi} \quad (1)$$

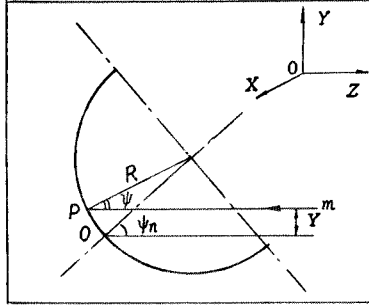


Fig. 2 Diffraction Geometry of a Notch

By Fig. 2  $\psi = \arcsin(\sin \psi_n - y/R)$  (2)

From the conventional X-ray residual stress formula

$$2\theta_\psi = \sigma/K \sin^2 \psi + 2\theta_0 \quad (3)$$

$$dI_\psi = I_0 \cos \psi dS$$

Here,  $2\theta_0$  is the diffraction angle for stress free specimen

$I_0$  is the diffraction intensity for unit area normal to the X-ray beam

$\sigma$  is the residual stress of tested specimen

Combining equ. (1), (2) and (3), one can get equ. (4) and (5).

$$2\theta = \sigma/K \overline{\sin^2 \psi} + C \quad (4)$$

$$\overline{\sin^2 \psi} = \frac{\int \int_S \cos \psi (\sin \psi_n - y/R)^2 dS}{\int \int_S \cos \psi dS} \quad (5)$$

The integral area S is the X-ray irradiation area on notch surface

In the experiment, some sample displacements are inevitable. Assume the center of X-ray beam apart from the notch root center by  $y_0$ . In order to simplify the problem, assume the notch geometry uniform along X direction, X-ray beam is a rectangle, S,  $Y(y_0-a, y_0+a)$ ,  $X(-b, b)$ . The integrals in equ.(5) are carried out in above area S, we can get equ.(6).

$$\overline{\sin^2 \psi} = \frac{N1 \Big|_{\psi_1}^{\psi_2}}{M1 \Big|_{\psi_1}^{\psi_2}} \quad (6)$$

Here,  $M1 = +1/2 * (\psi + \sin \psi * \cos \psi)$

$$N1 = 2/3 * (\sin \psi * \cos \psi + (3/8 * \psi + 1/2 * \sin \psi * \cos \psi + 1/8 * \sin \psi * \cos \psi * (1 - 2 * \sin^2 \psi))) / 2$$

$\psi$  value range,  $(\arcsin(\sin \psi_n - (y_0+a)/R), \arcsin(\sin \psi_n - (y_0-a)/R))$

If weight function factor  $\cos \psi$  in equ.(5) equals one, equ.(6) can be simplified into equ.(7).

$$\overline{\sin^2 \psi} = \sin^2 \psi_n - 2 \times y_0 / R \times \sin \psi_n + (y_0^2 + a^2 / 3) / R^2 \quad (7)$$

## DISCUSSION

### Influence Factors for Curvature $\psi$ -Splitting

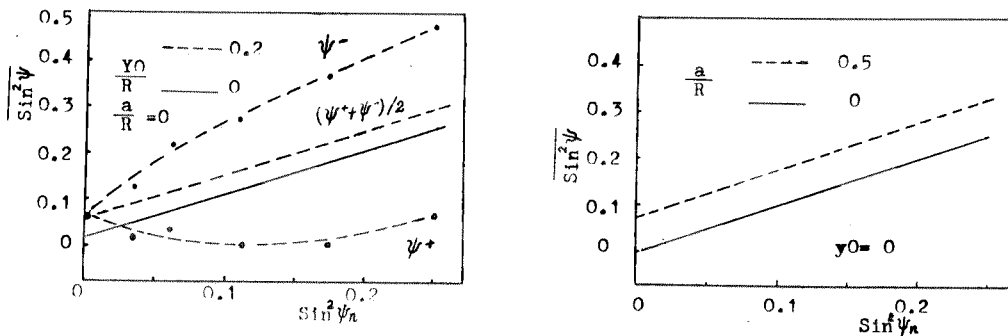
The data calculated by equ.(6) and (7) had been compared [3], the approximate trends had been shown, and the values computered by equ.(7) are slightly larger than that by equ. (6), so the equ. (7) may be used to qualitatively illustrate some associate problems. Equ.(8) can be obtained from equ.(4) in terms of equ.(7).

$$2\theta = \sigma / K \times (\sin^2 \psi_n - 2 \times y_0 / R \times \sin \psi_n + (y_0^2 + a^2 / 3) / R^2) + C \quad (8)$$

There are three terms in equ.(8),  $\sin^2 \psi_n$  term,  $\sin \psi_n$  term and the term having nothing to do with  $\psi_n$ . If the notch radius  $R \rightarrow \infty$ , i.e. for the smooth specimen, the linear correlation of  $2\theta$  vs.  $\sin^2 \psi_n$  can be got. The term " $(y_0^2 + a^2 / 3) / R^2$ " only affects the intercept of  $2\theta$  vs.  $\sin^2 \psi_n$ . The term " $2 \times y_0 / R \times \sin \psi_n$ " will cause the " $\psi$ -splitting" due to odd function  $\sin \psi_n$ , which is direct proportional to the sample axial displacement  $y_0$ , but inverse proportional to the notch's radius  $R$ , as shown in Fig.3. If the average value of  $+\psi$  and  $-\psi$  direction tests is taken, equ.(9) can be got, the term involved  $\sin \psi_n$  disappears from equ.(9), so  $\psi$ -splitting can be removed.

$$2\theta = ((2\theta)_{+\psi} + (2\theta)_{-\psi}) / 2$$

$$= \sigma / K \times (\sin^2 \psi_n + (y_0^2 + a^2 / 3) / R^2) + C \quad (9)$$



(a) (b)  
Fig.3 The Effect of  $y_0/R$  and  $a/R$  on " $\psi$ -Splitting"  
(a)  $y_0/R$  (b)  $a/R$

### Comparison of Two $\psi$ -Splitting Phenomena

The relationship between  $2\theta$  and  $\psi_n$  for triaxial state is expressed in equ. (10) [2].

$$\begin{aligned}
2\theta = & m \times \left( \frac{1+\nu}{E} \times (\sigma_{11} \cos \phi + \sigma_{12} \sin 2\phi + \sigma_{22} \sin \phi - \langle \sigma_{33} \rangle) \times \sin^2 \psi_n \right. \\
& + \frac{1+\nu}{E} \times \langle \sigma_{33} \rangle - \frac{\nu}{E} \times (\sigma_{11} + \sigma_{22} + \langle \sigma_{33} \rangle) \\
& \left. + \frac{1+\nu}{E} \times (\sigma_{13} \cos \phi + \sigma_{23} \sin \phi) \times \sin 2\psi_n \right) \quad (10)
\end{aligned}$$

The term  $(1+\nu)/E \times (\sigma_{13} \cos \phi + \sigma_{23} \sin \phi) \times \sin 2\psi_n$  will cause  $\psi$ -splitting in equ.(10), the corresponding term in equ.(8) is  $2 \times y_0 / R \times \sin \psi_n$ . The comparison between the  $\psi$ -splitting induced by triaxial stress and that by curvature is made as follows:

(a) One is caused by tangential stresses and depends on the states of materials ( $\sigma_{13}$ ,  $\sigma_{23}$ ); another by curvature and relies on the specimen geometry (curve radius  $R$ ).

(b) Their maximum effects appear at different positions, one at  $\psi_n = 45^\circ$  due to  $\sin 2\psi_n$ , but another at  $90^\circ$  due to  $\sin \psi_n$ .

(c) The effects of two conditions can all be removed by taking the average value of  $+\psi$  and  $-\psi$  direction tests. Besides, the  $\psi$ -splitting by curvature may be reduced by improving test technique, e. g. decreasing the sample axial displacement  $y_0$ , but that by triaxial stress can't.

#### Influence on Residual Stress Measurement

$$\sigma_r = \frac{\partial(2\theta)}{\partial(\sin^2 \psi_n)} \times K \quad (11)$$

Equ. (11) is widely used to measure the residual stress for smooth specimen in the conventional method, but if it is used for notched, the greater error may be caused [3]. The  $\sin^2 \psi_n$  must be replaced by  $\sin^2 \psi$  value (equ.(5)), moreover, the error caused by sample displacement can be removed mostly by taking the average value of  $+\psi$  and  $-\psi$  direction tests. Reducing the  $y_0$  is also an effective method since it can lessen the difference between  $\sin^2 \psi$  and  $\sin^2 \psi_n$ .

#### CONCLUSIONS

The  $\psi$ -splitting phenomenon can be observed in the residual stress measurement for curvature, which is different from that caused by high tangential stresses. It is direct proportional to the sample axial displacement, but inverse proportional to the curve radius. Much error for residual stress test at notch may be induced by this, which can be mostly eliminated by taking the average value of  $+\psi$  and  $-\psi$  direction tests. Reducing the sample axial displacement is also an effective method.

## REFERENCES

- [1] V.M.Hauk X-ray Methods for Measuring Residual Stress , In "Residual Stress and Stress Relaxation" Edited by E.Kuta and V.Weiss pp117 Plenum Press
- [2] The Japanese Society for Materials , Stress Measurement by X-Ray ( 1981 )pp148 (In Japanese)
- [3] Y.H.Yu,D.Q.Zhang and J.W.He Error Analysis for X-ray Stress Measurement at the Notch, Proceedings of ICRS-2 (Nov. 1988) France