DYNAMIC NONLINEARITY IN SHOT PEENING MECHANICS: SHAKEDOWN ANALYSIS

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ABSTRACT

The paper gives general steps for dynamic nonlinearity to shot peening mechanics. Three-dimensional constitutive material law expressed in stress invariants is used along with the nonlinear analysis based on the concept of initial stress. Residual stresses can be calculated using information in Gauss integration points. The corresponding shakedown modes can be interpreted as those which as state of equilibrium is no longer possible. The final results obtained the shakedown diagrams plotted in three dimensions.

KEYWORDS:

Shakedown, Residual stresses, Nonlinearity, Three dimensional Finite element.

INTRODUCTION

The work is concerned with the development of Three dimensional finite element analysis which cope with compex target geometry and allows for dynamic loading due to an impinging shot. The problem is envisaged to embrace a process of multiple indentation which covers the surface of the target progressively, thus allaowing for shakedown of each spot due to repeated impact.

For simplicity, the shot is assumed rigid and undeformable while the target material receives indentations. Low and high velocity ranges have been considered together with the contact time. The shot velocity ranges come from the data available from a series of tests carried out by the author. These have been simulated in order to process specific results.

It is essential to enlarge upon the basic criterion. The rigid steel shot is assumed to strike, with a gradually increasing load, on the steel target plate. This increasing load is imposed on each surface element, one at a time. The patch load in effect acts, during loading and unloading, on each surface element, exciting all the other elements through the conductivity model which links all the elements together. The second loading is then applied on the adjacent surface element, but this time all the residual displacements and residual stresses from the previous impact are allowed for. Therefore, the response of the target will be different. By storing and updating the state of the target the emmulative result is then plotted in a three-dimensional space. The process is repeated until the whole surface is covered more than once, so that each spot is loaded several times, the number of times depending upon the duration of the process, subsequently, the loading cycles and the surface target is said to shakedown.

STEPS FOR NONLINEARITY ANALYSIS

The dynamic equilibrium conditions [1-8] at the nodes of the descritized system of target elements, at a given time t, are given by

$$[M]\ddot{u}(t) + [c]\dot{u}(t) + [K]u(t) = R(t) \tag{1}$$

where [M] = mass matrix,

u = vectors of displacement,

R = vectors of load,

[C], [K] = damping and stiffness matrices.

These equations are supplemented with a system of initial conditions. In non-linear problems, the non-linear effects in the element stiffness matrix may be due to either large displacement effects or material-yielding behaviour, or a combination of both. In the present case of the target the non-linear stiffness effects are restricted to those related to material yielding. These matrices are dependent on the current displacement of target and its previous loading history.

The equation of motion may be written in increment form with modified [C] and [K]

$$[M]\{\ddot{U}(t)\} + [C_m]\{\dot{U}(t)\} + [K_m]\{U(t)\} = \{R(t)\} + \{P(t)\} \quad (2)$$

where $\{P(t)\}$ is the initial load = $\{[\Delta C]*\{\dot{U}(t)\}+[\Delta K]*U(t)\}\}$ (* indicates time $0 \rightarrow t$).

Solution at $t + \Delta t$

$$[M]\{\ddot{U}(t+\Delta t)\}+[C_m]\{\dot{U}(t+\Delta t)\}+[K_m]\{\Delta R(t+\Delta t)\}+\{\Delta P(t+\Delta t)\} \quad (3)$$

$\Delta P(t+\Delta t)$ represents the non-linearity during time increment Δt and is determined by interaction using the stress approach.

$$\{\sigma\} = [D_T](\{\varepsilon\}-\{\varepsilon_o\})+\{\sigma_o\} \quad (4)$$

The constitutive law is used with the initial stress and constant stiffness approaches throughout the non-linear and the dynamic iteration. For the iteration

$$\{U(t+\Delta t)\}_i = [K_m]^{-1}.\ \{R_{TOT}(t+\Delta t)\}_i \quad (5)$$

The strains are determined using

$$\{\varepsilon(t+\Delta t)\}_i=[B]\{U(t+\Delta t)\}_i \quad (6)$$

where [B] is the strain displacement. The stresses are computed as

$$\{\sigma(t+\Delta t)\}_i = [D_T]\{\varepsilon(t+\Delta t)\}_i +\{\sigma_o(t+\Delta t)\}_{i-1} \quad (7)$$

where $\{\sigma_o(t+\Delta t)\}$ is the total initial stress at the end of each iteration. All calculations for stresses and strains are performed at the Gauss points of all elements. The initial stress vector is

$$\{\sigma_o(t+\Delta t)\}_i = f\{\varepsilon(t+\Delta t)\}_i -[D_T]\{\varepsilon(t+\Delta t)\}_i \quad (8)$$

Using the principle of virtual work, the change of equilibrium and nodal loads $\{\Delta P(t+\Delta t)\}_i$ are calculated as

$$\{\Delta P(t+\Delta t)\}_{iTOT} = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} [B]^T\{\Delta P_o(t+\Delta t)\}_i\ d_\xi\ d_\eta\ d_\zeta$$

$$\sigma_o(t) = \{\sigma_o(t+\Delta t)\}_i = 0 \quad (9)$$

where d_ξ, d_η, d_ζ are the local co-ordinates.

The integration is performed numerically at the Gauss points.

Effective load vector P(t) is given by

$$\{\Delta P(t+\Delta t)_{iTOT} = -[\Delta\sigma(t)_o](\{U(t+\Delta t)\}_i * \{U(t)\})$$

$$= -\ [\Delta C(t+\Delta t)_i\{U(t+\Delta t)\}_i[\Delta K(t)_o(\{U(t+\Delta t)\}_i-\{U(t)\}_i)$$

$$= -\ [\Delta K(t+\Delta t)_i\{U(t+\Delta t)\}_i \quad (10)$$

Von Mises criterion is used and together with transitional factor (Fig. 1) f^*_{TR} form the basis of the plastic states such that

$$f^*_{TR} = \frac{\sigma_y(t) - \sigma_{y-1}(t)}{\sigma(t+\Delta t)_i - \sigma(t+\Delta t)_{i-1}} \tag{11}$$

The elasto-plastic stress increment will be

$$\{\Delta\sigma_i\} = [D]_{ep}\{\sigma(t+\Delta t)_{i-1}(1-f^*_{TR})\{\Delta\varepsilon\} \tag{12}$$

If $\sigma(t+\Delta t)_i < \sigma_y(t)$, it is an elastic limit and the process is repeated. The equivalent stress is calculated from the current stress state. Where stresses are drifted they are corrected from the equivalent stress-strain curve.

The residual load vector is calculated as

$$\{R\} = \{P_n\} - \int_v [B]^T\sigma(t+\Delta t)_i \, d\, vol \tag{13}$$

SHAKEDOWN ANALYSIS

The shakedown load factor is determined using again a piecewise linearized convex yield surface in the dynamic finite element displacement formulation. The constitutive low is maintained.

When the load F_i are applied on an elastic perfectly plastic steel target plate, it will shakedown if time independent, residual stresses σ_R can be found, such that

$$f[\sigma_R(\bar{X},t) + \sigma_R] \leqslant 0$$

for every $\bar{X}$ and t $F(\sigma) \leqslant 0$ being the yield condition and $\sigma_e(\bar{X},t)$ is the linear elastic response of the plate to F_i, where $\bar{X}$, t are spatial coordinates of the loading to be represented by the n-loading parameter confined in a prescribed region denoted by

$f(F_i)=0$ in the load space. The loading domain can be represented by a unit loading region $f(F_i)=0$ times α_p for proportional loading case. The actual loading history F_i can pass through any point inside the loading domain $f(F_i)=0$, as a prescribed function of time. The shakedown problem is to find the maximum value of α_p when the time function of the loading is known subject to the condition that

$$f[\alpha_p \sigma_{eij}(\overline{x},t) + \sigma_{Rij}] \leq 0$$

The above dynamic analysis is carried out with prescribed initial conditions such that the response becomes periodic due to external loading F_t. The stress response vectors are maximised with respect to the assumed linearized yield places.

RESULTS AND DISCUSSION

A simple model of a steel target plate, Fig.2 is chosen for dynamic indentation studies when it is subjected to steel spherical shots with wide range of impacting speeds. A theoretical solution which endeavours to model the shot peening which discussed elsewhere [9-11] has been associated with short and long time dependent condition, as mentioned above. Before giving an explanation to the results produced from the three-dimensional finite element analysis, it is essential to dilate upon the basic criterion. The rigid steel shot is assumed to strike with a gradually increasing load, the steel target plate. Beyond the elastic limit, plastic flow occurs in the target plate, resulting in a permanent indentation.

During that state - plastic deformation - when residual stresses occur in the target plate, they inhibit further plastic flow when the same plate is subjected to repeated impact of the same spot. After a few cycles, a steady-state is reached when the deformation is entirely elastic. This process as given above is known as the shakedown and the maximum load under which it occurs is the limit of the shakedown. The condition for the shakedown is maintained such that the systems of residual stresses together with the normal stresses due to loads do not exceed the yield criterion.

Figures 3-5, shows results of the shots with 30, 40 and 100 m/sec, respectively. The same spot is chosen for all these single impacts with these variable velocities. The reductions of the curvatures from the bottom in each indicates the eventual elastic deformation. In all cases with the increases in the number of impacts, there is a spreading effect of the plastically deformed regions. These regions tend to spread further beyond the rim of indentation rather than deeper into the target plate. The plots indicate a greater piling up of the metal at the edge of indentation. The maximum values of hardness increase in all cases up to 8-10 impacts.

It is interesting to note that some layers below the indentation undergo certain amounts of softening. The only explanation can be offered is perhaps a change in the direction of straining and/or the recovery strains causing further plastic deformation resulting in the reduction of hardness.

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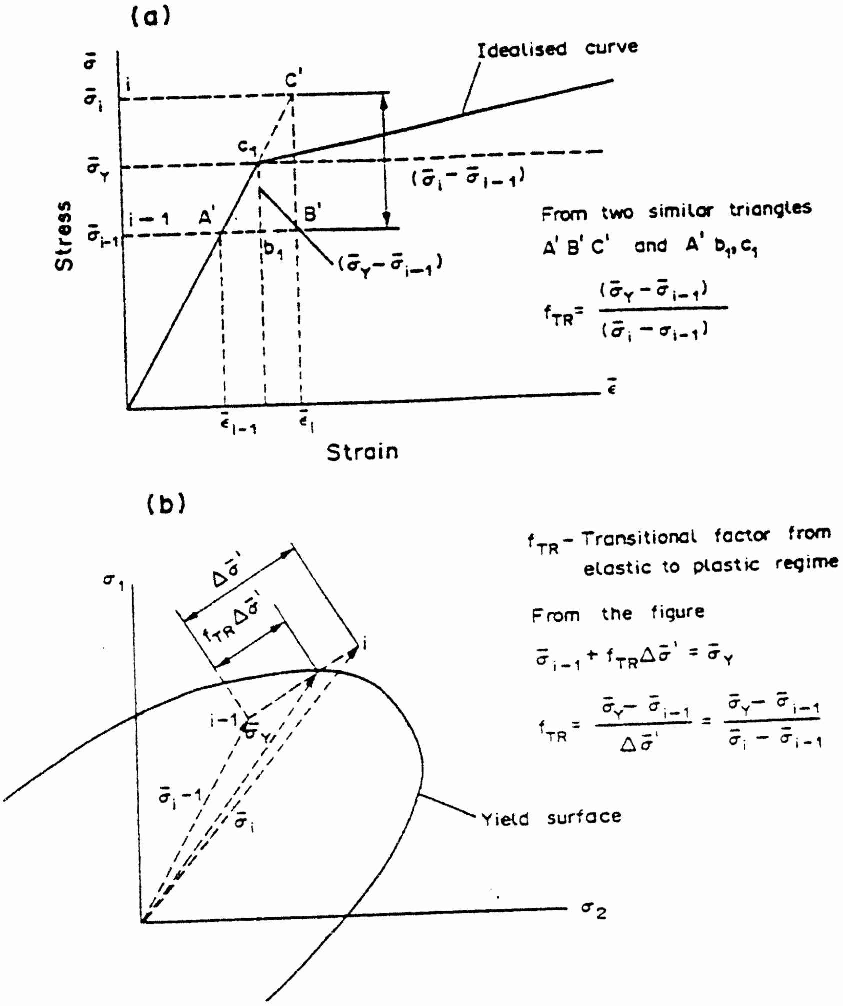

Fig. 1.(a): Equivalent stress-strain curve for steel plate. (b) Yield surface in the principal stress axes

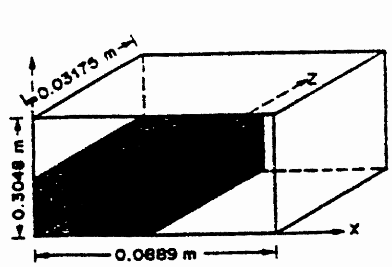

Fig. 2: Steel target plate actual plate size shaded area (1/4 of plate size

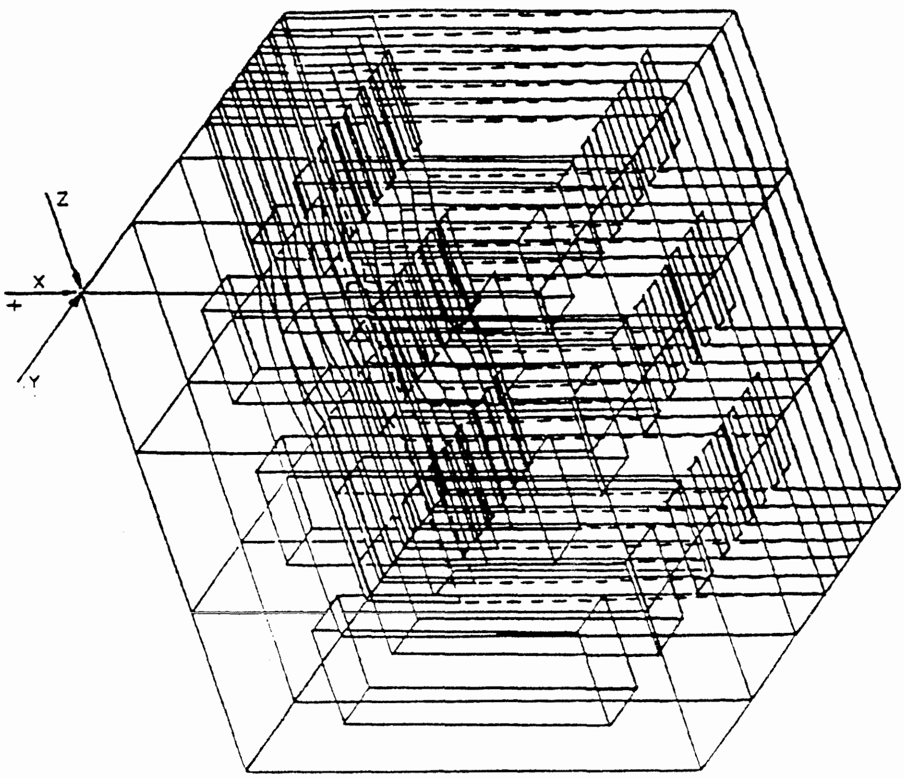

Fig. 3: Shakedown Analysis Type 1 (30m/sec)

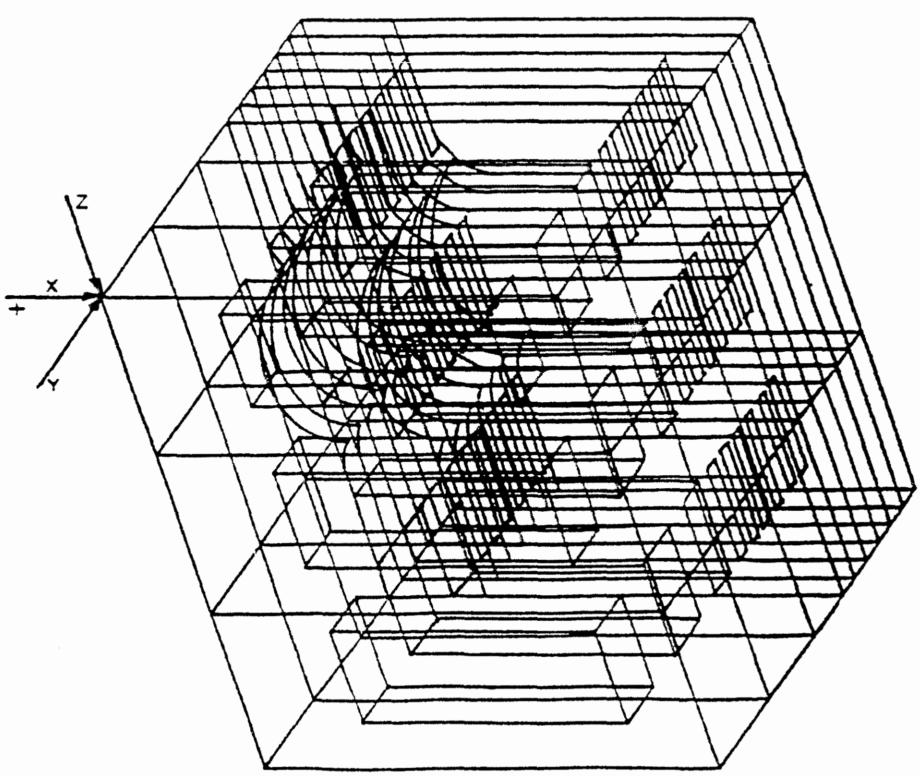

Fig. 4: Shakedown Analysis Type 2 (40m/sec)

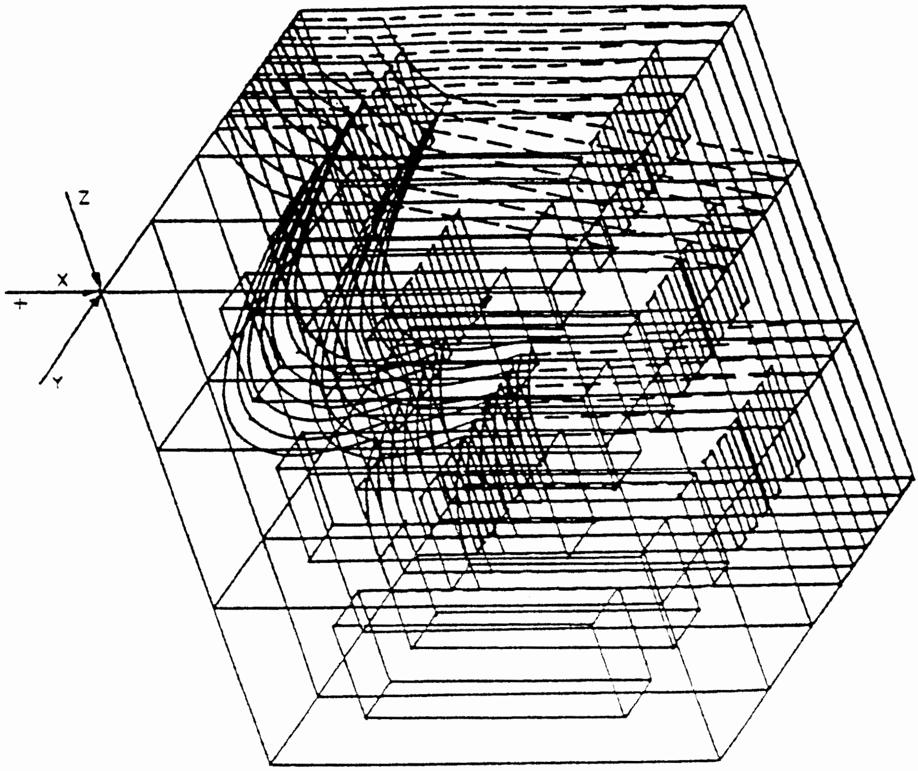

Fig. 5: Shakedown Analysis Type 3 (100m/sec)