Example of the Computer Simulation of Shot Peening Process

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1 Materials for simulation

40 HM steel was taken as the considered material. For the need of simulation of shot impacts at the same position, a cylindrical form of specimen was assumed. In other cases cuboidal specimens were considered. Cast iron shot of the diameter of 0.4 mm thrown with the velocity of 80 m/s was considered for calculations. Material data of specimens and shot assumed for simulation are shown in Table 1.

<table>
<thead>
<tr>
<th>Material data</th>
<th>Shot</th>
<th>Specimen material</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST170</td>
<td>ST170</td>
<td>40HM</td>
</tr>
<tr>
<td>Young’s modulus $E$ [MPa]</td>
<td>2.15 $\cdot 10^3$</td>
<td>2.06 $\cdot 10^3$</td>
</tr>
<tr>
<td>Tensile strength $R_m$ [MPa]</td>
<td>750</td>
<td>1079.0</td>
</tr>
<tr>
<td>Yield point $R_c$ [MPa]</td>
<td>380</td>
<td>882</td>
</tr>
<tr>
<td>Coefficient $k$</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Modulus of strain hardening $E_u$ [MPa]</td>
<td>19.35 $\cdot 10^3$</td>
<td>14.42 $\cdot 10^3$</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>0.295</td>
<td>0.29</td>
</tr>
<tr>
<td>Density $\rho$ [kg/m$^3$]</td>
<td>7800</td>
<td>7800</td>
</tr>
<tr>
<td>Friction coefficient $\mu$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

It was assumed that after the onset of plastic deformation, materials show a decrease of longitudinal modulus of rigidity defined by the coefficient $k$, and when stresses attain the value of immediate tensile strength $R_m$ material shows an ideal plastic flow.

2 Mathematical models

For determination of the displacements, strains and stresses the Finite Element Method was used for dynamic and static problems in elastic-plastic states with area contact \cite{1}. The search function were the nodal displacements $\mathbf{u}$, from which the individual components of nodal strain and stress vectors were determined. Non-linear equation of motion was solved by Newton-Raphson’s method \cite{1} using at each iterative step the linearized equation of motion \cite{1}:
\[
[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F^a\}
\]

where:
- \([M]\) - mass matrix,
- \([C]\) - damping matrix
- \([K]\) - rigidity matrix
- \(\{u\}\) - nodal displacement vector
- \(\{\dot{u}\}\) - nodal velocity vector
- \(\{\ddot{u}\}\) - nodal acceleration vector
- \(\{F^a\}\) - loading vector

with boundary conditions (Figs. 1 and 2) and initial conditions for time \(t = 0\) and for individual partial specimen volumes:

\[
\{u\} = \{0\}; \quad \{\dot{u}\} = \{0\} \quad \text{for specimen}; \quad [n]\{\ddot{u}\} \quad \text{for shot} = -V; \quad \{\ddot{u}\} = \{0\}
\]

where:
- \(l\) - normal vector to the specimen surface
- \(V\) - initial velocity of shot ball

Unknown values of nodal velocities \(\dot{u}_{n+1}\) and nodal accelerations \(\ddot{u}_{n+1}\) at the end of each iterative step were expressed with the aid of Newmarke’s scheme in function of the known values of \(u_n\), \(\dot{u}_n\) and \(\ddot{u}_n\) at the beginning of step and the search value of displacement \(u_{n+1}\) at the end of step.

**Figure 1:** 2-D symmetrical-axis model

**Figure 2:** 3-D model
Two models were used for calculations:

- a two-dimensional, symmetrical-axis model for static and dynamic analyses of shot impacts at the same position on specimen surface (Fig. 1)
- a three-dimensional model for dynamic analysis of successive adjacent shot impacts (Fig. 2).

In the majority of calculations the damping was neglected because of lack of data. Replacement possibility of dynamic "transient" type problem by a static one could essentially shorten the calculation time. In the equation of motion (1) it would mean the neglecting of elements related to the velocities and accelerations:

\[
[K]\{u\} = \{F^a\}
\]  

(2)

Two possible solutions of the static problem were assumed:

- forced action as a force acted on the shot ball
- displacement forced action.

At the beginning, the dynamic two-dimensional, symmetrical-axis model with a single shot ball was calculated. Afterwards from the relevant temporal sub-steps the following values were read off:

- maximum force value acted on specimen
- maximum ball displacements from nodes, which were not subjected to plasticizing.

These values were applied in several nodes on the ball surface.

Effective plastic strains (6) and stresses according to the Huber-Mises-Hencke’s hypothesis (7) were averaged in nodes. The grow of plastic strains \( \Delta \varepsilon_{\text{pl}} \) in the equation (5) was determined from the associated flow law (4), where:

- \( Q \) – plasticity potential,
- \( \lambda \) – plasticity coefficient (indexes mean: el – elastic state, pl – plastic state, tot – total, n – step number, \( \varepsilon_x, \varepsilon_{xy}, \ldots \) - components of strain tensor, \( \nu' \) – reduced Poison’s ratio)

\[
\begin{align*}
\{d\varepsilon_{\text{pl}}^n\} &= \lambda \frac{\partial Q}{\partial \sigma} \\
\{\varepsilon_{\text{pl}}^n\} &= \{\varepsilon_{\text{pl}}^{n-1}\} + \{\Delta \varepsilon_{\text{pl}}^n\} \\
\varepsilon_{\text{eq}} &= \frac{1}{\sqrt{2(1+\nu')}} \left[ (\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{xz}^2) \right]^{\frac{1}{2}} \\
\sigma_{\text{eq}} &= E\varepsilon_{\text{eq}} \\
\nu' &= \frac{1}{2} - \left( \frac{1}{2} - \nu \right) \frac{\varepsilon_{\text{el}}^n}{\varepsilon_{\text{tot}}^n} 
\end{align*}
\]  

(3) \hspace{1cm} (4) \hspace{1cm} (5) \hspace{1cm} (6) \hspace{1cm} (7)