State of the Art in Shot Peening Simulation

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ABSTRACT
Simulation models describing the fundamental processes taking place during a shot peening treatment are being proposed since the last four decades in order to investigate, understand, explain, and predict the correlation between the influencing factors of shot peening and the process results. This article provides a broad and extensive literature survey on simulation models of the shot peening process that have been developed and applied in the past decades and summarizes the knowledge that has been gained. The simulation models reviewed are subdivided into two main groups of models with different purposes. The first group of models deals with the dynamic shot behavior during shot peening or its related processes and is commonly known as Particle Dynamics Simulation. The second and largest group addresses the quantitative description and prediction of the deformation processes and the residual stress development during a single and/or multiple particle impact. This group of models can be separated in the analytical models, which are based on the fundamental approach of Hertz, and numerical models, such as the Finite-Element method, which has become the method of choice in recent years.

KEY WORDS
Literature Survey, Simulation, FEM, Particle Dynamics, Hertzian Pressure

INTRODUCTION

The claim for a better understanding of the shot peening process itself and steadily increasing computation power lead to a vast field of different shot peening simulation publications in the last decades. This review article summarises the approaches and conclusions of the different groups working on this subject and may be a helpful overview for those scientists and engineers dealing with shot peening simulation. It contains on the one hand Particle Dynamic Simulations dealing with the shot dynamics during the peening process and on the other hand those simulation models either analytical or numerical (FEM) that describe the deformation processes in order to predict the residual state after shot peening.

PARTICLE DYNAMICS MODELS

Particle dynamics models have the purpose to describe and analyze flow behavior during different manufacturing processes and are applied to a wide range of engineering problems. In the past years these models, originating in geotechnical and granular flow applications in the 1970s (Cundall, 1979), have gained attention from the shot peening community for their capability to characterize and analyze the impact of process parameters on the dynamics of the shot media during air blast and ultrasonic shot peening treatments. (Pile, 2005) investigated the shot velocity distribution during an ultra sonic shot peening treatment with a simple 1 D particle dynamics model taking into account inelastic shot – sonotrode interaction. The restitution coefficient, which is the ratio of impact to rebound velocity of a shot after an impact, was considered to be dependent on the interaction speed. It was found that the shot velocity distribution is governed by a Maxwell-Boltzmann distribution and the results agreed well with the experimental data obtained from (Chardin, 1996). The major limitation of their model due to its simplicity was that shot-wall interactions were excluded. This shortcoming could be eliminated by a 3D model proposed by (Micoulaut, 2005, 2007) taking into account shot – shot, shot – sonotrode, shot – sample and shot – chamber interactions. Simulations showed that inelastic side-wall collisions and associated dissipation, characterized by the restitution coefficient c_w, play a key role in the impact profile of the treated component and the sonotrode. With increased dissipation, an increased heterogeneity of the impacts arising from the accumulation of the shots on the side walls was found (cp. Fig. 1). Furthermore the shot quantity in the chamber was found to have a significant influence on the shot velocity distribution.

a) b) c) d)

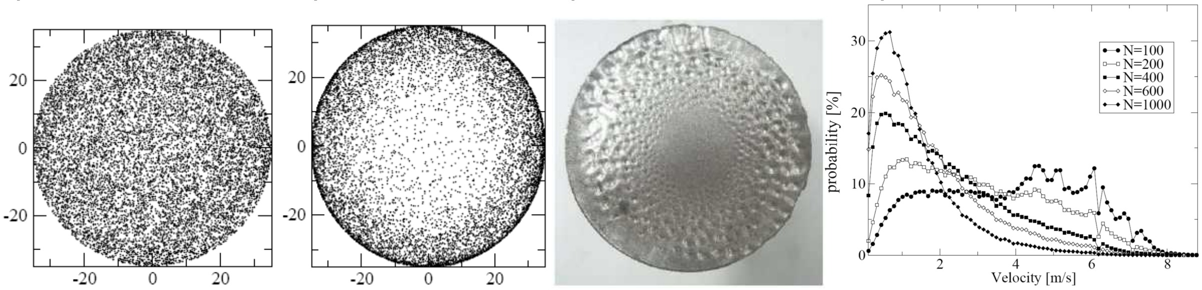

Figure 1: Distribution of impacts on chamber top wall for a) c_w = 0.91 and b) c_w = 0.2. c) sonotrode after several hours of use. d) Normalized vertical impact velocity distribution on the chamber top wall for different number N of shots (Micoulaut, 2007).

A numerical investigation on the shot velocity distribution during air blast shot peening was conducted by (Hong, 2005) with focus on the energy dissipation occurring in the shot stream due to shot-shot interactions. They found that an increasing mass flow leads to a higher amount of energy dissipation and hence to a reduction of the average impact velocity of the shots in the stream. Furthermore it was shown that shots rebound from the target surface and interact with incoming shots leading to an additional reduction of the average impact velocity. This phenomenon is important for small peening angles when the nozzle is adjusted almost perpendicular to the target. A peening angle of 62.5° was found to be more efficient than a peening angle of 90°.

MODELS FOR THE RESIDUAL STATE PREDICTION AFTER SHOT PEENING

The great majority of works on the field of shot peening simulation models are dealing with shot – work piece interaction and aim to quantitatively describe the process and predict the corresponding process results like the residual stress and work hardening state. The quantitative process descriptions presented in literature can be grouped into analytical and FEM models.

Analytical Models

The analytical process descriptions are based, in essence, on the fundamental quasistatic approaches of (Hertz, 1881) and (Tabor, 1947), assuming linear elastic or elastic-perfectly plastic deformation behavior. First, they regarded the impact of a single shot only (Li, 1991). Further on temporally and locally constant distributions of strains or stresses across the surface of the workpiece were imposed according to analytical depth-resolved equations derived for single shot impacts (Flavenot, 1977) leading to shot peening induced changes which are viewed as location and sequence independent. These models were further developed reflecting the cyclic deformation behavior and simplified equilibrium conditions (Guechichi, 1985). In (Cao, 1995) these approaches were implemented into an easily usable software package, which allows residual stress distributions to be inferred from the Almen intensity and the shot diameter. (Fathallah, 1996) introduced a similar model which takes into account friction effects, different impact angles and the hardness ratio of the shot to the workpiece. The model assumptions were first introduced by (Cao, 1994) as a software named "Shotpeen", whose successor, "Peenstress", has seen commercial use in recent years (LeGuernic, 1996)

FEM Models

Though some of the analytical models presented in literature reveal a realistic prediction of the residual stress state after shot peening FEM simulation models provide a much wider potential in analyzing the stress development during single or multiple shot impacts taking into account complex material deformation behavior.

2D Simulation Models

2D axis-symmetric FEM models are a very popular approach for the simulation of a single impact on a cylindrical or a semi infinite target body. (Hardy, 1971), (Voyiadjis, 1983), (Follansbee, 1984), (Sinclair, 1985), (Kral, 1993, 1995a, 1995b) investigated the contact stress distribution for different elasto-plastic material models. The influence of the shot deformation behaviour was analysed by (Mori, 1996), (Rouhaud, 2002), (Hirai, 2005), and (Zion, 2006). Friction was taken into account by (Mori, 1996), (Han, 2000b) and (Zion, 2006) and determined as an influencing parameter on the residual stress state as well as the assumed type of strain hardening, which was investigated by (Rouhaud, 2005) and (Ould, 2006). Model verification is commonly realized by a comparison of the contact pressure with Hertz's solution (Voyiadjis, 1983), (Follansbee, 1984), (Sinclair, 1985), (Kyriacou, 1996), (Baragetti, 1997, 2000, 2001), (Guagliano, 1999), (Schiffner, 1999), (Cochennec, 2006). Since material effects can be assumed to have a significant influence on the surface layer development (Al-Hassani, 1981) model verification beyond the pure elastic case is essential and therefore persecuted by comparing topography (Mori, 1996), (Hirai, 2005) or surface residual stresses (Ould, 2006) with experimental measurements.
A direct comparison of the residual stress depth profile along the axis of symmetry of a 2D model with experimental depth measurements on a shot peened component is questionable, since common measurement techniques only provide macroscopic and averaged stress values, whereas the measurement area is easily a magnitude larger than a dimple size. Furthermore, the fact that inclined and multiple shot impacts and associated coverage effects can not be taken into account with a 2D axis symmetric model leads to the application of 3D models.

3D Simulation Models

3D shot peening simulation models can be subdivided into micro models, focusing on inclined single (Hong, 2005) or multiple shot impacts (Al-Obaid, 1990a, 1990b, 1991, 1993), (Kyriacou, 1996), (Schiffner, 1999), (Al-Hassani, 1999), (Guagliano, 1999, 2001), (Meguid, 1999a, 1999b, 2002, 2007), (Baragetti, 2000, 2001), (Han, 2002), (Schwarzer, 2002, 2003), (ElTobgy, 2004), (Klemenz, 2005, 2006a, 2006b), (Majzoobi, 2005a, 2005b), (Frija, 2006), (Zimmermann, 2008), or macro models, where beneath the stress development in the surface near region also the component deflection due to the induced compressive stresses is taken into account. In order to minimize the number of elements many 3D micro models used symmetric boundary conditions on the lateral model faces, expanding the target body to infinity. (Zimmermann, 2008) showed that this type of boundary condition leads to unbalanced in-depth residual stress states. This is of minor interest when the profile of the compressive residual stresses in thick walled components is predicted and the magnitude of tensile residual stresses is negligible. However for thin walled components (Grasty, 1996) showed that a certain amount of deflection occurs after every shot impact. (Zimmermann, 2008) incorporated this knowledge into a sophisticated boundary

condition tolerating deflection to occur during the shot impacts, which lead to realistic predictions of the residual stress state of thin walled components.
Macro models are mostly used for the simulation of shot peen forming in order to predict the component curvature depending on the fundamental process parameters like shot diameter, velocity and peening time. In order to minimize computational costs simplified approaches or approaches equivalent to multiple shot impact loading are usually chosen, such as a "squeeze pressure" used by (Grasty, 1996) or thermal loads applied by (Levers, 1995, 1998) and (Zeng, 2002).

Thermal Effects
Contrary statements are reported about the question, whether thermal effects like adiabatic heating and associated thermal strains and/or thermal dependent mechanical deformation behavior have to be considered in a realistic FEM simulation model. On the one hand (Evans, 2002) estimated in a theoretical worst case study, where no thermal conduction takes place and all induced heat energy is retained in the deformed zone, the ratio between thermal to plastic strains induced due shot peening. Based on calculations for a stainless steel, an aluminum and a titanium alloy he concluded that thermal strains due to adiabatic heating effects make an insignificant contribution to residual stresses. (ElTobgy, 2004) confirmed this with a coupled thermal-mechanical and an isothermal single shot impact simulation. On the other hand (Rouquette, 2005) predicted strong temperature changes at the surface up to 100 °C after a single impact of a rigid shot. However, a comparison of the numerical results with temperature measurements at the bottom of a test sample revealed a certain overestimation of the calculated temperatures, which was attributed to the neglect of shot deformations during the impact. A systematical FEM study on this topic is still missing.

Explicit Dynamic vs. Implicit Quasi Static Analysis
Impact simulations can basically be carried out using an explicit or implicit dynamic analysis procedure, where the initial shot velocity is given as a boundary condition and inertia effects and stress wave propagation are taken into account, or an implicit quasi static analysis procedure, where the displacement of the shot and consequently also the indentation depth of the shot into the material have to be specified. Both methods are widely used for the shot peening simulation and have their particular assets and drawbacks. Generally the dynamic characteristics of shot peening like inertia effects as well as dynamic stress and strain propagation during a shot impact on the target material can only be captured using dynamic solution techniques. A major advantage of a dynamic simulation is that for the numerical solution of the problem an explicit time integration scheme can be used, which is better suited to handle nonlinear contact problems in comparison to implicit methods, used in static or dynamic analyses. Furthermore iterative calculations are not used and the tangent stiffness matrix is not formed, making explicit integration schemes numerically and computationally more efficient for the analysis of large models with relatively short dynamic response times. However a dynamic simulation leads to the problem that due to its nature and the usually used small model dimensions stress waves are reflected at the model boundaries back into the system leading to a certain stress oscillation in the system. (Guagliano, 2001) solves this problem by averaging the stress oscillation, whereas many other workers apply damping techniques such as stiffness and mass proportional or “numerical (bulk viscosity)” damping in order to achieve a static equilibrium (ElTobgy, 2004), (Meguid, 2002). Another common alternative is the usage of non-reflecting boundary surfaces in LS-DYNA (Meo. 2003) or infinite elements in ABAQUS (Al-Hassani, 1999), (Schwarzer, 2002, 2003), (Klemenz 2005, 2006a, 2006b), (Zimmermann, 2008) for minimized stress wave reflections at the model

boundaries. The combination of a dynamic and a subsequent static analysis was also shown to be a suitable approach achieving the equilibrium state (Wang, 2002). A potential error source in explicit dynamic analyses using first-order continuum elements with reduced-integration is their proneness to zero energy modes or “hourglassing” modes. Hourglassing was reported by (Baragetti, 2001) and could be identified in the work of (Guagliano, 2001). Generally the most effective method to minimize hourglassing is an appropriate mesh refinement in the contact zone.
An implicit quasi static impact simulation has the advantage that the mentioned damping problems do not occur. The indentation depth of the shot, which replaces the initial velocity boundary condition in static analyses, was determined by (Rouhaud, 2005), (Rouquette, 2005), and (Ould, 2006) using an energy criterion, for which the induced strain energy equals the initial kinetic energy of the shot. Another method was used by (Deslaef, 2000), who determined the indentation depth on basis of previous dynamic simulation or by (Evans, 2002), who used an analytical model.
Even though the implicit static method is often used for the simulation of a single impact or a small number of multiple shot impacts the grand advantages of the explicit dynamic method to handle highly nonlinear problems of especially large models robustly and efficiently made it to the first choice for 3D multiple impact simulations of shot peening.

Target Material Modeling
The material models used for the shot peening simulation range from elastic-perfectly plastic to elasto-plastic material models with isotropic, kinematic, or combined isotropic kinematic hardening behavior. (Kral, 1993) investigated the influence of elastic-perfectly plastic and elasto-plastic material deformation behavior with isotropic strain hardening on the residual stress development during a single shot impact. (Kyriacou, 1996) found that with an increasing strain hardening modulus the compressive residual stress maximum decreases. (Rouhaud, 2005) and (Ould, 2006) showed that the usage of a kinematic hardening model can lead to significantly smaller maximum compressive residual stresses in comparison to an isotropic hardening model due to reversed plastic flow occurring during unloading, which can not be captured by an isotropic hardening model. Experimental works from (Hasegawa, 1996) and (Kobayashi, 1998) showed that a quasi static and a dynamic ball indentation produces different residual stress states. (Kobayashi, 1998) supposed strain rate sensitivity to be the source of this phenomenon. Hence several workers investigated the influence of strain rate dependent material behavior on the residual stress state. (Al-Hassani, 1999) also found numerically significant differences between residual stress depth profiles calculated with a quasi static analysis neglecting and a dynamic simulation incorporating strain rate sensitivity by a Cowper-Symonds’ power law. (Meguid, 2002) showed that during a shot impact plastic strain rates up to $6 * 10^5$ 1/s are present. For AISI 4340 the incorporation of strain rate effects dramatically reduced the amount of induced plastic deformations in the surface near region whereas the maximum compressive residual stresses were increased by up to 32 %. In contrast (ElTobgy, 2004) found that incorporating strain rate sensitivity leads to a reduction of the maximum compressive residual stresses. (Baragetti, 2001) incorporated strain rate sensitivity by a power law leading to unrealistic high compressive residual stresses. It was concluded to ignore strain rate sensitivity in order to achieve more realistic results. (Schwarzer, 2002, 2003) also achieved a strong overestimation of the compressive residual stresses using an elasto-viscoplatic material model with isotropic hardening incorporating temperature and strain rate sensitivity of the flow stress. However, not the incorporation of strain rate effects was the reason of the overestimation but the neglect of cyclic deformation behavior. This was shown by (Klemenz, 2006a, 2006b) in a subsequent study. With the same simulation model and a combined isotropic

kinematic material model capable to accurately describe cyclic deformation behavior and the associated Bauschinger effect, a very good accordance with experimental results was achieved.

Shot Material and Shape

In many shot peening simulation models the shots are modeled as rigid bodies. This assumption, however, is questionable for shot impacts on especially hard materials, where the yield strength of both shot and work piece are of the same order and beside the elastic shot deformation a certain amount of plastic deformation might occur. Systematical analysis on this topic were carried out by (Mori, 1996), (Rouhaud, 2002), and (Hirai, 2005). (Mori, 1996) found that for a yield stress ratio of 2 between shot and target material no plastic deformations occurs in the shot. (Meguid, 2002), (El-Tobgy, 2004), and (Rouhaud, 2002) reported congruently that a rigid modeling of the shot leads to an overestimation of the compressive residual stress field in magnitude and depth, when an elasto-plastic deformation of the shot would actually take place. (Meguid, 1999a) reported that also the shot shape strongly affects the residual stress state.

Contact and Friction

In most commercial FEM software packages contact between shot and work piece can be modeled by several interaction laws, which can be grouped into normal and tangential interaction laws, whereas the latter addresses friction and is in almost every study applied either with the classic or an extended version of the isotropic Coulomb friction model. For the dynamic impact of a rigid shot on an elasto-plastically deformable body (Han, 2000a, 2000b) paid attention to normal contact penalty based interaction laws, their numerical stability and computational efficiency under the small contact deformation assumption and determined appropriate values for the so called penalty stiffness. (Han, 2000b) showed that there is an influence of the penalty stiffness on the contact force time history and on the residual stress state. (Meo, 2003) used a penalty stiffness factor of 0.4 for their 2D axis symmetric single shot impact model in LS-DYNA. The influence of tangential friction was investigated by (Han, 2000b), (Meguid, 2002), (ElTobgy, 2005), and (Frija, 2006). (Han, 2000b) analyzed the impact of the friction coefficient μ for Coulomb friction on the residual stress state after a single shot impact under an impact angle of 75° with the result that the absolute surface and maximum compressive stresses increase with an increasing friction coefficient. However, for friction coefficients higher than a certain threshold value no further influences could be observed, see also (ElTobgy, 2005), (Meguid, 2002). (Meguid, 2002) more profoundly investigated the influence of friction on the residual stress state, finding that the presence of friction leads to smaller plastic deformations and lower compressive residual stresses at the surface and larger plastic deformations and higher compressive residual stresses in depth. The reason for smaller plastic deformations at the surface could be attributed to the friction dependent higher deformation at the surface and the corresponding higher strain rates leading to a higher flow stress and consequently to smaller plastic deformations.

Impact Order, Shot Arrangement, Coverage, and Stress Evaluation Methods

Usually shot velocity, shot diameter, and impact angle are used as constant input parameters in a shot peening simulation, even though large variations can be present in reality. A further important input variable is coverage or in other terms the number of shot impacts on a certain area and their arrangement. However no comprehensive investigations on how to model coverage accurately have been carried out yet. In fact most of the 3D multiple impact simulation models developed in recent times did not focus on coverage but the general understanding of how the stress state develops during successive impacts. Simulations carried out by (Kyriacou, 1996) showed that tensile residual stresses can remain at the surface for large distances of separation of the indenters, which was confirmed by (Kobayashi, 1998) experimentally. Several workers analyzed the stress changes occurring after a first impact beneath the produced dimple when subsequent shot successively impacted in the vicinity. (Baragetti, 2001) reported that after a second impact only the surface stresses changed and the magnitude of the maximum compressive stress and the further stress depth profile from the single impact remained almost constant. Similar results were shown by (El-Tobgy, 2005), who additionally varied the spacing between the first and the second shot. He attributed work hardening effects to be the reason of the little changes occurring concluding that also with a single shot model similar results can be obtained as with a multiple shot impact model with far fewer computations. In contrast (Klemenz, 2008) discovered already a reduction of maximum compressive stresses after a second impact and proved it experimentally. Strong changes were also predicted by (Schwarzer, 2002) and (Majzoobi, 2005a). The latter observations are confirmed experimentally by (Kobayashi, 1998). (Meguid, 1999), (Schwarzer, 2002) and (Majzoobi, 2005a) pointed out that after multiple shot impacts the residual stress field is strongly heterogeneous, whereas (Majzoobi, 2005a) also showed that a kind of relative homogenization of the stress field is reached after 25 shot impacts (cp. Fig. 2).

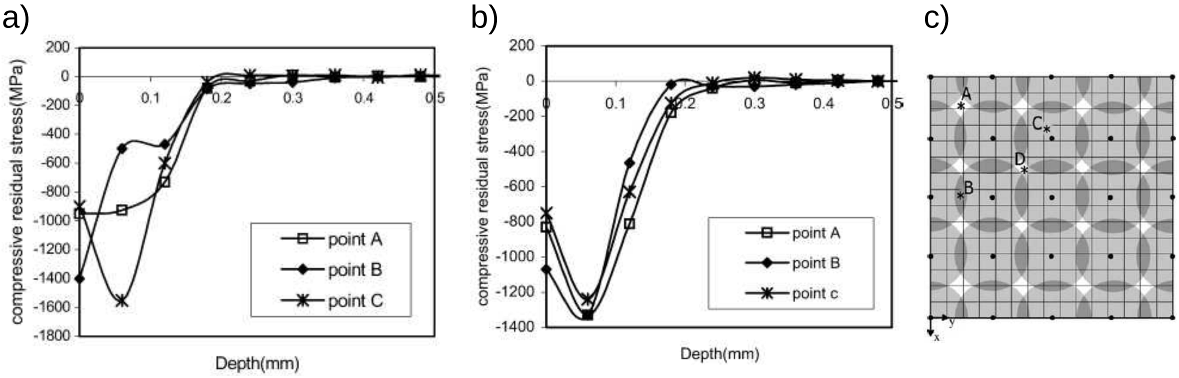

Figure 2: Residual stress depth profiles after a) 13 and b) 25 shot impacts evaluated at the positions indicated in c) (Majzoobi, 2005a).

In order to extract residual stress depth profiles from the heterogeneous residual stress field predicted in multiple impact 3D shot peening simulations (Schwarzer, 2002, 2003), (Klemenz, 2005), (Meguid, 2007), and (Zimmermann, 2008) use an averaging technique.

The influence of process parameters such as shot velocity, shot diameter, and impact angle have been frequently studied with 2D and 3D simulation models. In general the qualitative dependencies known form experimental works could be successfully captured.

Analytical Prediction Tools on Basis of FEM Simulation Results

(Baragetti, 2000, Guagliano, 1999) correlated characteristic values of the residual stress depth profile such as the maximum compressive residual stress with a non-dimensional number $\overline{N} = v \cdot \sqrt{\rho / R_m}$ representing the peening parameters shot velocity v, shot density ρ, and the ultimate tensile strength of the work piece R_m. On basis of dynamic axis-symmetric 2D FEM simulation of a single impact the analytical correlation functions were developed. This approach was implemented into a Visual Basic program, permitting the inverse calculation of the needed Almen intensity and the type of shot for a desired residual stress state. Furthermore the correlation between Almen intensity and shot velocity for a given shot diameter was established on basis of single shot FE simulations. The correlations found were in good agreement with experimental observations. (Evans, 2002) used the response surface methodology (RSM) (Box, 1951) in order to derive a linear correlation function between process parameters and process results. 2D FEM simulations provided the basis of the determination procedure for the coefficients of the correlation function. A similarity mechanics approach for the fast prediction of surface layer characteristics for arbitrary process parameters was presented by (Klemenz, 2005, 2006b). In this approach analytical correlation functions between dimensionless influence parameters and dimensionless result values on basis of few FEM simulations are used to get an immediate quantitative estimation of the influence of arbitrary process parameters on the residual stress state of the work piece.

CONCLUSIONS

Simulation models proposed in literature help to understand the complex physical processes taking place during shot peening. Addressing the dynamic shot behavior during shot peening or its related processes, Particle Dynamics Models showed their capability to realistically describe the shot-shot and shot-target interactions during shot peening as well as the shot-sonotrode and shot-wall interactions during ultrasonsic shot peening. It can be expected that these models will gain more attention to the shot peening community in near future, because of their potential to optimize shot peening process parameters. The prediction of the residual state after shot peening is focused by analytical and FEM simulations models, where the latter have become the method of choice in recent years. 2D axis symmetric and 3D simulation models showed their capability to describe the dependency between shot velocity, diameter and the corresponding residual stress state. 3D models hereby overcome the shortcomings of single impact 2D axis symmetric models, to take into account inclined and multiple shot impacts, coverage, and deflection effects and should accordingly be preferred when quantitatively realistic results are expected. For a realistic prediction of the residual stress state the targets deformation behavior and corresponding material effects like strain rate sensitivity and the Bauschinger effect have to be taken into account by an appropriate material model. Friction between shot and target was pointed out to influence the residual stress state up to a certain threshold friction coefficient. For a better comparability with experimental residual stress measurements averaging techniques of the heterogeneous stress field calculated with 3D multiple impact simulation models have been proven to be necessary. Analytical prediction tools on basis of FEM simulation results were demonstrated to be a useful extension for an instant estimation of the residual stress state after shot peening.

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