

X-Ray Residual Stress Measurements of Shot Peened Components

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INTRODUCTION

It is a paradox that we cannot measure residual stresses directly. Measurements of strain can be made, which are then converted into stress using available elastic constant values. This procedure is familiar to anyone involved in strain-gage analysis. Tiny changes in some physical property (that is proportional to strain) are monitored – such as resistance changes in the fine wire of strain gages or of interplanar spacing for x-ray analysis.

There is a vast literature on x-ray stress analysis. The intention with this article is to present only the basic engineering principles that are relevant for understanding how residual stresses can be measured in shot peened components. An example is given to show how x-ray studies can reveal variations in residual stress with distance from a peened area.

INTERPLANAR SPACING CHANGES

X-ray stress analysis relies upon the fact that tiny changes in interplanar spacing of crystalline materials can be measured accurately. Fortunately virtually all shot peened components are crystalline. With a crystalline material we have a direct relationship between interplanar spacing and the angle of x-ray reflection. That relationship is embodied in the famous Bragg equation that can be expressed in the form:

$$n\lambda = 2d_{hkl.}\sin\theta \tag{1}$$

where λ is the wavelength of the x-radiation used, d_{hkl} is the interplanar spacing of the crystal planes being monitored, n is an integer and θ is the angle of reflection.

Differentiation of equation (1) yields:

$$\Delta \theta = -\Delta d_{hkl}/d_{hkl}.tan\theta \tag{2}$$

Equation (2) shows that for a residual stress causing a strain, $\Delta d_{hkl}/d_{hkl}$, there will be a movement, $\Delta \theta$, of the diffraction angle. The magnitude of $\Delta \theta$ depends directly upon tan θ , so that large diffraction angles are necessary. For peened high-strength steels there will be maximum compressive residual lattice strains of about -0.01. By substitution in equation (2), we find that there will be a shift of x-ray diffraction angle equal to +2.138° if the diffraction angle used is 75°. At a diffraction angle of 15° the same lattice strain would give a shift of only +0.154°. Fig.1 shows a simplified representation of the x-ray diffraction situation.

If we have no residual stress then the unstrained interplanar spacing, d_u , can be measured – as in (a). A compressive residual stress, σ , decreases the diffraction angle and the corresponding increased interplanar spacing, d_n , can be measured. The measured lattice strain is then $(d_n - d_u)/d_n$ which corresponds to the vector quantity, ϵ_n , shown in (b).



Fig.1 Decrease of diffraction angle, $\theta,$ with application of compressive stress, $\sigma.$

STRESS-STRAIN RELATIONSHIPS

The penetration of x-rays is so small that we can consider the situation for peened surfaces to be one of simple two-dimensional stressing. For any stressed component there will be two principal stresses, σ_1 and σ_2 , acting along x and y axes that are both perpendiculars to each other and to a z axis (which is normal to the peened surface). With strain-gage analysis we can use a rosette of three gages to determine the directions, x and y, which the principal stresses make relative to some known direction, ϕ , see fig.2. Three strains are measured which are all parallel to the component's surface. With x-ray analysis we cannot measure strain parallel to the component's surface. Instead we rely upon strain measurements made at various angles, ψ , to the surface. Two such strain directions are illustrated in fig.2, being ε_z and $\varepsilon_{\phi, \psi}$. As will be shown later, the residual stress, σ_{ϕ} , for the known direction ϕ is deduced from these measured strains.



RELATIONSHIP BETWEEN MEASURED LATTICE STRAINS AND RESIDUAL STRESS

The classical theory of isotropic elasticity, as enunciated by such legendary figures as Timoshenko, gives that:

$$\varepsilon_{\phi,\phi} = \frac{\upsilon + 1}{E} \cdot \sigma_{\phi} \cdot \sin^2 \psi - \frac{\upsilon}{E} (\sigma_1 + \sigma_2)$$
(3)

where $\boldsymbol{\nu}$ is Poisson's ratio and \boldsymbol{E} is elastic modulus.

Equation (3) forms the basis of x-ray stress analysis. From a mathematical point of view, it is the simplest of all types of equation – a straight-line, y = m.x + c. As we vary the angle ψ the only parameter that changes is the lattice strain, $\epsilon_{\phi, \psi}$. We can therefore write equation (3) as:

$$\mathbf{\varepsilon}_{\mathbf{\phi},\,\psi} = \mathbf{m}.\,\mathbf{sin}^2\psi + \mathbf{c} \tag{4}$$

where m, the slope of the straight line, is equal to $\sigma_{\phi} (v+1)/E$ and c, the intercept of the straight line with the lattice strain axis, is equal to $-(v/E).(\sigma_1+\sigma_2)$.

The best way to understand any equation is to use it. Consider the hypothetical example illustrated in fig.3.



Fig.3 Peened disc showing orientation of significant directions.

A circular steel disc has been uniformly peened so that a surface residual stress of – 500MPa has been induced. In this particular situation the residual stress does not vary with angle ϕ , so that we have that $\sigma_1 = \sigma_2 = \sigma_{\phi} = -500$ MPa. The elastic constants for this steel disc are known to be such that E = 210GPa and v = 0.30. $\sigma_{\phi}(v+1)/E$ then equals -0.0031 and $-(v/E).(\sigma_1+\sigma_2)$ equals +0.00143. Putting these values into equation (4) gives that $\epsilon_{\phi, \Psi} = -0.0031$. $\sin^2 \psi + 0.00143$. This equation is plotted as fig.4.

Four points have been marked on the plot corresponding to ψ angles of 0, 30, 45 and 60°. The so-called "sin² ψ " technique



Fig.4 Linear relationship between lattice strain and $\sin^2 \psi$.

involves a set of such points and a least-squares straight line fitted to the points to establish the equation. The so-called "twoexposure" technique employs just two points and therefore does not require a least-squares fitting procedure to determine the straight-line relationship.

With x-ray stress analysis the parameter actually measured is the angular position, θ , of the diffraction peak. That is then converted into interplanar spacing, dy, using the Bragg equation. A vital quantity is the interplanar spacing for unstressed material, du. We cannot, however, measure du directly for a stressed component. Instead we use an average of measured d-spacings. This procedure is illustrated by fig.5 for which the lattice strains of fig.4 have been converted into interplanar spacings. The known interplanar spacing for unstressed material, du, is given a value of 1.000000. Relative to that unit value the maximum and minimum measured spacings would be 1.001430 and 0.999105 respectively. The average of those two values is 1.0002675. Using 1.0002675 for du (instead of 1.000000) introduces an error of only one part in four thousand. That error is insignificant relative to the accuracy with which we know such factors as the elastic constants.



Fig.5 Data from fig.4 plottted in terms of interplaner spacings.

Within the phrase "classical theory of isotropic elasticity" the one beguiling word is "isotropic". The bulk modulus, E, for a component manufactured from a given material can vary by more than 50%. That is because the elastic 'constants' of individual crystals vary with crystallographic direction. With x-ray stress analysis one specific crystallographic direction is involved – the perpendicular to the set of crystal planes undergoing

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reflection. It is therefore necessary to use values for E and ν that have been measured using x-rays for the specific crystallographic direction and component material being studied.



Fig.6 Part of ordered array of 0.67mm diameter indentations on mild steel plate.

SURFACE RESIDUAL STRESS STUDY

A perfectly-ordered array of peening indentations was produced on an as-rolled plate of mild steel using a 2.00mm ball indenter and an X-Y table, see fig.6. The array involved 6 rows of 21 indentations in a square array. Each indentation was 0.67mm diameter and centred on a 1mm grid.Chromium K α radiation was used to determine the surface residual stress. The x-ray beam was constrained so as to irradiate a 12mm by 1.5mm rectangular area of the specimen. With the specimen mounted on a micrometer table, measurements of stress were made at intervals of 2.00mm along the measurement line shown in fig.7. Three of the ten measurement positions are shown.



Fig.7 Schematic scale drawing of peened area and x-ray beam movement.

The primary objective was to determine the variation of induced residual stress with distance. Measurement values are shown in fig.8 together with a superimposed representation of the peened area – to scale. The as-rolled plate had an initial tensile residual stress of 50MPa.

It can be deduced from the measurements that peening has produced a maximum compressive residual stress of about 175MPa (some 40% of the U.T.S.) with a 'coverage' of only 34.7%. Compressive surface residual stress extends away from the peened region to a distance equivalent to about ten times the indentation diameters (far greater than previously supposed). The array of indentations used for this experiment is atypical, but can be justified for research purposes. Further studies will be carried out using 'ordered peening' in the light of the useful results achieved.



Fig.8 Variation of surface residual stress with distance across peened mild steel plate.