Accuracy of computerized saturation curve analysis

by David Kirk

INTRODUCTION

Accuracy has two components: **precision** and **bias**. Attention has to be paid to both components if we are to achieve high levels of accuracy.

Imagine that we have a digital thermometer, sensitive to changes of 0.001˚C, which *always* reads 104.167˚C±0.001˚C when its probe is immersed in boiling pure water at atmospheric pressure. That would represent perfect precision but would have a large bias of 4.167˚C and therefore poor accuracy. On the other hand, we could have a simple mercury thermometer that was only sensitive to ±0.1˚C but always read 100±0.1˚C when immersed in pure boiling water. That would represent low precision but zero bias and therefore better overall accuracy.

Our primary objective in saturation curve analysis is to determine the Almen arc height that satisfies the "10% criterion". The accuracy with which we achieve this objective depends upon three factors: the accuracy of our arc height measurements, data set characteristics and finally saturation curve analysis procedure. All three contain elements of variability (converse of precision) and bias. This article considers, quantitatively, the causes and effects of these elements with respect to the overall accuracy of saturation curve analysis.

ACCURACY OF ALMEN ARC HEIGHT MEASUREMENTS

(a) Variability

All arc height measurements have some degree of variability. Consider the two hypothetical sets of Almen arc height data, A and B, given in Table 1. These are for sets of twelve identical strips peened using the same conditions but by different operators. The objective in both cases was to impose an arc height of 0.0063". It can be seen that both operators were successful *on average*. The variability of arc heights for operator A was, however, much less than that for operator B. This difference is *quantifie*d by the respective standard deviations of 0.0001" and 0.0003". (Standard deviations are easily calculated using Excel. We highlight a cell and insert, for example, "=STDEV(A1:A12)" where A1:A12 contains our twelve arc height values.)

We do not need to understand the mathematical basis of 'standard deviation' in order to use it effectively (we can drive within speed limits without knowing how a speedometer works). The term standard deviation refers to the 'spread' to be expected from a set of values that are 'normally distributed'. 'Normal' distributions for standard deviations of 0.0001" and 0.0003" are shown in fig.1. The *area* under both curves is the same (1.000) representing the 100% probability of recording a value somewhere. We can see that the probability of obtaining a value very close to the mean 0.0063", is much greater with the smaller standard deviation than it is with the larger one. The data in Table 1 agrees with that prediction. Conversely, the probability of obtaining a value well away from the mean is much smaller with a lower standard deviation. Table 2 presents a useful quantification of that effect.

The significance of the probabilities given in Table 2 is twofold. On the one hand we should *expect* about one in three values to be one standard deviation or more away from the mean value. If someone regularly reports a lower probability than the

Table 1
Variability displayed by two sets of Almen arc height data.

Strip No.	Arc heights (inch x 1000) Set A	Set B
1	6.2	6.3
2	6.3	6.5
3	6.3	5.9
4	6.2	6.7
5	6.5	6.0
6	6.3	5.9
7	6.3	6.4
8	6.4	6.3
9	6.2	6.2
10	6.3	6.5
11	6.3	6.7
12	6.1	5.9
AVERAGES	**6.30**	**6.30**
STANDARD DEVIATIONS	**0.10**	**0.30**

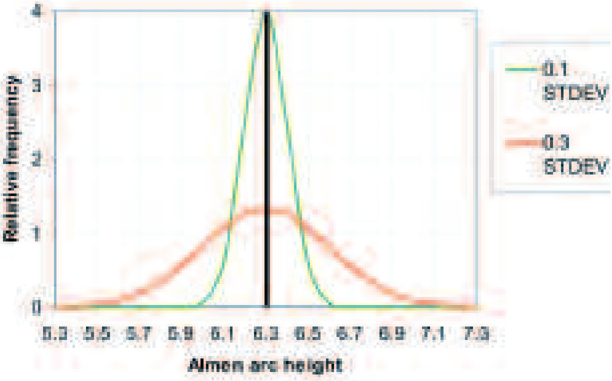
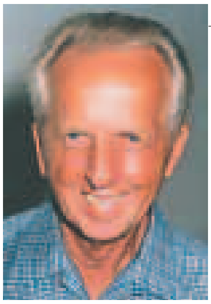


Fig.1 Spread of arc heights about a mean value of 0.0063".

Continued on page 26



Dr. David Kirk, our "Shot Peening Academic", is a regular contributor to The Shot Peener. Since his retirement, Dr. Kirk has been an Honorary Research Fellow at Coventry University, U.K. and is now a member of their Faculty of Engineering and Computing. He is currently writing a book "The Science of Shot Peening". We greatly appreciate his contribution to our publication.

Continued from page 24

Table 2
Probability of obtaining a specific value for normally-distributed measurements.

No. of standard deviations away from the mean value.	Probability of obtaining value.
1	One in three
2	One in twenty
3	One in four hundred

'norm' then we should be suspicious! On the other hand if someone regularly reported values more than three standard deviations from the 'norm' we should be worried!

We can usefully quantify the *origin* of different values of standard deviation for Almen arc height determinations. In order to do that we use a term called "variance". Variance is simply σ^2, where σ is the standard deviation. The advantage of using variance is that total variability is simply the sum of the variances of the contributory factors. The total variability of *repeated* Almen arc height values, σ^2_T, is made up of the separate variances due to strip variability, measurement errors and variations in applied peening parameters. Hence we have that:

$$\sigma^2_T = \sigma^2_S + \sigma^2_M + \sigma^2_{AP} \qquad (1)$$

where **S**, **M** and **AP** refer to strip, measurement and applied peening respectively.

Almen strips are produced to very close tolerances so that the σ^2_S contribution should normally be very small. 'Premium grade' strips will produce a smaller variance than 'standard grade' strips (other factors being equal). The σ^2_M contribution depends upon the quality of the Almen gage and the operator's skill/assiduousness. With good equipment and careful attention to detail, σ^2_M should also be relatively small. The major factor contributing to variability would then be σ^2_{AP}.

During actual shot peening there will always be some variation of the parameters that would affect strip deflection. Examples are: air pressure fluctuation, variations in flow rate and shot size (as when a batch of new shot is working its way through).

Equation (1) quantifies Almen strip measurement variability. Consider, by way of illustration, two examples A and B reflecting

Table 3 Effect of separate variabilities on total variability of Almen strip measurement.

		σ_S strip variability	σ^2_M measurement variability	σ^2_{AP} peening variability	Total variability
A (Good)	Standard deviation	0.01	0.03	0.095	**0.10**
	Variance	0.0001	0.0009	0.009	**0.01**
B (Poor)	Standard deviation	0.04	0.10	0.280	**0.30**
	Variance	0.0016	0.01	0.078	**0.09**

good and poor combinations of factors respectively. Table 3 shows the results of applying equation (1) to *hypothetical* values (expressed in units of thousandths of an inch).

It is important to appreciate that variability of data cannot be completely avoided. Data variability can, of course, be minimized by careful attention to *all three* contributory factors.

(b) Bias

One obvious source of bias is the original strip curvature or 'prebow'. This can be allowed for by 'zeroing' the gage with the slightly curved strip in place. The origins and minimization of bias with strip measurements are well-documented and therefore will not be discussed here. These include support ball wear, zero error and gage calibration over the full working range.

DATA SET CHARACTERISTICS

Important data set characteristics are: variability, number and range of the points that make up the set. All three characteristics affect the selection of fitting curve and the subsequent accuracy of saturation curve analysis. To make life complicated they all interact with one another. 'Regression' curve fitting is designed to accommodate data variability ('interpolation' merely joins data points and ignores variability). The larger the number of data points the greater will be the accuracy of the final analysis. **Data range has a profound effect on the accuracy of saturation curve solutions.** In spite of that, there is virtually no published information on the subject. Consider, for example, SAE Data Set No. 8 from Sheet 3 of the Almen Solver (downloadable from www.shotpeener.com). This data set is presented in fig.2 where a three-parameter equation has been fitted. That equation is (h = a(1-exp(-b*t^c))) where h is arc height , t is peening time and a, b and c are the three parameters. There are six data points, covering a time range from 0.25 to 4. If, however, the operator had been restricted to only four data points then a variety of saturation points would have been obtained – depending on the range covered by the four points!

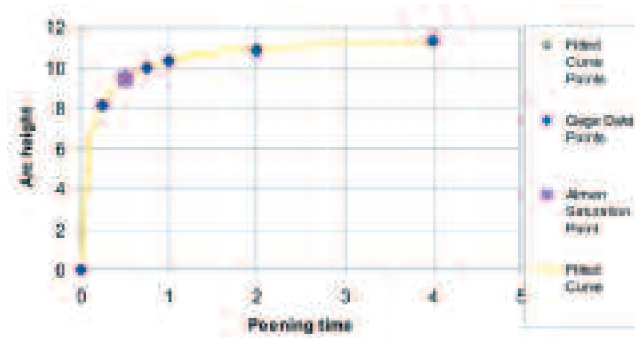


Fig.2 Three-parameter saturation curve fitted to SAE Data Set No.8.

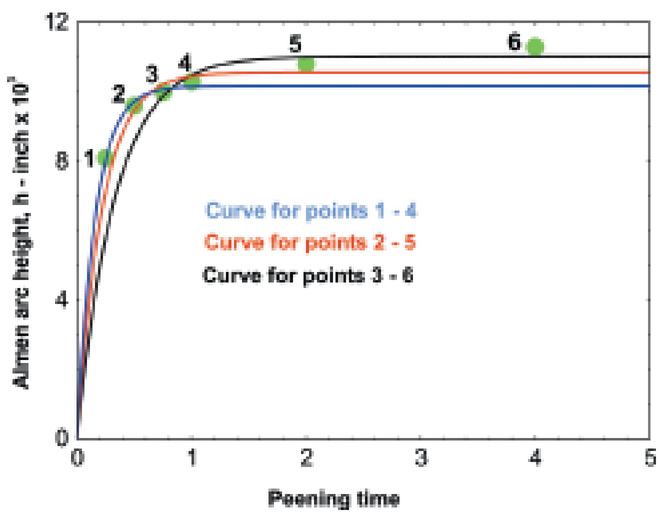


Fig.3 Two-parameter saturation curves for different ranges of SAE Data Set No.8.

This phenomenon can be illustrated by selecting different ranges of four points from Data Set No.8; for example the first four, middle four and last four. The corresponding fitted curves are shown

Continued on page 28

Continued from page 26

Table 4 Saturation points for different ranges of points from SAE Data Set No.8.

Strip No.	Peening Time	Arc Height
1	0.25	8.1
2	0.5	9.6
3	0.75	10.0
4	1	10.3
5	2	10.8
6	4	11.3
Range	Saturation point height	Saturation point time
All six, 1 - 6	9.58	0.43
First four, 1 - 4	9.15	0.37
Middle four, 2 - 5	9.51	0.51
Last four, 3 - 6	9.93	0.77

in fig.3 where computer-optimized saturation curves have been drawn using a two-parameter exponential equation [h = a (1 –exp (-b*t))].

Data Set No.8 is given in Table 4 together with the saturation point values corresponding to the four curves shown in figs.2 and 3.

The consequence of using only four data points with different time ranges is that we introduce a bias to our saturation point. That bias (which is the second factor reducing accuracy) depends on the position of the four data points relative to the saturation point. The magnitude of the bias will also be affected by the curve-fitting equation that is being used. Bias can never be eliminated but having more than four data points in each set can minimize it.

The variability of the data, together with the range and number of points, affects the overall accuracy. An unrealistic exception would be if every data point had zero variability. The larger the variability of individual data points the greater is the need to have more points in each set – provided that they cover an appropriate region of the saturation curve.

SATURATION CURVE ANALYSIS PROCEDURE

(a) Curve Finding versus Curve Fitting

A clear distinction has to be drawn between curve *finding* and curve *fitting*. Curve finding involves trying to find an equation that gives a 'best fit' to our set of data points. There are computer programs incorporating a facility that allows such an equation to be found from hundreds of 'library' examples. Manual curve sketching has a similar approach in that it involves the brain in trying to *find* a 'best fitting' curve. Both bias and lack of precision are unavoidable with manual curve fitting to any given data set. Computerized curve *fitting*, on the other hand, is when a program fits our data points to a pre-determined equation. The parameters of our pre-determined equation are adjusted until the differences between our data points and those of the curve are minimized. The program, being automatic, ensures perfect precision in terms of data analysis but cannot, of course, remove bias and variability from our data.

It is only curve *fitting* that is appropriate for saturation curve analysis.

(b) Curve Equation

Our first problem is to find an appropriate equation. The general shape of a saturation curve is well known – qualitatively. Arc height initially rises quickly, then slows down rapidly and finally becomes almost constant. This observed shape is caused by the mechanisms involved in peening the strip. Each indentation causes a tiny increase in arc height, largely due to plastic deformation. Arc height should therefore *approximate* to the shape of a coverage curve (which has the well-known form: C = 100(1-exp(-b*t))) which interprets as: **h = a(1-exp(-b*t))**. The parameter "**a**" increases with size and velocity of the shot particles. Parameter "**b**" increases with shot flow rate. As peening progresses the strip work hardens so that successive indentations induce less plastic deformation and so are less effective in producing increments of arc height. Adding a "**c**" parameter to the peening time accommodates this work hardening effect. We then have that: **h = a(1-exp(-b*t^c))**. A third, minor, mechanism is a complex combination of factors that include self-annealing and work softening. This combination progressively offsets the effect of work hardening and can be represented by a linear component, **d*t**, added to the preceding equation. A 'true shape' for saturation curves can therefore be expressed in the form: **h = a(1-exp(-b*t^c)) +d*t.**

We could only expect to have a data set that gave a 'true shape' curve if the variability effect of the individual data points was virtually eliminated. That is possible if we average hundreds of repeat measurements for each set of peening conditions. Fig.4 shows a 'true shape' distribution of data points based on Wieland's published summary of hundreds of measurements. The three curves just described are included (in x-y format), together with a two-parameter 'saturation growth' equation, enshrined in the French Specification: NF L 06-832, **y = a*x/(x+b)**. It can be seen that all four equations are reasonably good fits to the 'true shape'. The two-parameter exponential equation has a slight negative bias and the two-parameter growth rate equation has a slight positive bias – yielding saturation intensities of 0.00130 and 0.00126" respectively.

(c) Interaction between curve equation and data set characteristics

The next problem is to allow for the interaction between curve equation and data set characteristics. If a relatively low accuracy for the saturation point is acceptable then we can use the bare minimum of four points for the data set size. A four-point data set size prescribes the use of a two-parameter equation (such as h = a(1-exp(-b*t))). Greater accuracy will be achieved, for given data point variability, by using five-point data sets and a

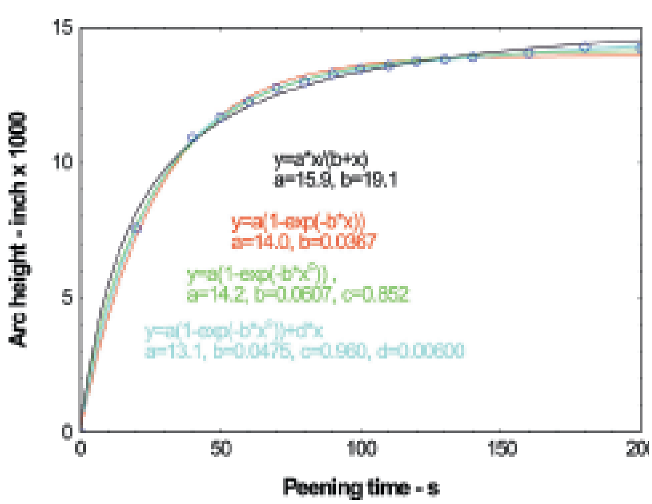


Fig.4 True shape data points fitted to a variety of equations.

Continued on page 30

ACCURACY OF COMPUTERIZED SATURATION CURVE ANALYSIS
Continued from page 28

three-parameter equation (such as h = a(1-exp(-b*t^c))). A six-point data set size will result in an even greater level of accuracy. The four-parameter equation, h = a(1-exp(-b*t^c)) +d*t, *can* be used with six-point data sets and might be expected to yield the most accurate result. In practice, however, three-parameter equations are more 'robust' and should normally be the preferred choice for six-point (or more) data sets.

(d) Accuracy of Curve Analysis Procedure

We do not need to be statisticians to appreciate the effects that our unavoidable arc height variability has had on our curve analysis. Visual examination of the plotted curve together with the data points will give us a *qualitative* impression of data set integrity. Computerized curve analysis allows *quantitative* impressions to be realized. Consider, for example, the results for the four-point SAE Data Set No. 6 shown in fig.5 and Table 5. Of the four points in the set the first two deviate substantially from the curve and the last two are very close to the curve.

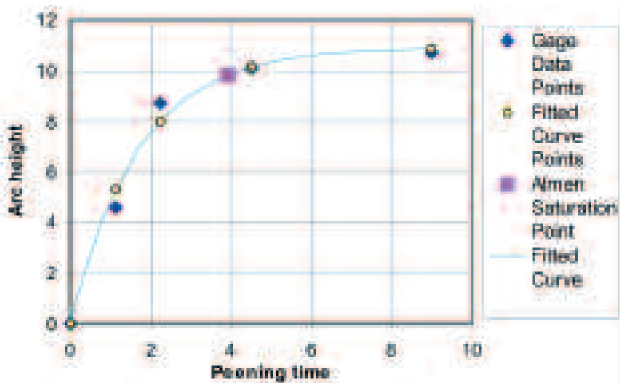


Fig.5 Two-parameter curve fitting of SAE Data Set No.6.

The curve fitting procedure used here is the "least-squares" method. This means that the 'MINSUM' of 0.99 in the last column of Table 5 is the smallest sum of differences-squared that can be found. Applying the same curve-fitting procedure to the six other four-point SAE data sets gives corresponding MINSUM values of 0.05, 0.09,0.05, 0.03,0.02 and 0.08. We can therefore equate the calculated MINSUM with the level of integrity for a given data set and hence question the integrity of set no.6 (MINSUM of 0.99). The "MINSUM" depends upon the number of points so that an alternative parameter, favored by statisticians, is the "RMS" as defined in Table 5.

Table 5 Values for two-exponent curve fitting of SAE Data Set No.6.

Strip No.	Time	Measured Arc Height	Curve point at same time	Difference	Difference -squared
1	1.13	4.6	5.28	+0.69	0.48
2	2.25	8.7	8.00	-0.70	0.49
3	4.5	10.1	10.13	+0.03	0.00
4	9	10.7	10.85	+0.15	0.02
[RMS is the "Root Mean Square" and is the square root of the average of the difference-squared values. Hence: RMS = SQRT(0.99/4)].					MINSUM = 0.99 RMS=0.498

DISCUSSION

All measurements have a degree of accuracy. That accuracy can be quantified in terms of precision and bias. The accuracy of saturation curve analysis depends primarily on two factors: data set characteristics (variability, number of points and peening time range) and curve equation. Increasing the number of data points offsets unavoidable variability of data. If the data set contains only four points then a two-parameter curve is appropriate but some bias of the analysis is unavoidable. With six points in a data set, bias is virtually eliminated - by being able to apply either a three- or four-parameter curve equation and by covering a wider time range. Data variability within the data set will also be indicated more clearly.

Almen arc height measurements are valuable pieces of information. Their value should not be diminished by subjective treatments such as manual saturation curve fitting. Computerized saturation curve analysis is a very simple operation and is completely objective. The computer program used should indicate which curve equation is being employed. Computed differences between data points and the fitted curve yield quantified measures of accuracy.

The value of Almen arc height data can be enhanced if it is computer-stored together with the computerized saturation curve analysis results and job setup details. A standard spreadsheet can accommodate all of the corresponding job setup details (shot type, machine type and number, machine settings etc.). We can then accumulate enough data to establish the standard deviations (of saturation intensity, saturation time, 'MINSUM' etc.) for each combination of job setup details. Then we can quantitatively assess the significance of subsequent 'unexpected results'. We could, for example, have a situation where a particular job setup was known to yield a saturation intensity of 0.0063" at 4.0 seconds with corresponding standard deviations of 0.0002" and 0.05 second. If, with the same job setup, the saturation intensity were subsequently found to become 0.00069" (three standard deviations away from the mean) at 4.0 seconds we would know that that was highly unlikely to be a purely random occurrence (Table 2 indicates a 'one in four hundred' possibility). Something has *probably* changed that has raised the 'indentation potential' of the shot stream – such as increased shot velocity or shot properties. Saturation times, though not substantially affecting saturation intensities, are useful indicators of shot flow rates and can be similarly analyzed. Machine controls have to be set in order to impose specified Almen intensities and coverages. Those controls have their own levels of accuracy. Peened strip arc heights can be used to assess the accuracy of each separate control. The accuracy of our saturation curves is a measure of our ability to control the relevant peening parameters.

CONCLUSIONS

1. The overall accuracy of a saturation curve analysis depends primarily on the number, range and accuracy of the arc height values in a given data set.
2. Computerized saturation curve analysis is more accurate than manual saturation curve fitting.
3. Arc height data should be fitted to an equation that reflects the 'true shape' of Almen saturation curves.
4. Computerized saturation curve analysis findings form a valuable control feature when used in association with job setup details.