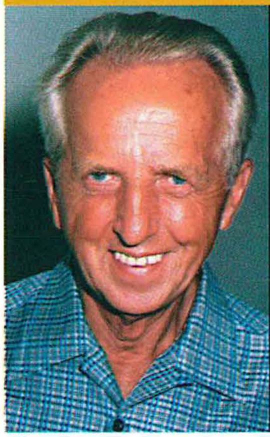

Relationships between Almen Strip Thickness and Arc Heights

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Introduction

When clamped Almen strips are being peened, a complex curvature is introduced (see Fig.1) where the vertical scale has been deliberately exaggerated. 'Curvature' is defined as 1/R where R is the radius of a curved surface. The maximum as-clamped deflection is shown as h.

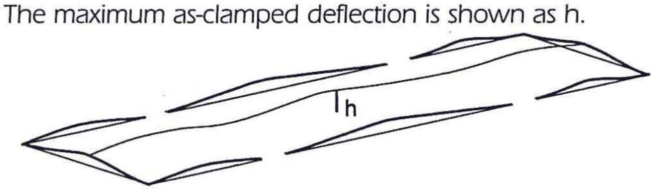

Fig.1 Complex curvature of peened, clamped Almen strip

On releasing the hold-down screws, the strip will deflect further and adopt a simpler shape such as that shown in Fig.2. Points A to D represent the positions of the balls on an Amen gage. It is this simpler shape that is measured for total deflection, h, where h = h1 + h2. This total deflection is the 'Almen arc height' and increases with the severity of peening.

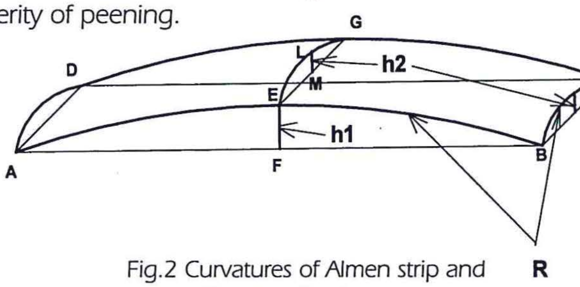

Fig.2 Curvatures of Almen strip and corresponding contributions to arc height

It is a common mistake to believe that the cause of the deflection is entirely due to a residual stress system that is introduced by peening. If we stress-relieve peened Almen strips, a substantial proportion of the curvature remains. The deflection is, in fact, due to a combination of plastic, permanent, deformation and adjustment of the peening-induced residual stress system on release from the holding jig.

Common sense tells us that, for a given severity of peening, thinner Almen strips will deflect more than will thicker strips. Intuitively we know that the thinner strips are more flexible (less rigid) than the thicker strips.

Standard Almen test strips, N, A and C have nominal thicknesses of 0.79, 1.30 and 2.39 mm respectively. The standard A strip should be used for peening intensities that produce arc heights of '0.15 mm A' to '0.61 mm A'. For intensities below '0.15 mm A', the use of N strips is recommended (ref. SAE J443), whereas for intensities above '0.61 mm A' the use of C strips is recommended. It is known that an N strip will deflect more than will an A strip if both were given identical peening treatments. Similarly, an A strip will deflect more than will a C strip – again if both were given identical peening treatments. The approximate relationships between readings of test strips N, A and C (for identical peening conditions) are as follows (ref. SAE J442):

C strip reading x 3.5 = A strip reading, hence A/C = 3.5 and A strip reading x 3.0 = N strip reading so that N/C = 3.0

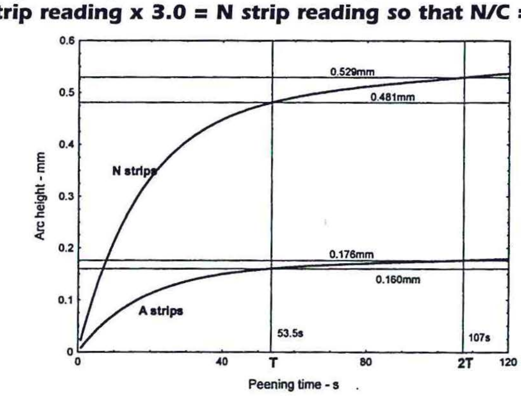

Fig.3 Strict three-to-one ratio Almen saturation curves for N and A strips

The relationships given above can be expressed graphically, as shown in Fig.3. The curve for the N strips in Fig.3 was produced from actual data, whereas the curve presented for the A strips was produced by dividing all of the N-strip data by three. Hence, for this artificial situation, there must always be a strict three-to-one ratio of arc height for all peening times. The saturation time, T, must be the same for both curves. The arc height at 2T is required to be 10% greater than that at T. These differences are 0.048mm (0.00189") and 0.016mm (0.00063") respectively for the N and A saturation curves in Fig.1. For the particular peening conditions used (S170 shot, 2 bar air pressure, 332mm gun-to-strip distance, 8.12g/s shot flow) the N strips are much more suitable than A strips would have been. That is because accurate detection of a 0.016mm arc height increase would require extremely careful attention to all aspects of arc height measurement.

The origin of the 3:1 and 3.5:1 approximate relationships is examined - using a combination of strip rigidity and the bending moment induced by peening. Rigidity is proportional to the cube of the strip thickness whereas bending moment is related to the residual stress distribution. Peening one face of an Almen strip whilst it is held down introduces a compressively-stressed surface layer with a corresponding force parallel to that surface. For equilibrium, that force must have an equal and opposite force. The distribution of that balancing force affects the net bending moment being applied to the strip. The effect of the balancing force is directly related to the sub-surface tensile residual stress profile. This profile is considered in terms of two types of sub-surface tensile residual stress distribution.

Experiments on pairs of N and A and A and C strips have been carried out to test both the validity of the models and the accuracy of the approximate relationships. It was found that the measured ratios of deflections (N/A and A/C) were lower and higher, respectively, than the 3:1 and 3.5:1 published relationships. These findings are explained in terms of the proportions of the constituent balancing residual stress factors.

Rigidity of Almen Test Strips

Almen test strips are, prior to peening, flat rectangular beams having the major dimensions of: Width, W, = 19.05mm and Length = 76.2mm. The rigidity, I, of a flat rectangular beam is given by the classic elasticity equation:

$$I = \text{Width} \times \text{thickness}^3/12 \text{ or}$$
$$I = Wt^3/12 \qquad (1)$$

Note that the rigidity depends only on width and thickness cubed and is independent of the length of the beam. If we substitute the known values of width and thickness for N, A and C strips into equation (1) we get that:

$$I_N = 19.05 \times 0.79^3 /12 \text{ mm}^4,$$
$$I_A = 19.05 \times 1.3^3 /12 \text{ mm}^4 \text{ and}$$
$$I_C = 19.055 \times 2.39^3 /12 \text{ mm}^4.$$

Hence:

$$I_N = 0.78\text{mm}^4, \quad I_A = 3.49\text{mm}^4 \text{ and } I_C = 21.67\text{mm}^4 \qquad (2)$$

We see in equation (2) quantification of what common sense tells us—that the thicker the strip is, the more rigid it is. If we change the width to 76.2mm in equation (1) we get rigidity values that are exactly four times as large as those in equation (2). That is showing us that it is four times as difficult to bend an Almen strip 'spanwise' as compared with 'lengthwise'. Again that agrees with common observation e.g. when trying to bend a ruler along as compared with across its length.

Bending Moments

A bending moment, M, is defined as a force, F, multiplied by the distance, D, from that force to a neutral axis. Fig.4 shows a simple example of a force being applied to a beam that is being used as a lever about a fulcrum at O. The force, F, at a distance, D, from the fulcrum balances a force 10F at a distance D/10 from the fulcrum. At O we have a bending moment of F.D. and an upward force of 11F. The application of a bending moment to a rectangular beam will cause it to bend. The degree of bending is conveniently expressed using the term 'curvature'. Curvature is the reciprocal of the radius of bending, R. Hence, the induced curvature is 1/R and is directly proportional to the magnitude of the bending moment. In other words, "the greater the bending moment the greater is the induced curvature".

It is important to note that:
(a) For equilibrium there must be a balance of 'clockwise' and 'anti-clockwise' bending moments and
(b) It is bending moments, not forces, that cause bending.

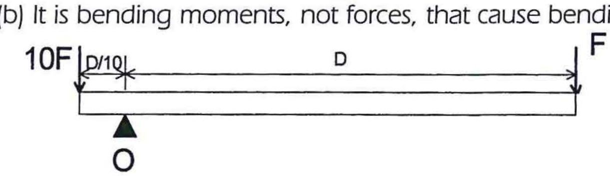

Fig.4 Simple beam with a force, F, at a distance D from the fulcrum, O, and a balancing force, 10F, at a distance D/10 from O

Bending Moments Induced in Almen Strips

When an Almen strip is being peened a compressive force, F, is induced parallel to, but not exactly at, the peened surface. That force exerts a bending moment on the strip that is initially restricted by the hold-down screws. The restriction is removed when the strip is released for measurement. Therefore the Almen strip bends further, giving the displacement that is subsequently recorded as Almen arc height.

Force is the product of stress multiplied by the area over which the stress acts. The major stress that we are concerned with is the residual compressive stress induced, by peening, within the surface layer. The area over which that stress acts is, approximately, the depth of the compressed layer multiplied by the width of the strip.

In order to quantify the bending moments induced during peening of Almen strips, we need make calculations involving residual stress distributions, forces and distances. These calculations are much easier if we use 'models' of the situation. The use of models is common in science and engineering as a means of either simplifying the mathematics involved or of producing a situation where solutions to problems are then feasible using available mathematical procedures. A solution to the problem of quantifying the bending moments induced during peening is greatly simplified if we use models of the residual stress profile. Two models of residual stress profile are considered based on two types of subsurface tensile stress distribution.

Model A

Fig.5 shows a representation of one model of the situation where the upper surface has been peened, inducing a compressed layer to a depth, d, in a strip of thickness, t. A force, F, induced by peening one face, is assumed to act parallel to, but not at, the extreme surface. It may be assumed that the force acts at some distance, $(t/2 - d/2)$, from the neutral axis of the strip. That force must be balanced by an equal and opposite force, F, acting below the peened surface. This force comes from a uniform level of tensile stress, Type A. With this model, the balancing tensile force acts on the opposite side of the neutral axis to the compressive peening force. The two forces both exert a bending moment on the strip. Because the two forces are on opposite sides of the neutral axis, they will produce bending moments that act in the same direction. The model assumes that the tensile residual stress below the compressed peened surface is virtually constant.

The next problem is to estimate the bending moment that will be present in the strip.

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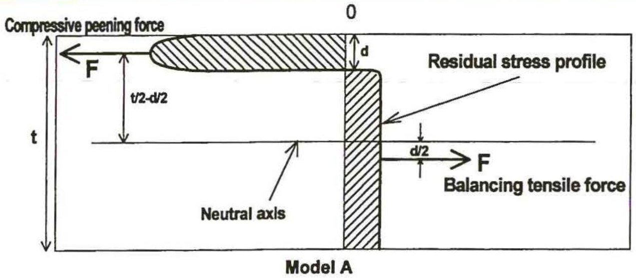

Fig.5 Origin of bending moment in clamped, peened Almen strips using Type A tensile residual stress distribution

The bending moment, M_A, acting on the strip shown in Fig.5 is given by:

$M_A = F(t/2-d/2) + F.d/2$ which simplifies to:

$$M_A = F.t/2 \qquad (3)$$

Equation (3) tells us that, using Model A, the induced bending moment depends directly on the magnitudes of both the force and on the strip thickness. Substituting the known thicknesses of Almen strips into equation (3) gives:

$$M_N = F.0.79\text{mm}/2,\ M_A = F.1.30\text{mm}/2 \text{ and } M_C = F.2.39/2 \qquad (4)$$

Hence, for a constant induced force, F, the bending moment increases linearly with strip thickness.

Model B

Fig.6 shows a different model of the residual stress distribution during peening. Instead of the balancing force being distributed evenly over the unpeened section, it is assumed that it is adjacent to the compressed surface layer and at a depth, m, below the surface, Type B. With this model, both the compressive peening force and the balancing tensile force act on the same side of the neutral axis. Because the two forces are on the same side of the neutral axis, they will produce bending moments that act in opposite directions.

The bending moment, M_B, acting on the strip shown in Fig.6 is given by:

$M_B = F\ (t/2-d/2) - F\ (t/2 - m)$ which simplifies to:

$$M_B = F.(m - d/2) \qquad (5)$$

Equation (5) tells us that, using Model B, the induced bending moment depends directly on the magnitude of the force but is independent of the strip thickness.

Curvature Caused by Bending Moment

The relationship between bending moment, M, and curvature, 1/R, for elastically bent beams is governed by the famous formula:

$$M = E.I.1/R \qquad (6)$$

where E is the modulus of elasticity and I is the rigidity.

From an educational point of view equation (6) is a classic example of how we can visualize the inter-relationships between variables. The left-hand-side, M, must be equal to the right-hand-side (which is the product of three variables, E, I and 1/R). Hence, if M increases then one or more of the right-hand-side variables must increase to preserve equality. For example, with a given type of Almen strip both E and I are constant (hopefully). Therefore, any increase in peening severity (which increases M) must be accompanied by an equal increase in curvature.

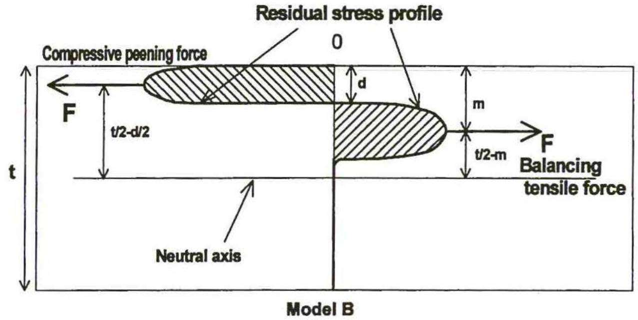

Fig.6 Origin of bending moment in clamped, peened Almen strips using Type B tensile residual stress distribution

We can re-arrange equation (6) to give:

$$1/R = M/(E.I) \qquad (7)$$

If we substitute $I = Wt^3/12$ into equation (7) we get that:

$$1/R = M/(E.Wt^3/12) \qquad (8)$$

Model A

If we substitute $M = Ft/2$ into equation (8) we get that:

$1/R = (Ft/2)/(E.Wt^3/12)$ which simplifies to:

$$1/R = 6F/(E.Wt^2) \qquad (9)$$

Now we can assume that F, E and W are all constant for Almen strips given a fixed intensity of peening. The situation then simplifies even further to give:

$$1/R = K/t^2 \qquad (10)$$

where K is a constant equal to 6F/(E.W)

Equation (10) is saying that the curvature of an Almen strip given a fixed intensity of peening is inversely proportional to its thickness. Substituting the known thicknesses of Almen strips N, A and C into equation (10) yields that:

$1/R_N = K/(0.79\text{mm})^2$, $1/R_A = K/(1.30\text{mm})^2$ and $1/R_C = K/(2.39\text{mm})^2$. Hence:

$$1/R_N/1/R_A = 2.71 \text{ to1 and } 1/R_A/1/R_C = 3.38 \text{ to } 1 \qquad (11)$$

Model B

If we substitute the value of bending moment, M_B, given by equation (5) into equation (8) we get that:

$1/R = F.\ (m - d/2)/(E.Wt^3/12)$ which simplifies to:

$$1/R = C/t_3 \qquad (12)$$

where C is a constant equal to 12F (m-d/2)/E.W

Substituting the known thicknesses of Almen strips N, A and C into equation (10) yields that:

$1/R_N = C/(0.79\text{mm})^3$, $1/R_A = C/(1.30\text{mm})^3$ and $1/R_C = C/(2.39\text{mm})^3$. Hence:

$$1/R_N/1/R_A = 4.46 \text{ to } 1 \text{ and } 1/R_A/1/R_C = 6.21 \text{ to } 1 \qquad (13)$$

Relationship Between Curvature and Arc Height

It remains for us to examine the connection between curvature and Almen arc height, h. Fig.1 represents the part of a peened Almen strip that rests on the four gage balls, touching them at A, B, C and D. Both longitudinal and transverse curvatures have been imposed on the strip. Hence, the Almen arc height is given by:

$$h = h1 + h2 \qquad (14)$$

If we assume that the curvature is a constant 1/R then the arcs AEB and ELG are circular. The heights, h1 and h2, can then be estimated using the 'intersecting chord theorem'. We have that $h1 = (AB^2/4).\ 1/R$ and $h2 = (EG^2/4).1/R$. Hence:

$$h1 + h2 = (AB^2/4 + EG^2/4).1/R \text{ or}$$

$$h = (AB^2/4 + EG^2/4).1/R \qquad (15)$$

Now AB is the fixed major distance between the balls of an Almen gage. Also EG is fixed, being equal to BC which is the fixed minor distance between supporting balls. Therefore, $(AB^2/4 + EG^2/4)$ has a constant value, K. We can therefore express equation (15) as:

$$h = K.1/R \qquad (16)$$

It follows from equation (16) that equation (11) can be expressed as:

$$h_N/h_A = 2.71 \text{ to } 1 \text{ and } h_A/h_C = 3.38 \text{ to } 1 \qquad (17)$$

Equation (13) can similarly be expressed as:

$$h_N/h_A = 4.46 \text{ to } 1 \text{ and } h_A/h_C = 6.21 \text{ to } 1 \qquad (18)$$

Experimental Verification of Almen Arc Height Ratios

The ratios given in equations (17) and (18) are below and above, respectively, the empirical values generally quoted (3 and 3.5 to 1). Those empirical values are not qualified in terms of the severity of peening that has been applied – other than that the same severity must be applied to any given pair of strip thicknesses. The author is not aware of substantial data that has been published relating Almen arc height to strip thickness. It was therefore decided to investigate the presumed relationships experimentally.

The ratios shown in Table 1 are for eight pairs of strips peened for a range of air pressures and shot flow rates. All of the strips were peened for 120 seconds using S170 cast steel shot and with a fixed gun-to-strip distance of 332 mm. The average N/A ratio is close to the Model A prediction whereas the A/C ratio is substantially above the Model A prediction. Both average ratios are substantially below those that would be predicted using Model B.

Discussion and Conclusions

It has been shown that the major reason for the difference in deflections when using different Almen strip thicknesses is the combination of bending moment and rigidity.

The predicted ratio of deflections based on those two parameters, assuming a Type A tensile stress distribution, is 2.71 and 3.38 for N/A and A/C strips respectively. These are similar to, but not identical with, the 3.0 and 3.5 ratios that have been presented in various standards publications. If a Type B tensile residual stress distribution is assumed then the predicted curvature ratios are much greater – 4.46 and 6.21 for N/A and A/C strips respectively. It follows that the Type A stress distribution gives values that are much closer to the published values than does a Type B distribution.

The measured ratio, given in Table 1, is 2.753 for N/A deflections. That value is close to that predicted using Model A and indicates that Type A tensile residual stress distribution predominates. The measured ratio for A/C deflections, 4.198, is significantly greater than the 3.38 prediction using Model A. This can be explained by assuming that the actual tensile residual stress distribution is a composite of Types A and B.

Table 1 Arc heights achieved on peening Almen strips for 120 seconds

Strip	Air Pressure - bar	Flow rate - gs^{-1}	Arc height - mm	Ratio
N	2	4.18	0.491	2.940
A	2	4.18	0.167	
N	2	8.12	0.536	2.777
A	2	8.12	0.193	
N	3	4.18	0.709	2.686
A	3	4.18	0.264	
N	3	8.12	0.717	2.607
A	3	8.12	0.275	
		Average of N/A ratios = 2.753		
A	5	8.12	0.428	4.038
C	5	8.12	0.106	
A	5	14.59	0.435	4.223
C	5	14.59	0.103	
A	6	8.12	0.498	4.256
C	6	8.12	0.117	
A	6	14.59	0.500	4.274
C	6	14.59	0.117	
		Average of A/C ratios = 4.198		

That assumption follows from the fact that the peening treatments applied for the A/C study were all much more severe than those applied for the N/C study. With high peening severities the surface work-hardening is greater. Fig.7 shows a schematic representation of a composite stress distribution. The distribution shown in Fig.7 is close to that which is commonly encountered in residual stress profile measurements. F_A and F_B represent the effective forces associated with the two components, with F_A being much greater than F_B.

It must be appreciated that the models used to predict deflection are very simple and are based on formulae that govern purely elastic behavior - rather than the mixed elastic/plastic behavior that actually occurs with peened Almen strips. Nevertheless, the models give reasonable agreement with practical observations. It would be useful to examine the application of the models to a much larger set of data. In particular the effect of shot size needs to be investigated.

Future studies will include examining changes of N/A and A/C with peening time and the effects of strip thickness on 'saturation times, T'.

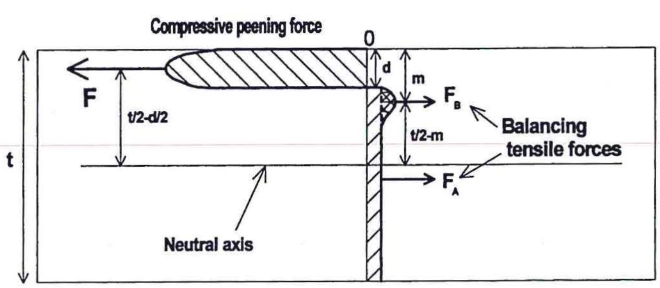

Fig. 7 Origin of bending moment in clamped, peened Almen strips with combined Types A and B tensile stress distributors