THE DEFORMATION AFFECTED NON STATIONARY DIFFUSION

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ABSTRACT

Today's dynamic simulations of mass transport processes are powerful and widely used, e.g. by the space industry and in the advanced control systems on modern plants. Yet, we still make little use of dynamic modeling of complex thermochemical processes. Apparently, there is a growing demand for the more advanced modeling of the real, practical problems such as the three-dimensional flow fields and multi phase flows. Despite their close relevancy to numerous technologies they are relatively little known. Such processes are common and are widely used, e.g., in metall and ceramic industry, to form and/or finish various elements that may later undergo the thermal and/or ageing treatments. Modelling of the flow in such processes represents several major challenges since the flow is inherently transient, includes a free surface and different forms of transport, e.g., diffusion.

This work will show the simulation of such complex mass transport process namely, the deformation (flow) of high viscosity compressible phase in forming unit or as a result of impact. The mathematical model of the process allows to examine the effects of the nonuniform distribution of the diffusing element at the external boundaries, the geometry and the influence of flow rate. An obvious simplification is an assumption that phase is compressible and amorphous, i.e., the Newtonian compressible fluid.

KEY WORDS:

Diffusion, simulation, flow, compresible phase, Navier-Lame equation

INTRODUCTION

Today's dynamic simulations of transport processes are powerful and widely used, e.g., by the space industry and in the advanced control systems on modern plants. Yet, we still make little use of dynamic modeling of complex thermochemical processes. Apparently, there is a growing demand for the more advanced modeling of the real, practical systems. Contrary to an extensive research on modeling of injection molding [1], the modeling of three-dimensional flow fields is relatively little known. Such processes are common, e.g., in metallurgy and ceramic, to form various elements that later undergo the thermal and/or ageing treatments. The medium is usually multi component and multi phase non homogeneous phase, that shows very complex properties. Modeling of the flow in such process represents several major challenges since the flow is inherently transient, includes a free surface and material is moving – changing its geometry.

MATHEMATICAL STATEMENT AND PHYSICAL DESCRIPTION

This work will show the simulation of complex mass transport process namely, the diffusion affected flow of high viscosity phase in the transient press or as a result of its deformation. The mathematical model of the process allows to examine the effects of the nonuniform distribution of the diffusing element and/or temperature dependent viscosity. Moreover the geometry of the deformed element and the influence of lubricant on the surface friction can be studied. An obvious simplification is an assumption that the phase is Newtonian compressible fluid.

We will study the behavior of fluid contained in time dependent domain $\Omega(t) \subset \Re^3$ during the period $t \in [0, t_{limit}]$. Its boundary $\partial\Omega(t)$ is a disjoint sum $\partial\Omega(t) = \partial^k_{rig}\Omega(t) \cup \partial^k_{free}\Omega(t) \cup \partial^u_{free}\Omega(t)$ of three regular surfaces: known rigid boundary, known free boundary and unknown free boundary respectively.

We are looking for the following unknowns:

$\upsilon : \mathrm{U}_t \{t\} \times \Omega(t) \to \Re^3$ velocity of the medium and

$p, \rho, T, g : \mathrm{U}_t \{t\} \times \Omega(t) \to \Re$ which represent pressure, density, temperature and density of the diffusing element respectively. They should satisfy three basic conservation laws:

$$\frac{\partial \rho}{\partial t} + div(\rho \upsilon) = 0 \qquad \text{Mass conservation law,}$$

$$\rho\left(\frac{\partial \upsilon}{\partial t}+\partial^{(\upsilon)}\upsilon\right)=Div\Im+\rho b,$$
$$\Im=\left\{\frac{\nu}{2}\left(\left(\upsilon_{i,j}+\upsilon_{j,i}\right)-\frac{2}{3}(div\,\upsilon)\delta_{ij}\right)-p\delta_{ij}\right\}$$

Momentum conservation law (the Navier-Stokes system),

$$\frac{\partial g}{\partial t}+div(g\upsilon)-div\left(D\,grad\,g\right)=0$$

Conservation law of the diffusing element

where $\nu=\nu(T(t,x))$ and $D=D(T(t,x))$ are viscosity and diffusion coefficient of the of the diffusing element in the phase, $b=b(t,x)$ denotes external mass forces (e.g. gravity) assumed to be well known functions of time $t\in[0,t_{\text{limit}}]$ and position $x\in\Omega(t)$. Moreover:

$$p=p(\rho,T)=C_1(\rho-\rho^*)^\gamma+C_2\rho T,$$
$$\gamma>1,\ C_1,C_2,\rho^*>0$$

Will be utilized as the equation Of state.

We assume the following initial conditions: $\upsilon(0)=\upsilon_0$, $p(0)=p_0$, $\rho(0)=\rho_0$, $T(0)=T_0$, $g(0)=g_0$ on $\Omega(0)$; and for each $t\in[0,t_{\text{limit}}]$ the following boundary conditions:

i) $g(t)=g_\partial(t)$ on $\partial\Omega(t)$,

ii) $\dfrac{\partial g}{\partial \mathbf{n}_t}(t)=0$ on $\partial^k_{rig}\Omega(t)$,

iii) $(\upsilon(t)|\mathbf{n}_t)$ vanish on $\partial^k_{rig}\Omega(t)$ in coordinate system which is stiffly attached to the press wall,

iv) $\Im(t)\mathbf{n}_t=-p_{ext}(t)\mathbf{n}_t$ on $\partial^u_{free}\Omega(t)$,

v) $\Im(t)\mathbf{n}_t=-(\Im(t)\mathbf{n}_t|\mathbf{n}_t)\mathbf{n}_t$ is perpendicular to $\upsilon(t)-(\upsilon(t)|\mathbf{n}_t)\mathbf{n}_t$ on $\partial^k_{rig}\Omega(t)$.

where $\mathbf{n}_t$ denotes the unit outward normal to the surface $\partial\Omega(t)$. Detailed formulation of the above problem can be found elsewhere [2].

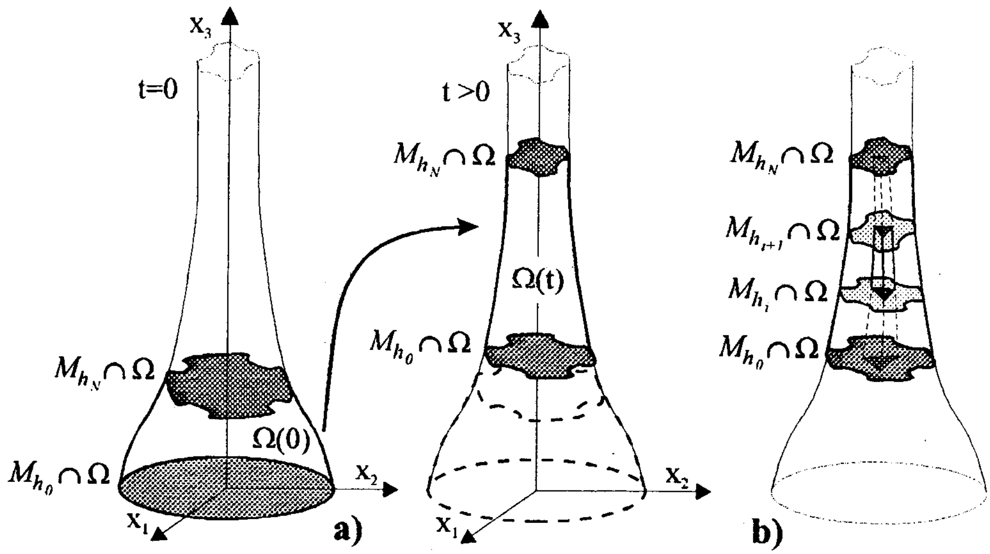

Figure 1 Schematic view of: a) the evolution of the medium shape,
b) the basic cell of triangulation in Ω(0).

DISCRETE FE/FD APPROACH

We consider an arbitrary domain $\Omega \subset \Re^3$ with regular boundary such that $\Re e_3 \subset \Omega$ and one-parameter family $\{M_h\}_{h\in\Re}$ of planes which are orthogonal to the axis $\Re e_3$ such that $M_h := \{x \in \Re^3 : (x - he_3 | e_3) = 0\}$. Moreover we assume that

$$\forall h \in \Re^3, \forall \xi \in \Omega \quad T_\xi M_h + T_\xi \partial\Omega = \Re^3$$

where $T_\xi \partial\Omega$ is a plane tangent to $\partial\Omega$ in the point ξ. We assume that we know two functions $h_{\min}, h_{\max} : [0, t_{\text{limit}}] \to \Re$ such that $h_{min}(t) < h_{max}(t)$, for each $t \in [0, t_{\text{limit}}]$ (see Fig. 1.a). Let us define

$$\Omega(t) := \Omega \cap \{x \in \Re^3, x = (x_1, x_2, x_3) : h_{\min}(t) \le x_3 \le h_{\max}(t)\}, \quad t \in [0, t_{\text{limit}}] \qquad (1)$$

and distinguish the sequence of points $h_{\min}(0) =: h_0 < h_1, \ldots, < h_{N-1} < h_N := h_{\max}(0)$. Using the algorithm of two dimensional (flat) Delunay triangulation [3, 4] we attempt to triangulate the first layer $M_{h_0} \cap \Omega$. Next we copy this triangulation to remaining layers $M_{h_i} \cap \Omega, i = 1, \ldots, N$ · solving the Dirichlet problem for two-dimensional Laplace equation. Then, using nodal points from two consecutive layers we build basic cells of triangulation (see Fig. 1.b). Each of these cell is spread to six simplexes of cubic triangulation in a canonical way. The initial mesh is transformed to the mesh covering $\Omega(t)$ for an arbitrary time instance $t > 0$. Mesh nodes take new positions, but triangulation topology keeps unchanged. Let us denote by $\wp$ the set of node labels, associated with the initial mesh. Thanks to the constant topology, labels remaind valid at $t > 0$.

To solve the presented differential problem we use the Faedo-Galerkin method with respect to the spatial variables (see e.g. Thomee [5]). We will use the family $\{X_t\}, t \in \Re_+$ of finite dimensional spaces spanned by Lagrange 1^{th} degree splines $\varphi_P : \bigcup_t \{t\} \times \Omega(t) \to \Re$ which are afine on every simplex and takes 1 at $x_P \in \Omega(t), P \in \wp$ (the current position of P^{th} node) and 0 at $x_Q \in \Omega(t)$, $Q \in \wp$, $P \neq Q$ for each $t \in [0, t_{\text{limit}}]$. We look for approximate solution in a form:

$$\sum_{P \in \wp} \lambda_P(t) \varphi_P(t, x) \in X_t, \quad t \in [0, t_{\text{limit}}] \tag{2}$$

where $\lambda_P(t) \in \Re^7$ represent approximate nodal values of $\{\upsilon_i(t)\}, i = 1,2,3$, $p(t)$, $\rho(t)$, $T(t)$, $g(t)$.

After spatial integration we may pass to the initial problem for the system of ODE's:

$$L(t) \bullet \Lambda(t) = F(t, \Lambda(t)), \Lambda(0) = \Lambda_0, \Lambda(t) := \{\lambda_P(t)\}_{P \in \wp} \tag{3}$$

Next we can solve the above system using one of the multi-step methods such as Adams-Bashforth method of third degree:

$$L(t_i)(\Lambda_{i+1} - \Lambda_i) = \frac{h}{12}\left(23F(t_i, \Lambda_i) - 16F(t_{i-1}, \Lambda_{i-1}) + 5F(t_{i-2}, \Lambda_{i-2})\right) \quad (4)$$
$$\Lambda_i := \Lambda(t_i), \quad i = 0,1,2,\ldots, \quad 0 < t_i < t_j \quad \text{for } i < j$$

Finally we consecutively solve the linear algebraic system

$$L_i \Lambda_{i+1} = R_i, \quad i=1,2,\ldots, \quad (5)$$

where

$$L_i := L(t_i), \; R_i = \frac{h}{12}\left(23F(t_i, \Lambda_i) - 16F(t_{i-1}, \Lambda_{i-1}) + 5F(t_{i-2}, \Lambda_{i-2})\right) + L(t)\Lambda_i .$$

Various combinations of initial and boundary conditions have been tested and for simple geometry of the formation process. Complex simulation system will be applied in the future for a process showing more complicated geometry, which cause significant growth of the computation and memory complexity. The results show the great potencial of the model in describing simultaneous flow and diffusion. Task migration can dramatically improve control effect in rapid changes of computer performance during one time iteration step [6].

CONCLUSIONS

1. The simulation of the forming process will give the ultimate answer for the best manufacturing parameter combinations to reduce the production cost.
2. The increased speed of computations will allow to implement process simulation as the guideline for mixing, process parameters' adaptation and optimalization of the process conditions in a production plant.
3. Task migration can dramatically improve control effect in rapid changes of computer performance during one time iteration step.

REFERENCES

1. Manzione L. T. ed.; Application of Computer Aided Engineering in Injection Molding, Hanser Publ., 1987.
2. Danielewski M, Holly K, Bożek B, Bednarz S, Golec S and Filipek R; Dynamics of the graphite electrode forming process, Univ. of Mining and Metallurgy, Cracow, 1998, Rep. 1246/98.

3. Bożek B, Holly K, Jaskólski J; Variance methods for thermo load of elements of IC engine, 20th International On Combustion Engines (CIMAC 1993), London 1993, D74.
4. Holly K, Mosurski R; An automatic triangulation of an arbitrary flat domain, Opuscula Mathematica, Vol. 17, 1997, pp. 23 - 32.
5. Thòmee V; Galerkin Finite Element Methods for Parabolic Problems, Springer Verlag 1984.
6. Barragy E, Carey G.F, Van de Geiju R; Performance and Scalability of Finite Element Analysis for Distributed Parallel Computation, Journal of Distributed and Parallel Computing 21, 1994, pp. 202-212.