DEVELOPMENT OF FATIGUE LIMIT ANALYSIS DIAGRAM

Fumio Takahashi 1, Kotoji Ando 2, Koji TAKAHASHI 2, Hideki OKADA 1, Masayuki OOHASHI 3, Hironori EBASHI 4

1 NHK SPRING Co.,Ltd., 3-10 Fukuura Kanazawa-ku Yokohama 236-0004, Japan
2 Toyo Seiko Co., Ltd. , 3-195-1 Umaganji Yatomi City Aichi 490-1412, Japan
Yokohama National University 79-1 Tokiwadai Hodogaya-ku Yokohama 240-8501, Japan
3 Keihin Hatsujo Co., Ltd., 5-2931 Uragou-cho Yokosuka Kanagawa 237-0062, Japan
4 Horikiri, Inc, 1827-4, Kamikouya Yachiyo Chiba 276-0022, Japan

ABSTRACT
Development of Fatigue Limit Analysis Diagram: FLAD was conducted to estimate the fatigue limit of materials with a surface defect like a surface flaw. To make the FLAD, new criterion of cyclic plastic zone, based on Dugdale model, and cyclic Crack Tip Opening Displacement : CTOD, were proposed. The FLAD in this study considers the effect of stress ratio. Therefore, assuming the residual stress as the stress ratio, the shot peening effect was considered. The FLAD using new criterion was compared with literature data. As a result, the proposed FLAD concurred with experimental data despite differences in materials and stress ratios.

KEY WORDS
Fracture analysis diagram, Plastic zone, Crack tip opening displacement, Shot peening, Stress ratio

INTRODUCTION
Suspension springs have been used under increasingly higher stress conditions to satisfy the weight saving needs of automotive manufacturers. Hardness has been increased in order to achieve higher strength. In general, high hardness materials have high notch sensitivity and even small flaws lead to a significant decrease in fatigue strength. Therefore, it is important to develop the fracture criterion by considering the defect effect.
In the case of static fracture, the Fracture Analysis Diagram: FAD, which is one of the fracture criterion, is well known. As the FAD is visible to determine whether a fracture exists or not, it would be industrially useful if it can be applied to fatigue fracture criterion. To improve fatigue strength, applying surface compressive residual stress to the surface in automobile parts, like suspension springs, is often conducted. It would be more useful if FAD would consider the effect of the surface compressive residual stress.
However, only a few studies [1] concerning estimation of fracture criterion using FAD exist, and no studies concerning residual stress to the FAD in the case of fatigue fracture have been published. Therefore, the development of FAD considered the phenomenon of fatigue or Fatigue Limit Analysis Diagram: FLAD was studied. To make the FLAD, new criterion of cyclic plastic zone, based on Dugdale model [2], and cyclic Crack Tip Opening Displacement : CTOD, were proposed.

MODEL

Development of fatigue limit analysis diagram by cyclic plastic zone

Consider a center crack of which the length is 2*a* in an infinite plate. In fatigue, reverse yielding is generated on the crack tip by stress concentration in the process of unloading from $\sigma=2\sigma_a$ to $\sigma=0$ shown in Fig. 1. In the model in this section, the crack growth limit will be estimated by cyclic plastic zone, r_p^c generated by the reverse yielding, which is compressive yielding. We assume that fatigue crack growth starts when each cyclic plastic zone, r_p^c, of large scale yielding and that of small scale yielding reaches critical value, $r_p^c{}_c$. This crack growth inception condition is supposed to depend on the cyclic plastic zone size without reference to the yielding state (small or large) and equal between the two (cyclic plastic zone size in small scale yielding and large scale yielding). As a result, in the case that the stress ratio, R, equals zero, the FLAD based on cyclic plastic zone for arbitrary yielding scale is shown as the following equation.

$$\frac{\Delta K_{th(a),R=0}}{\Delta K_{th\infty,R=0}} = \frac{\alpha\sigma_{w(a),R=0}}{\sigma_Y} \cdot \left(\frac{8}{\pi^2} \left\{ \sec\left(\frac{\pi}{2} \cdot \frac{\alpha\sigma_{w(a),R=0}}{\sigma_Y} \right) - 1 \right\} \right)^{-\frac{1}{2}} \tag{1}$$

$$a = \frac{1}{\pi} \left(\frac{\Delta K_{th(a),R=0}}{2\sigma_{w(a),R=0}} \right)^2$$

Where. $\Delta K_{th(a),R=0}$ is threshold stress intensity factor when the stress ratio equals zero, $\Delta K_{th\infty,R=0}$ is the threshold stress intensity factor of large scale crack when stress ratio equals zero, α is material constant, $\sigma_{w(a),R=0}$ is fatigue limit with a crack and σ_Y is yield stress.

In the case of $R \geqq 0$, we consider the following equation between $\Delta K_{th\infty,R=R}$ and $\Delta K_{th\infty,R=0}$ at arbitrary stress ratio.

$$\Delta K_{th\infty,R=R} = (1-R)\Delta K_{th\infty,R=0} \tag{2}$$

Suppose that the relationship between $\sigma_{w0,R=R}$ and $\sigma_{w0,R=0}$ is true on the Soderberg diagram and equation (2) is true for arbitrary crack length, *a*. When equation (2) is substituted with equation (1), the FLAD for $R \geqq 0$ is the following equation.

$$\frac{\Delta K_{th(a),R=R}}{\Delta K_{th\infty,R=0}} = \{(1-R)\alpha + 2R\} \cdot \frac{\sigma_{w(a),R=R}}{\sigma_Y} \cdot \left(\frac{8}{\pi^2} \left[\sec\left\{ \frac{\pi}{2} \cdot \left(\alpha + \frac{2R}{1-R} \right) \cdot \frac{\sigma_{w(a),R=R}}{\sigma_Y} \right\} - 1 \right] \right)^{-\frac{1}{2}} \tag{3}$$

On the other hand, the following relationship is supposed to be true between $\Delta K_{th\infty,R=R}$ and $\Delta K_{th\infty,R=0}$ in R<0.

$$\Delta K_{th\infty,R=R} = \Delta K_{th\infty,R=0} \tag{4}$$

When equation (4) is substituted with equation (1), the FLAD for R<0, the following equation in the same as when R≧0.

$$\frac{\Delta K_{th(a),R=R}}{\Delta K_{th\infty,R=0}} = \frac{(1-R)\alpha + 2R}{1-R} \cdot \frac{\sigma_{w(a),R=R}}{\sigma_Y} \cdot \left(\frac{8}{\pi^2} \left[\sec\left\{ \frac{\pi}{2} \cdot \left(\alpha + \frac{2R}{1-R} \right) \cdot \frac{\sigma_{w(a),R=R}}{\sigma_Y} \right\} - 1 \right] \right)^{-\frac{1}{2}} \tag{5}$$

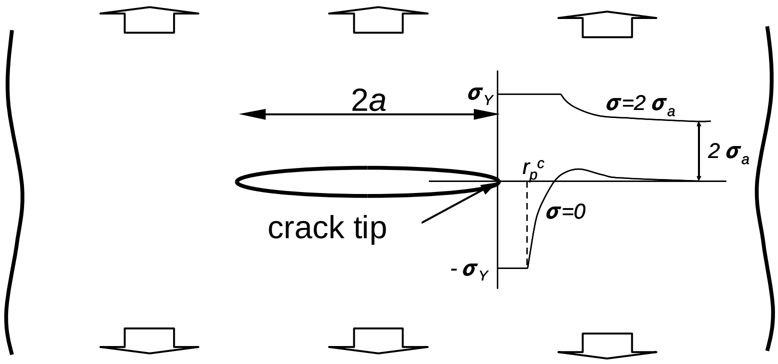

Fig. 1 Stress distribution near crack tip subject to unilateral fatigue

Development of fatigue limit analysis diagram by cyclic clack opening displacement

In this model in this section, we consider both cyclic Crack Tip Opening Displacement: CTOD of large scale yielding (Dugdale model[2]) and that of small scale yielding. Crack growth will start when each cyclic CTOD, δ_c reaches critical value, δ_c^c. This crack growth inception condition is supposed to depend on the cyclic CTOD without reference to the yielding scale and equal between the two. As a result, in the case of R=0, the FLAD based on cyclic CTOD for arbitrary yielding size is shown as the following equation.

$$\frac{\Delta K_{th(a),R=0}}{\Delta K_{th\infty,R=0}} = \frac{\pi\alpha}{\sqrt{8}}\left(\frac{\sigma_{w(a),R=0}}{\sigma_Y}\right)\left(\ln\left[\sec\left\{\left(\frac{\pi\alpha}{2}\right)\left(\frac{\sigma_{w(a),R=0}}{\sigma_Y}\right)\right\}\right]\right)^{-\frac{1}{2}} \tag{6}$$

We can derive the equation for cyclic CTOD with the assumption that was used when the FLAD for cyclic plastic zone was derived. The FLAD for $R \geqq 0$ is the following equation.

$$\frac{\Delta K_{th(a),R=R}}{\Delta K_{th\infty,R=0}} = \{(1-R)\alpha + 2R\}\frac{\pi}{\sqrt{8}}\left(\frac{\sigma_{w(a),R=R}}{\sigma_Y}\right)\left(\ln\left[\sec\left\{\left(\frac{\pi}{2}\right)\left(\alpha + \frac{2R}{1-R}\right)\left(\frac{\sigma_{w(a),R=R}}{\sigma_Y}\right)\right\}\right]\right)^{-\frac{1}{2}} \tag{7}$$

The FLAD for R<0 is equation (8).

$$\frac{\Delta K_{th(a),R=R}}{\Delta K_{th\infty,R=0}} = \frac{(1-R)\alpha + 2R}{1-R}\frac{\pi}{\sqrt{8}}\left(\frac{\sigma_{w(a),R=R}}{\sigma_Y}\right)\left(\ln\left[\sec\left\{\left(\frac{\pi}{2}\right)\left(\alpha + \frac{2R}{1-R}\right)\left(\frac{\sigma_{w(a),R=R}}{\sigma_Y}\right)\right\}\right]\right)^{-\frac{1}{2}} \tag{8}$$

Figs. 2 and 3 show the FLADs by cyclic plastic zone and cyclic CTOD respectively. Each curve is Failure Assesement Curve :FLC which indicate fracture limit. We can estimate that fracturing will not happen inside FLC and fracturing will happen outside FLC. As decreasing R, the safty zone that fracturing will not happen spreads in the case of R>0 and does not spread to the vertical axis in the case of R<0 with each FLAD.

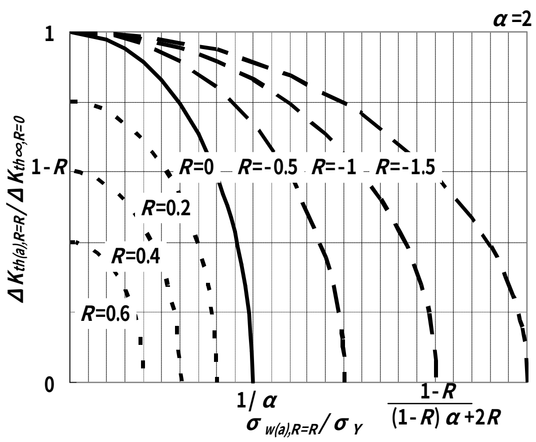

Fig. 2 FLAD based on cyclic plastic zone criterion

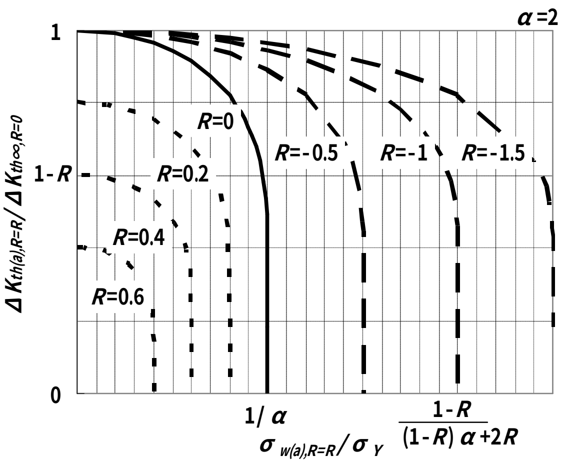

Fig. 3 FLAD based on cyclic CTOD criterion

APPLICATION OF THE MODEL

Comparison between calculation equation results and literature data

We validate the assessment equation with literature data. The quoted material and the references [(3)-(6)] are shown in Tab. 1. The results of comparison between the FLAD and literature data for each material are shown in Fig. 4. Assessment using FLAD had not applied to spring steel which is high strength material. The FLAD in this study considers stress ratio, so it can consider the effect of shot peening by considering that the residual stress affects stress ratio. SUP9(SP) in Tab. 1 is the data with shot peening. Application method of the FLAD to shot peened material is shown in the next section. The residual stress of shot peening has compressive residual stress of 500MPa. We assume that this stress affects stress ratio.

The lower limit of quoted data approximetly agrees with FAC of cyclic plastic zone without reference to material and the stress ratios. It was confirmed that the FLAD in this study indicats the fracture limit of material.

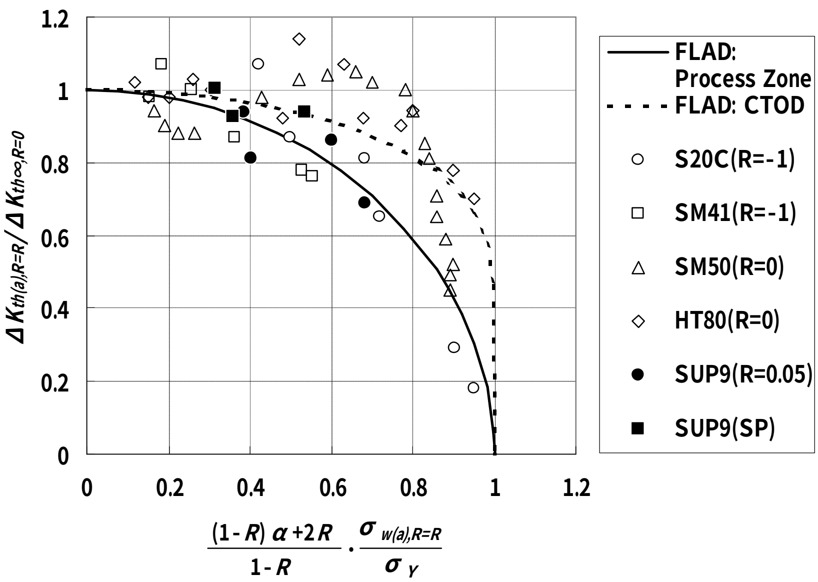

Fig. 4 Comparison between FLAD and the data quoted from Tab. 1

Tab. 1 Material database

Material	σ_Y MPa	$2\sigma_{w0,R=R}$ MPa	$\Delta K_{th,R=R}$ MPa·m$^{1/2}$	R	Ref.
S20C	366	235	5	-1	(3)
SM41	251	177	7	-1	(4)
SM50	373	378	5	0	(5)
HT80	726	549	5	0	(5)
SUP9	1250	880	7.5	0.05	(6)
SUP9	1250	880	7.5	SP※	(6)

※: In case of SP, stress ratio depends on the applied stress.

Assessment method using FLAD for shot peened specimens

Fig. 5 shows an example of the assesment method using the FLAD for shot peened specimens. Let us consider a shot peened specimen having a semi-circuler crack of a=0.2mm half surface length. Suppose that the stress amplitude, σ_a of 350MPa and R=0 is applied to the specimen in case 1. Compressive residual stress by shot peening is 500MPa based on past experimental results. Without shot peening, the data of the specimen having an a=0.2mm semicircular crack, which is indicated by a solid circle, ● in Fig. 5, is outside the FAC and the specimen is predicted to fracture. The data regarding shot peened material is indicated by an open circle, ○ due to decreasing $\Delta K_{th(a),R=R}$. This is because we can consider only the stress which causes crack opening, that is the tensile stress equals residual stress subtracted from applied stress. The FAC also changes to dashed-dotted line from the solid line with changing stress ratio at the same time. The data indicated by an open circle,○ is predicted not to fracture because it is inside the FAC.

Suppose that stress amplitude, σ_a of 400MPa and R=0 is applied to the specimen in case 2. Considering case1, the assessment is a comparison between open triangle, △ and dotted line. The amplitude stress, σ_a of 400MPa can be estimated to be the fatigue limit of the shot peened specimen having an a=0.2mm crack because the data of open triangle, △ is just on the dotted line. These result agree with our past experimental data. As just described, even the fracture limit of the shot peened material can be estimated using the FLAD.

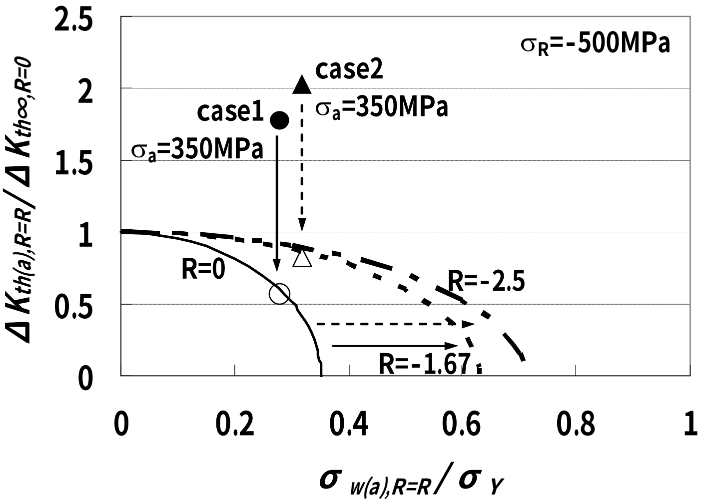

Fig. 5 Assesment method using FLAD for shot peened specimens

CONCLUSION

Development of fatigue limit analysis was conducted and FLADs were derived. The comparison between FLADs and literature data was conducted to check the validity of assessment equation.

(1)It is confirmed that the proposed FLAD is even applicable to spring steel which is high strength material.

(2)The FLAD agrees well with experimental data without reference to material and stress ratio.

(3)Considering residual stress by Shot peening stress ratio, it is confirmed that the FLAD agrees well with the fracture data of shot peened specimens.

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