SHOT PEENING AND ROBUST DESIGN FOR FATIGUE PERFORMANCE

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ABSTRACT

Fatigue data usually display substantial scatter. The goal of this paper is to demonstrate how simulated variations in shot peening parameters and material properties affect the predicted fatigue performance (mean life and scatter) of a component. This is achieved by applying robust design principles to fatigue life evaluation methods. Analyzing changes in fatigue performance due to variations in the peening parameters and material properties identified levels of the controllable factors which maximize the mean fatigue life and minimize its scatter. These simulation predictions are consistent with past experimental observations concerning effects of compressive residual stresses on fatigue performance. In addition, the compressive zone depth is identified as a possible off-line production quality check relating directly to the component fatigue performance. This study is an initial step in the development of a generalized methodology to aid engineers with design for robust fatigue performance for other manufacturing processes as well.

INTRODUCTION

Motivation

For many mechanical components, fatigue performance is an important aspect of product quality. In practice, design for reliable fatigue performance is often experience-based. Use of surface treatment processes to enhance fatigue performance also depends, to a large extent, on experiential knowledge. To seek further improvements in fatigue performance, this paper develops the foundation for a systematic robust fatigue design methodology that combines manufacturing process models and fatigue life models to relate manufacturing process parameters to component fatigue performance. Shot peening serves as a representative surface treatment process whose existing input/output relationship has largely been empirical.

One goal of this study is to relate shot peening process parameters and component material parameters to the fatigue life of the component. The role of robust design is to use these relationships to understand not only how to increase the mean component life, but how to reduce the variability of that life subject to the inherent variability in process, material and geometric parameters. Both Miller (1993) and Schijve (1994) give comprehensive arguments for the development of systematic design methods to accommodate variability in fatigue analysis and design. Figure 1 qualitatively illustrates the goal of our effort, where the curve optimized for both mean, μ, and variability, σ, represents our target.

The study also seeks to demonstrate that the designer has increased flexibility by considering fatigue performance enhancement processes in addition to the material properties and component geometry. The application of the robust design methodology to fatigue design will enable existing computer-based fatigue analysis and surface treatment process simulations to predict variability in fatigue performance as well as its mean value.

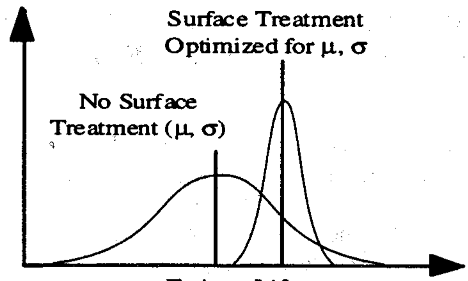

Figure 1. Robust Fatigue Design Goal

Previous Work

As described by Almen (1963), the benefits to fatigue performance of compressive residual stress treatments such as shot peening have been known for about 50 years. Developments in fatigue and fracture mechanics theory over the past few decades have led to improvements in the predictive tools for fatigue life evaluation. Numerous researchers such as Beghini *et al* (1994), Farrahi *et al* (1995), Hammond *et al* (1990) and Underwood (1994) have empirically shown the benefits of compressive residual stresses on fatigue performance, with Beghini *et al* and Underwood showing agreement between predicted and observed fatigue life using fracture mechanics considerations.

Other researchers have studied the Taguchi method (Taguchi, 1978 and 1987) and its applications in mechanical design. Kackar (1985) and Hunter (1985) promoted the application of statistical experiments and parameter design to quality control. d'Entremont *et al.* (1988) adopted the concept of quality loss and developed a nonlinear programming code. Sundaresan *et al.* (1989, 1991) adapted Taguchi's method and incorporated a Sensitivity Index in the optimization procedure to seek a robust optimum. Sundaresan *et al.* (1993) compared the efficiency of three different methods that incorporate variations in constraints. Yu *et al.* (1994) dealt with manufacturing errors that affect design variables with specific characteristics, *Manufacturing Variation Patterns (MVP).*

Research Approach

Figure 2 summarizes the steps used in this study to compute component fatigue life. Given a set of process control parameters, the sub-surface residual stress distribution in a component is determined from a regression model generated from empirical data. This residual stress distribution, along with the applied stress distribution and material fatigue properties, lead to the fatigue life from a model based on linear elastic fracture mechanics (LEFM) and crack growth prediction methods. In this paper, the behavior of steel alloy 4340 is considered. Dowling (1982), Lankford (1985) and Nelson *et. al.* (1994) have demonstrated that the use of LEFM techniques to predict the fatigue behavior of small (e.g., sub-millimeter) initial cracks is reasonable for that alloy, but is not necessarily valid for other metallic alloys. Details of the crack growth computations will be given later.

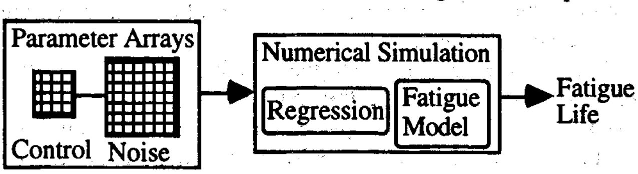

Figure 2. Fatigue Life Computation Steps

An extensive (but not exhaustive) set of controllable (design) and uncontrollable (noise) parameters for relevant material properties, peening parameters and size of initial crack-like microdiscontinuities was identified. These parameters were pared down based on the available data and the capabilities of the numerical models. The final set of control and noise parameters led us to construct standard Taguchi orthogonal control and noise arrays.

These arrays vary the control and noise space to generate the data needed to compute mean fatigue life and fatigue life variation.

Subsequent sections of this paper provide further details about process parameter selection, the residual stress and fatigue models, the robust design procedure and results of the robust design simulations.

MODEL DEVELOPMENT

Overview of Shot Peening

Figure 3 shows a simplified schematic of the shot peening process. Shot media of a given diameter impinge the target surface at high velocity, dimpling the surface and generating compressive residual stresses. Although Hammond *et al.* (1990), Farrahi *et al.* (1995) and others have demonstrated that these residual stresses may diminish through the course of the service loading history due to stress relaxation mechanisms, this paper assumes that the residual stress distributions remain constant throughout the fatigue life. However, we realize that stress relaxation can have a significant influence on the fatigue performance of materials, especially if the sum of the applied stress and the residual stress approaches or exceeds the yield stress. It is also assumed that fatigue cracking originates at the surface of the components.

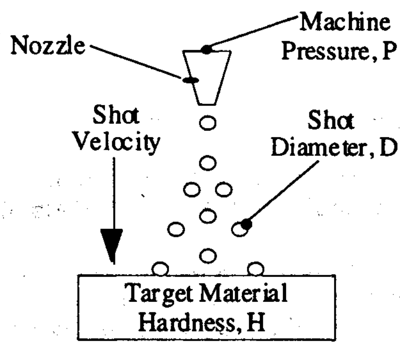

Figure 3. Shot Peening Process

Process and Material Parameters

Table 1 summarizes the key controllable and uncontrollable parameters used in this study given the restrictions of the available shot peening data and fatigue model.

Table 1. Controllable & Uncontrollable Parameters

Controllable		*Uncontrollable*	
Shot Peening Pressure	P	Pressure Variation	ΔP
Shot Diameter	D	Diameter Variation	ΔD
Component Hardness	H	Hardness Variation	ΔH
		Crack Growth Exponent Variation	Δn
		Crack Growth Constant Variation	ΔC
		Fracture Toughness Variation	ΔK_C
		Fatigue Threshold Variation	ΔK_{th}
		Initial Crack Size Variation	Δa_0

Regression Model

The shot peening residual stress data from Brodrick (1955) were the basis for our regression model relating the process parameters to the residual stress distribution. Figure 4 shows a characteristic plot of these data that represents residual stress as a function of depth into the workpiece of 4340 steel for particular values of workpiece hardness, shot size, machine air pressure, shot coverage and shot intensity. Since the data had to be manually read and entered for data fitting, only the specimens that received complete coverage were considered. In addition, intensity is a confounded measure of the shot peening process parameters and time. The lack of the peening time information which was embedded in the intensity measure prevented us from using it for analytical purposes. Therefore, for illustrative purposes, the regression model relates residual stress to machine pressure (P), shot diameter (D), workpiece hardness (H) and depth (Z) into the workpiece. Note that workpiece hardness is not a process parameter, but is a controllable design factor. Also, as a simplification for this analysis, only the compressive zone of the residual stress distribution was considered.

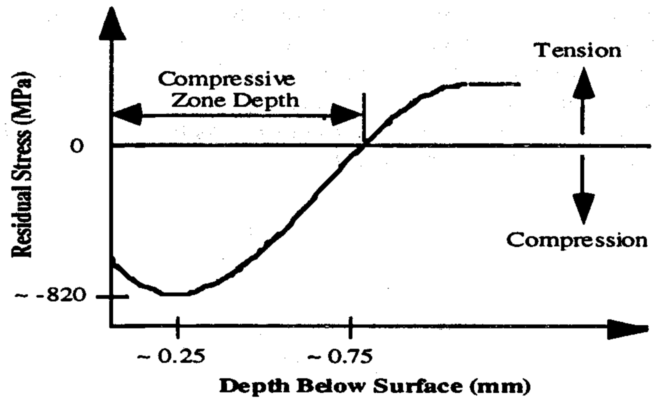

Figure 4. Typical Shot Peening Data Curve

Given that the general shape exhibited in Figure 4 is characteristic of all the data examined, and that the regression model only included the compressive zone data, it seemed natural to fit the data to at least a second order polynomial. A third order polynomial was the final choice to relate residual stress to the shot peening process parameters. The regression equation is shown below (Eq. 1). The coefficients are consistent with the units of the original data (i.e., Rockwell C hardness, psi, and inches). Note that the only dependence on machine pressure is in the interaction term with depth (P•Z).

$$\sigma_{res}(z) = -3318\mathbf{D} + 52533\mathbf{D}^2 - 227003\mathbf{D}^3 - 1.8\mathbf{H} + .0002\mathbf{H}^3 + 421871\mathbf{Z}^2 - 1730755\mathbf{Z}^3 - 105146\mathbf{DZ} - 60.5872\mathbf{PZ} + 263.972\mathbf{HZ} \quad (1)$$

Comparison of the residual stress values from the regression model with the actual residual stress data suggested restricting the ranges of the parameter values to be a subset of the ranges used to generate the original regression model. Use of this restricted range would provide a better fit than the full range of parameter values. Table 3 in the next section summarizes the ranges used for the fatigue simulations.

Crack Growth Evaluations

The numerical simulation model computed the fatigue life for specimens of 4340 alloy steel with an assumed initial semi-elliptical surface crack of depth a_0 and width of 2c as shown in Figure 5, for three different applied stress conditions shown in Fig. 6.

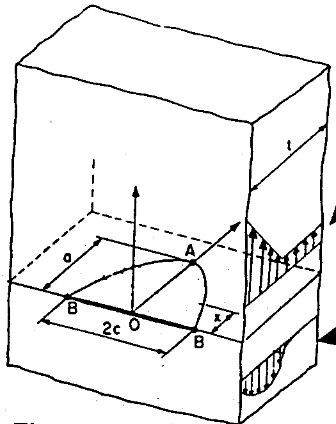

Figure 5. Crack Growth Specimen (not to scale)

The Forman equation was used to compute the incremental crack growth per load cycle:

$$\frac{da}{dN} = \frac{C(\Delta K_{net})^n}{(1-R)K_C - \Delta K_{net}} \tag{2}$$

where

$\frac{da}{dN}$ = increment of crack growth

C, n = crack growth constant and exponent

ΔK_{net} = $K_{net,\max} - K_{net,\min}$

R = $\frac{K_{net,\min}}{K_{net,\max}}$

K = stress intensity factor

K_C = fracture toughness

Combining the residual stress intensity factor, K_{res}, with the cyclic applied stress intensity factors, $K_{app,\min \text{ or } \max}$, results in the maximum and minimum net stress intensity factors, $K_{net,\min \text{ or } \max}$, as

$$K_{net,\max} = K_{res} + K_{app,\max} \tag{3}$$

$$K_{net,\min} = K_{res} + K_{app,\min} \tag{4}$$

The residual and applied stress intensity factors are calculated as functions of the instantaneous crack depth from

$$K_{res} = 2 \cdot \int_0^a \sigma_{res} \cdot m(z,a)dz \tag{5}$$

$$K_{app,m} = 2 \cdot \int_0^a \sigma_{app,m} \cdot m(z,a)dz \tag{6}$$

where

σ_{res} = residual stress distribution

$\sigma_{app,m}$ = applied stress distribution, max or min
$m(z,a)$ = semi-elliptical surface crack weight function (Shen et. al, 1991)
a = crack depth

Values of K_{res} and K_{app} were calculated for an initial crack depth, a_0, then the increment of crack growth was computed. Crack depth was then updated and new values of K_{res} and K_{app} were computed with the updated depth. The crack aspect ratio (a/2c) was kept constant at 0.5. This process was continued until $K_{net,max} \geq K_C$ or the denominator in Eq. 2 became zero or negative. In cases where $K_{net,max} \leq K_{th}$ (threshold stress intensity), crack arrest was predicted (infinite life condition). Numerical integration was used to determine K_{res} and K_{app} from Eqs. 5 and 6. Residual stress distributions from the regression model were used to evaluate K_{res}, while different assumed applied stress distributions shown in Fig. 6 were used to compute K_{app}.

ROBUST DESIGN PROCEDURE

Quality Characteristic

Since the goal of this study was to achieve infinite fatigue life we used the reciprocal of life as the quality parameter. Phadke (1989) defines quality loss, Q_L, as:

$$Q_L = k \cdot \left[(\mu - \mu_0)^2 + \sigma^2\right] \quad (7)$$

The coefficient k is a constant whose value does not change for a particular application. The ideal target is $\mu_0 = 0$ (i.e., N = infinite life). Note that by taking the reciprocal of life, our objective is to minimize the inverse of life, μ, and its scatter, σ. Therefore, equation (7) reduces to the quality characteristic:

$$Q = (\mu^2 + \sigma^2) \quad (8)$$

Noting $\mu^2 = [E(y)]^2$ and $\sigma^2 = E(y^2) - [E(y)]^2$ where E(y) = expected value of y

results in: $Q = E(y^2) = \frac{1}{N}\sum_{i=1}^{N} y_i^2$ where $y_i = \frac{1}{Life}$. (9)

Referring to Eq. 8, note that to minimize Q, μ and σ need to be minimized. A value of Q of zero means that all values of the quality parameter are zero, i.e., all cases analyzed exhibited infinite life. On the other hand, an infinite value of Q means that at least one of the cases had $K_{net,max} \geq K_C$ or the denominator in Eq. 2 was zero or negative as an initial condition, indicating that failure occurred on the first cycle.

Parameter Settings

Table 2 shows the settings of controllable parameters based on the limitations of the shot peening residual stress regression model.

Table 2. Controllable Parameter Settings

Parameter	*Low Value*	*High Value*
P (MPa)	207	620
H (Rc)	30	41
D (mm)	0.58	1.68

Figure 6 summarizes three hypothetical applied stress conditions as a function of depth into the workpiece. They all level off to a constant stress level of 700 MPa and all loading conditions are zero-to-tensile stress cycles (a stress ratio R=0). The first stress condition, a steep linear stress gradient, is representative of a stress concentration due to, for example, a sharp notch. The second loading stress distribution is not as steep, and is representative of a milder notch. The third applied stress distribution has an even milder gradient.

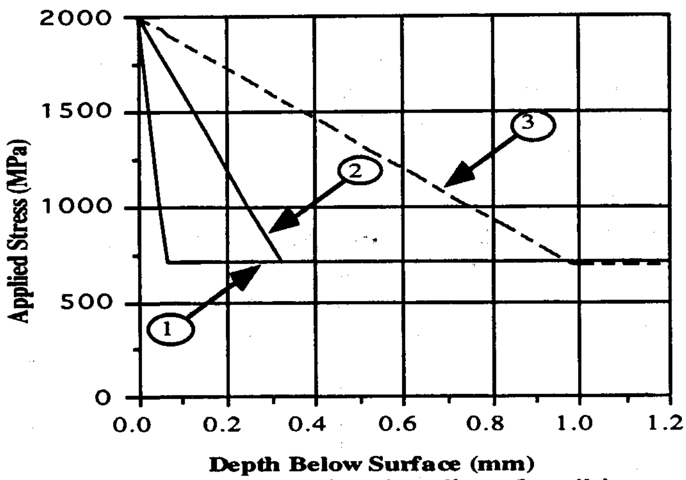

Figure 6. Three Service Loading Conditions

Table 3 shows the nominal values of the crack growth parameters for two hardnesses estimated from the literature (Damage Tolerant Design Handbook, 1983; Structural Alloys Handbook, 1986). These values are for 4340 steel. Note that at a hardness of 30, the value of K_C had had to be estimated. For this study, the nominal initial crack size, a_0, was taken as 0.125 mm.

Table 3. Fatigue Parameter Nominal Values

Hardness (Rc)	K_C $(MPa\sqrt{m})$	C	n	K_{th} $(MPa\sqrt{m})$
41	110	8.42×10^{-11}	2.22	4.0
30	150	1.64×10^{-11}	2.63	6.0

The uncontrollable noise parameters identified in Table 1 are variations in the shot peening process parameters and the Eq. 2 parameters. Table 4 summarizes the simulated ranges for the noise parameters based on typical variations along with the source of the estimates.

Table 4. Noise Parameter Ranges

Noise Parameter	*Range*	*Source*
ΔP	± 10%	Champaigne (1994)
ΔD	± 0.13 mm	estimated based on nominal D
ΔH	± 2 Rc	estimated from Brodrick (1955)
Δn	± 10%	estimates assumed for the sake of this analysis
ΔC	± 80%	•
ΔK_{Ic}	± 10%	•
ΔK_{th}	± 30%	•
Δa_0	± 60%	estimates assumed for the sake of this analysis

Design of Experiments

Inner Array (Design Parameters): The orthogonal array (Table 5) used for the control parameters was an eight run full factorial since there were only three parameters at two levels. The eight run full factorial arrays was repeated for each of the three different applied stress conditions.

Outer Array (Noise Parameters): Table 6 is an L16 orthogonal array of the eight noise parameters.

Table 5. Full L8 Array for Controllable Parameters

Run	*P*	*D*	*H*
1	L	L	L
2	H	L	L
3	L	H	L
4	H	H	L
5	L	L	H
6	H	L	H
7	L	H	H
8	H	H	H

L: Low
H: High

Table 6. L16 Array for Uncontrollable Parameters

Run	ΔP	ΔH	ΔD	Δn	ΔC	ΔK_{Ic}	ΔK_{th}	Δa_0
1	H	H	H	H	H	H	H	H
2	H	H	H	H	L	L	L	L
3	H	H	L	L	H	H	L	L
4	H	H	L	L	L	L	H	H
5	H	L	H	L	H	L	H	L
6	H	L	H	L	L	H	L	H
7	H	L	L	H	H	L	L	H
8	H	L	L	H	L	H	H	L
9	L	H	H	H	H	H	H	H
10	L	H	H	H	L	L	L	L
11	L	H	L	L	H	H	L	L
12	L	H	L	L	L	L	H	H
13	L	L	H	L	H	L	H	L
14	L	L	H	L	L	H	L	H
15	L	L	L	H	H	L	L	H
16	L	L	L	H	L	H	H	L

Simulation: The residual stress and fatigue models lead to a computer code to predict fatigue life for 3 stresses x 8 parameters x 16 noise factors = 384 different runs.

RESULTS

Without Shot Peening

Numerical simulation established the baseline condition for a workpiece at the two values of hardness for the three different applied stress conditions. The first eight rows of the L16 noise array simulated the variability since ΔP and ΔD do not factor into fatigue life for this case. Figure 7 compares the quality characteristic, Q, for the three different stress conditions. Note that the longest life and the least scatter occur at a hardness level of 30. Figure 8 illustrates the details of this result for Stress Condition 3. Also, note that the life for Stress Condition 3 is less than the life for stress condition 2 which in turn, is less than the life for stress condition 1. Since this is consistently the case, the remainder of this section will only show the analysis for the shortest life stress condition, case 3.

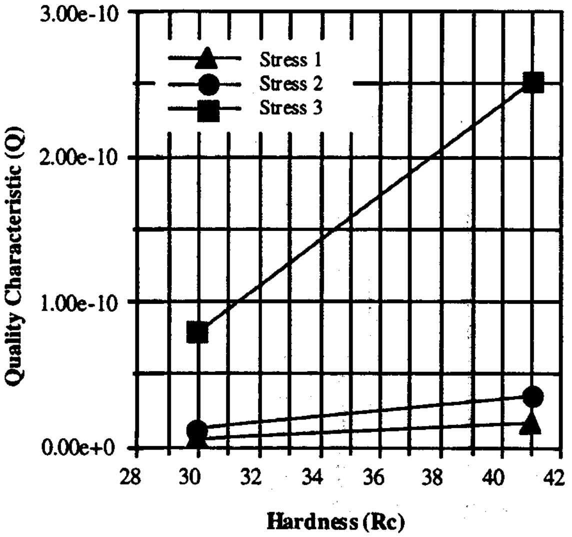

Figure 7. Quality Characteristic Comparison for No Shot Peening

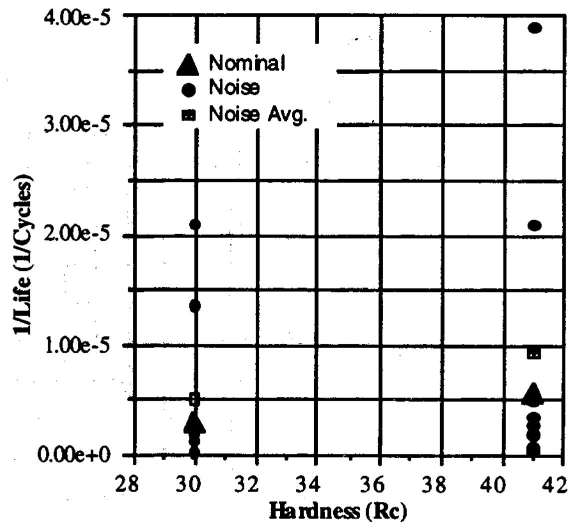

Figure 8. Reciprocal Life Data at Stress Condition 3

With Shot Peening

Figure 9 shows the sensitivity plots of the quality characteristic for pressure, hardness and shot diameter obtained from the full factorial arry run with the L16 noise array. Note that maximum life and minimum scatter occur at a hardness of 30 Rc, a pressure of 620 MPa, and a shot diameter of 1.68 mm. Figure 10 compares the effect of adding residual stress by plotting the quality characteristic against hardness for both cases. Clearly, the residual stress distribution generated during the shot peening process not only increased mean fatigue life, but reduced scatter. This can be seen for the three stress conditions in Figure 11.

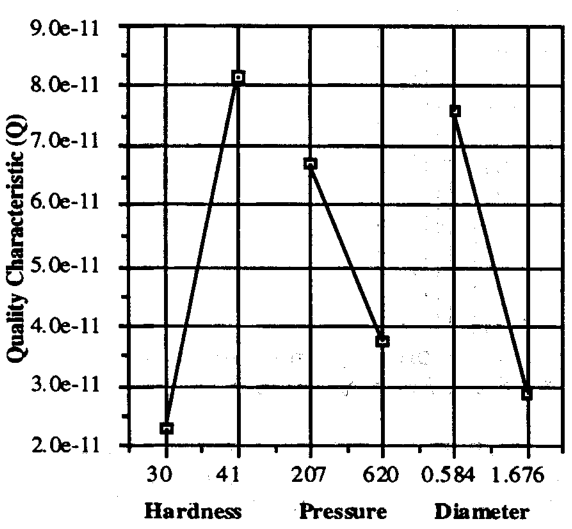

Figure 9. Q Sensitivity to Process Parameters (Stress 3)

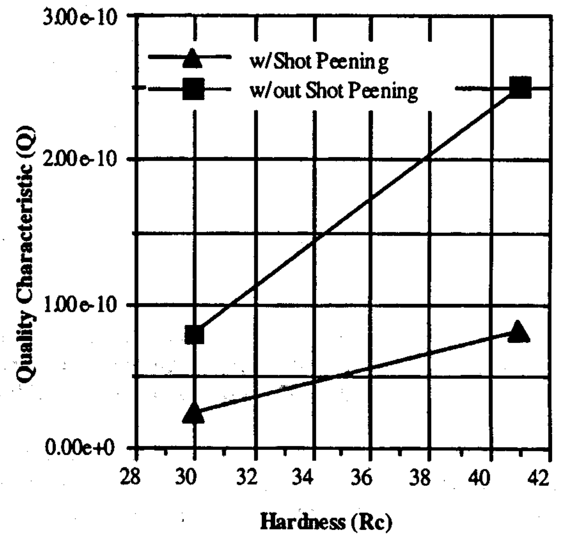

Figure 10. Shot Peening vs. No Shot Peening (Stress 3)

DISCUSSION

Robustness Trends

For this particular shot peening example over the range of parameters studied, significant improvements in fatigue life could be achieved using the following parameter settings: H = 30 Rc, D = 1.676 mm, P = 620 MPa. Also, the shot peening induced a residual stress distribution in the workpiece that not only improved mean fatigue life, but more importantly, also reduced scatter as shown in Figure 11.

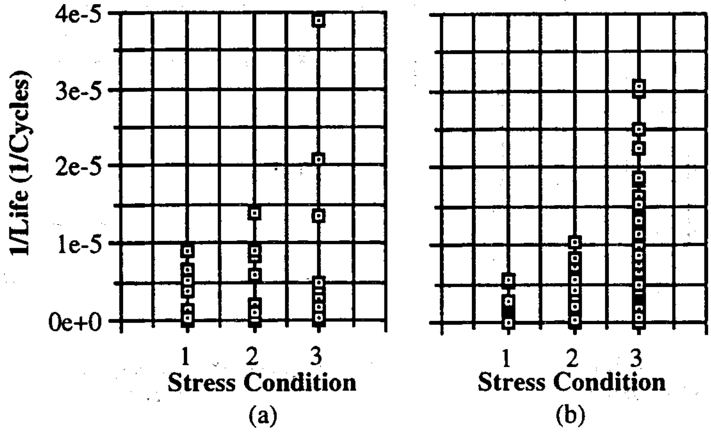

Figure 11. Fatigue performance as function of applied stress gradient (a) Without shot peening; (b) With shot peening

These results are consistent with residual stress fatigue results in the literature as shown in Figure 12. For our specific example the lower hardness specimen exhibited greater fatigue life because it had a lower crack growth rate and a higher fracture toughness. It also makes sense that the larger diameter shots with higher velocity will cause more plastic deformation. Larger shots will impart a deeper residual stress distribution into the workpiece, resulting in increased fatigue life.

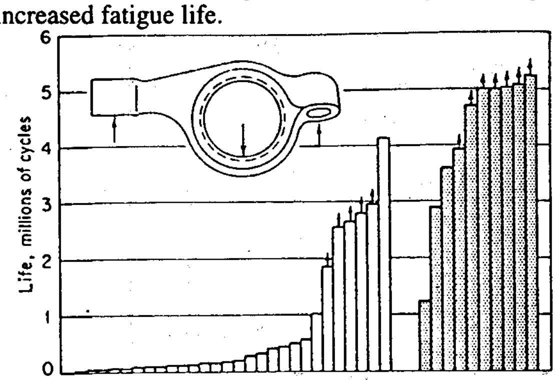

Figure 12. Fatigue performance for engine rocker arms (Almen *et al*, 1963) (Clear: Polished Arms; Shaded: Shot-Peened Arms)

Another interesting result is that, as the compressive zone depth increases, the scatter in the fatigue life decreases, shown in Figure 13 for the three stress conditions. This trend implies that the compressive zone depth could be used as an off-line production quality control parameter that relates directly to the fatigue performance of a component.

Figure 13 also shows a hypothetical reliability curve with some degree of assurance that a component will have that minimum fatigue life for a given compressive zone depth for the conditions considered here. This information is independent of the process that generated the residual stress; therefore, the designer would have flexibility in choosing a manufacturing process in addition to adjusting material parameters and geometry to attain the desired fatigue performance. Finally it should be noted the best improvements in fatigue performance for a given residual stress distribution are predicted for the steepest applied stress gradient, which is consistent with the empirical trends observed by others previously.

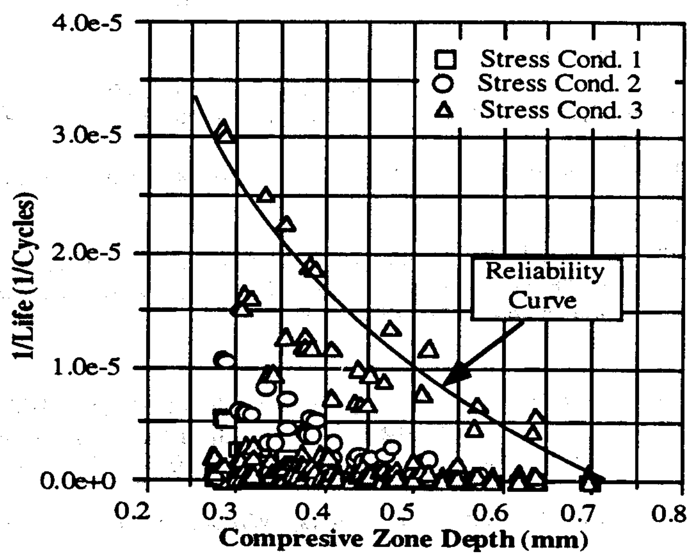

Figure 13. (1/Life) versus Compressive Zone Depth

Robust Design for Fatigue Performance

The example presented in this study applies specifically to the particular machine and material workpiece from which the data were generated, over a limited range of control parameters. However, there are generalities that we can extract from this example to form a foundation for a robust design methodology for fatigue performance. The steps for this proposed methodology are as follows.

1) Identify the pertinent parameters for the particular manufacturing process under consideration.
2) Generate a relationship between manufacturing process parameters and the residual stress distribution. This relationship may come from empirical data or from computational models.
3) Design orthogonal control and noise arrays for the key control and noise parameters.
4) Use existing or create models to predict fatigue life.
5) Use the quality characteristic, $Q = \frac{1}{N}\sum_{i=1}^{N} y_i^2$, where $y_i = \frac{1}{Life}$, to perform the analysis.

CONCLUSIONS AND FUTURE WORK

This study focused on a particular data set of shot peening results. For lower hardness 4340 alloy steel workpiece, we found that larger diameter shots at higher velocity setting increased the fatigue life and reduced its variability. Also, imparting compressive residual stresses increased the mean fatigue life and reduced the scatter of that life compared to a workpiece with no residual stresses.

Although the actual conclusions reached in this study are limited in scope, the process to arrive at those results is not. The process takes advantage of existing data or models to relate manufacturing process parameters to the workpiece fatigue life. This direct relationship between manufacturing process parameters and fatigue life benefits production quality control and gives designers added flexibility for fatigue performance design. Thus if the material and geometry specifications are constrained, we may be able to achieve the desired life and reduce scatter by an appropriate choice of manufacturing process and process parameters.

This study has developed a foundation for a methodology that will include the effects of manufacturing process on the fatigue performance of a component. The refinement of this methodology requires the following future tasks.

1) Seek better computational models for determining residual stress distributions from the process parameters.
2) Verify the results empirically.
3) Incorporate the costs associated with the different process parameters (mean and control of noise) and see how that affects the optimal selection of process and material parameters to achieve a desired fatigue life (mean and scatter).
4) Define parameters for use by designers as well as parameters for use in production quality control.

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REFERENCES

Almen, J.O., Black, P.H., (1963) *Residual Stresses and Fatigue in Metals*, McGraw-Hill, New York.

Beghini, M., Bertini, L., Vitale, E. (1994) "Fatigue Crack Growth in Residual Stress Fields: Experimental Results and Modeling," *Fatigue & Fracture of Engineering Materials & Structures*, Vol. 17, No. 12, pp. 224-234.

Brodrick, R.F. (1955) "Protective Shot Peening of Propellers," WADC Technical Report 55-56, Part 1, pp. 25-36.

Champaigne, J. (1994) "Shot Peening Process Tolerances," *The Shot Peener Newsletter*, Vol. 8, Iss. 3, pp. 1-5.

Damage Tolerant Design Handbook (1983), MC1-HB-01R, Vol. 2, Battelle Labs, Columbus, OH.

d'Entremont, K.L. and Ragsdell,K.M. (1988) "Design for Latitude Using TOPT." *ASME Advances in Design Automation*, DE-Vol. 14, pp. 265-272.

Dowling, N.E. (1982) "Growth of Short Fatigue Cracks in Alloy Steel," Scientific Paper No. 82-1D7-STINE-P1, Westinghouse R&D Center, Pittsburgh, PA.

Farrahi, G. H., Lebrun, J. L., and Couratin, D. (1995) "Effect of Shot Peening on Residual Stress and Fatigue Life of Spring Steel," *Fatigue & Fracture of Engineering Materials & Structures*, Vol. 18, No. 2, pp. 211-220.

Fuchs, H. O. (1988) "Approximate Analysis for Optimizing Prestress Treatments," *Analytical and Experimental Methods for Residual Stress Effects in Fatigue, ASTM STP 1004*, R.L. Champoux, J. H. Underwood, and J.A. Kapp, Eds., ASTM, pp. 13-20.

Hammond, D. W., Meguid, S. A. (1990) "Crack Propagation in the Presence of Shot-Peening Residual Stresses," *Engineering Fracture Mechanics*, Vol. 37, No. 2, pp. 373-387.

Hunter, J.S. (1985) "Statistical Design Applied to Product Design." *Journal of Quality Technology*, Vol. 17, No. 4, October 1985, pp. 210-221.

Kackar, N.R. (1985) "Off-Line Quality Control, Parameter Design and the Taguchi Method." *Journal of Quality Technology*, Vol. 17, No. 4, October 1985, pp. 176-188.

Lankford, J. (1985) "The Influence of Microstructure on the Growth of Small Fatigue Cracks," *Fatigue & Fracture of Engineering Materials & Structures*, Vol. 8, No. 2, pp. 161-175.

Miller, K. J. (1993) "Materials Science Perspective of Metal Fatigue Resistance," *Materials Science and Technology*, Vol. 9, pp. 453-462.

Nelson, D. V., Yuce, H., Chow, L. G. (1994) "A Study of the Growth of Small Fatigue Cracks in a High Strength Steel using a Surface Acoustic Wave Technique," *Fatigue & Fracture of Engineering Materials & Structures*, Vol. 17, No. 11, pp. 1357-1369.

Phadke, Madhav S. (1989) *Quality Engineering Using Robust Design.* Prentice Hall, New Jersey.

Schijve, J. (1994) "Fatigue Predictions and Scatter," *Fatigue & Fracture of Engineering Materials & Structures*, Vol. 17, No. 4, pp. 381-396.

Shen, G. and Glinka, G. (1991) "Weight Functions for a Surface Semi-Elliptical Crack in a Finite Thickness Plate," *Theoretical and Applied Fracture Mechanics*, Vol. 15, pp. 247-255.

Structural Alloys Handbook (1986), Vol. 1, Battelle Labs, Columbus, OH.

Sundaresan, S., Ishii, K., and Houser, D. R. (1989) "A Procedure Using Manufacturing Variance to Minimize Transmission Error in Gears." *Proceedings of the 1989 ASME Design Automation Conference*, September 1989, Montreal Canada, Vol. 2, pp. 145-152.

Sundaresan, S., Ishii, K., and Houser, D. R. (1991) "Design for Robustness using performance simulation programs." *Proceedings of the ASME Design Automation Conference*, Miami, FL. pp. 249-256.

Sundaresan, S., Ishii, K., and Houser, D.R. (1993) "A Robust Optimization Procedure with Variations on Design Variables and Constraints." *Proceedings of 1993 ASME Advances in Design Automation Conference.* September 20-22, 1993. Vol. 65-1 pp. 379-386.

Taguchi, G. (1978) "Performance Analysis Design." *International Journal of Production Research*, 16, pp. 521-530.

Taguchi, G. (1987) *System of Experimental Design.* Edited by Don Clausing, American Supplier Institute, Dearborn, MI.

Underwood, J. H. (1995) "Residual Stress Effects at a Notch Root in A723 Steel to Extend Fatigue Life," *Experimental Mechanics*, Vol 35, No. 1, pp. 61-65.

Yu, J. and Ishii, K. (1994) "Robust Design by Matching the Design with Manufacturing Variation Patterns," ASME Design Automation Conference, September 1994, Minneapolis, MN, DE-Vol. 69-2, pp. 7-14.