An analytical approach to relate shot peen forming parameters to resulting curvature with expanding cavity model

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Abstract
For shot peen forming process, it is essential to determine the forming parameters for forming a specific component shape and to predict the resulting shape with planned parameters. A large number of physical experiments are usually involved to determine the relationships between the parameters and the shape. An analytical approach with expanding cavity model is proposed to predict the resulting curvatures for various peen forming parameters. The predicted stress fields for one-shot impacting are in good agreement with simulated results. For multiple-shot impacting, the shots are considered in regular distribution and the stress fields induced by these shots overlap in the elastic regions. The peening coverage is calculated from the separation distance of the adjacent shots and the shot dimple size. The relationships between the resulting curvature of plate and different peening coverage values are predicted and compared with experiments. The calculated results are consistent with the experimental results.

Keywords Shot peen forming, analytical model, stress field, peening parameter, experiment.

Introduction
Shot peen forming is a well-established method for shaping contour of thin structures such as aircraft wing panels. In the forming processes, shots impact on the component surface with high speed producing numerous elastically-plastically deformed dimples resulting in a residual compressive stress distribution in the surface layer and a curvature deformation of the component.
Several theoretical works have been performed to calculate the residual stress profiles. Li et al. [1] developed a simplified analytical model to calculate the induced stress field for 100% peening coverage. Miao et al. [2] followed Li's model [1] to calculate the induced stress in a semi-infinite body and related peening parameters to Almen intensity. Al-Hassani [3] and Al-Obaid [4] expressed the induce stress profile with a cosine function and introduced a spherical model for shot peen processes.
In this paper, an analytical approach based on expanding cavity model is proposed to relate the peen forming parameters to the residual stress field and the resulting curvature of plate for various peening parameters.

Analytical model
The process of one shot with velocity V impacting onto a target is treated as quasi-static process. The internal stresses induced by the shot impacting are calculated with the expanding cavity model as shown in Fig. 1. The cavity radius a is equal to the projected radius of the shot dimple. The outer radius of the plastic zone and the target are r_c and r_o, respectively.
Gao et al. [5] presented the stress components in the target during loading for power-law hardening plastic material. In polar coordinates, the radial σ_r and hoop $\sigma_\theta, \sigma_\varphi$ stress components in the plastic zone ($a \le r \le r_c$) are given by

$$\sigma_r = \frac{2\sigma_s}{3}\left(\left(\frac{1}{n}-1\right)-\frac{1}{n}\left(\frac{r_c}{r}\right)^{3n}+\frac{r_c^3}{r_o^3}\right) \qquad (1)$$

$$\sigma_\theta = \sigma_\varphi = \frac{2\sigma_s}{3}\left(\left(\frac{1}{n}-1\right)+\left(\frac{3}{2}-\frac{1}{n}\right)\left(\frac{r_c}{r}\right)^{3n}+\frac{r_c^3}{r_o^3}\right) \tag{2}$$

where, n is the strain-hardening exponent and σ_s is the yield stress of the target material.

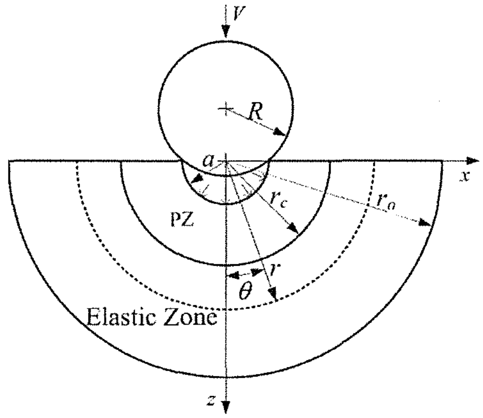


Fig. 1. Schematic view of shot indentation with cavity radius a and plastic zone outer radius r_c and thick sphere outer radius r_o.

The stress components in the elastic zone ($r_c \le r \le r_o$) are given by

$$\sigma_r = \frac{2\sigma_s}{3}\frac{r_c^3}{r_o^3}\left(1-\frac{r_o^3}{r^3}\right) \tag{3}$$

$$\sigma_\theta = \sigma_\varphi = \frac{2\sigma_s}{3}\frac{r_c^3}{r_o^3}\left(1+\frac{r_o^3}{2r^3}\right) \tag{4}$$

The mean contact pressure is

$$p = \frac{2}{3}\sigma_s\left(1+\frac{1}{n}\left(\frac{Ea}{4R\sigma_s}\right)^n-\frac{1}{n}\right) \tag{5}$$

Considering the initial kinetic energy of one shot is mostly converted into elastic-plastic work during the impact, the shot parameters are related to the shot dimple as

$$\frac{1}{2}kmV^2 = \int_0^{h^*}\pi p a^2 dh \tag{6}$$

where, k is an efficiency coefficient related to elastic and thermal dissipation and taken as 0.8 [6]. m is the mass of one shot. h^* is the finial depth of the shot dimple. The integral results is

$$\frac{\pi\sigma_s}{3kmR}\frac{n-1}{n}a^4 + \frac{4\pi\sigma_s}{3kmR}\left(\frac{E}{4R\sigma_s}\right)^n\frac{a^{n+4}}{n(n+4)} - V^2 = 0 \tag{7}$$

The shot dimple radius for certain peen forming parameters and conditions can be obtained numerically with Eq. (7). Then, the outer radius of plastic zone is

$$r_c = \left(\frac{Ea^4}{4R\sigma_s}\right)^{1/3} \tag{8}$$

Substituting r_c to Eq. (1)-(4), the stress distribution for one-shot loading is obtained. The stress components after shot unloading is calculated by considering (1) unloading is an elastic process, (2) the hydrostatic stress does not introduce plastic deformation. The equilibrium equations of transverse residual stresses is

$$\pi r^2\sigma_r^t(r)-\pi(r+dr)^2\sigma_r^t(r+dr)+\frac{\pi}{2}\left(\sigma_\theta^t(r)+\sigma_\theta^t(r+dr)\right)\left((r+dr)^2-r^2\right)=0 \quad (9)$$

where, σ_r^t and σ_θ^t are the transverse residual stresses, $\sigma_r^t(r)=\sigma_r(r)-\Delta\sigma_r(r)$, $\sigma_r^t(r+dr)=\sigma_r(r+dr)-\Delta\sigma_r(r+dr)$ $\sigma_\theta^t(r)=\sigma_\theta(r)-\Delta\sigma_\theta(r)=\sigma_\theta(r)+\Delta\sigma_r(r)/2$, $\sigma_\theta^t(r+dr)=\sigma_\theta(r+dr)-\Delta\sigma_\theta(r+dr)=\sigma_\theta(r+dr)+\Delta\sigma_r(r+dr)/2$. The transverse residual stresses are obtained with iterative solution of Eq. (9) with a tiny dr and initial condition $\Delta\sigma_r(a)=\sigma_r(a)$.

The transverse residual stress components can be transformed from polar coordinates to rectangular coordinates with

$$\begin{bmatrix}\sigma_x & \sigma_{xy} & \sigma_{xz}\\ \sigma_{yx} & \sigma_y & \sigma_{yz}\\ \sigma_{zx} & \sigma_{zy} & \sigma_z\end{bmatrix}=\begin{bmatrix}\sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi\\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi\\ \cos\theta & -\sin\theta & 0\end{bmatrix}\begin{bmatrix}\sigma_r & & \\ & \sigma_\theta & \\ & & \sigma_\varphi\end{bmatrix}\begin{bmatrix}\sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta\\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta\\ -\sin\varphi & \cos\varphi & 0\end{bmatrix}$$

For two shots impacting on the target with a separation distance d, when $2a\le d<2r_c$ there are three types of overlapping zones: elastic-elastic zone (EEZ), elastic-plastic zone (EPZ) and plastic-plastic zone (PPZ), as shown in Fig. 2. When $2r_c\le d\le 2r_o$, there are two types of overlapping zones: EEZ and EPZ. When $d>2r_o$, there is no overlapping zone.

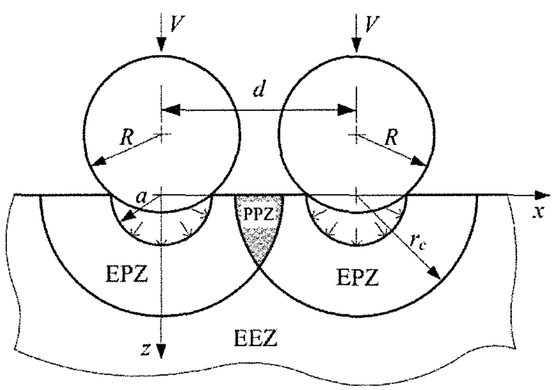


Fig. 2. Schematic view of two shots impacting onto the target with a separation distance d.

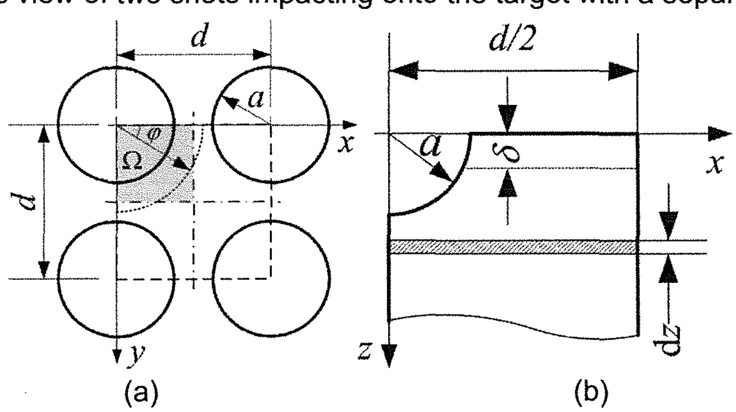


Fig. 3. (a) Schematic view of regularly distributed shot dimples and (b) lateral view of element Ω.

Assuming multiple shots impacting is imposed at the regular positions as shown in Fig. 3(a). The shot peening coverage C can be predicted as $C=\pi a^2/d^2$. The stress fields induced by adjacent shots impacting would overlap each other. Since the plastic zone of each impacting is relatively small and subsequent impacting is hard to yield the former hardened material, the overlapping of adjacent stress fields is assumed to only take place in EEZ. In EEZ, the resulting stress field is the sum of adjacent stress fields. In the EPZ and PPZ, the stress fields remain in the state of single shot impacting.

With square distribution of shot dimples, in the element Ω ($d \times d \times r_o$) shown in Fig. 3, the overlapping stress $\sigma_x^T(x,y,z)$ in the EEZ can be calculated as

$$\sigma_x^T(x,y,z) = \sigma_x^t(x,y,z) + \sigma_x^t(x-d,y,z) + \sigma_x^t(x,y-d,z) + \sigma_x^t(x-d,y-d,z) \quad (10)$$

and in the EPZ and PPZ $\sigma_x^T(x,y,z) = \sigma_x^t(x,y,z)$. Where $\sigma_x^t(x,y,z)$ denotes the stresses induced by the shot at the point (0,0,0) shown in Fig. 3, $\sigma_x^t(x-d,y,z)$ at the point (d,0,0), $\sigma_x^t(x,y-d,z)$ at the point (0,d,0) and $\sigma_x^t(x-d,y-d,z)$ at the point (d,d,0).

The overlapping stress $\sigma_x^T(x,y,z)$ is averaged in each tiny layer dz along z direction. The mean stress $\sigma_x^m(z)$ is expressed as $\sigma_x^m(z) = 4\int_S \sigma_x^T(x,y,z)\,\mathrm{d}x\mathrm{d}y / d^2$, $S \in \{0 \le x \le d/2, 0 \le y \le d/2\}$. For shots impacting on a plate, the peened surface is free of constraint. The stresses near $z=0$ plane are released in fact. Then, the stresses of the cavity model in the surface layer with thickness of δ as shown in Fig. 3(b) are treated to be zero.

The peening effect on the deformation of plate is represented by the bending moment induced by the overlapped stresses. The equivalent mean bending moment per unit width M_x is expressed as $M_x = \int_0^h (z - h/2)\sigma_x^m(z)\,\mathrm{d}z$, h is the thickness of the plate and is equal to r_o. The resulting curvature of the plate can be calculated as $\rho = 12(1-\nu)M_x/(Eh^3)$ for the spherical resulting shape, where ν is the Poisson's ratio of the target material.

Finite element simulation

To verify the analytical model for one shot impacting, a one-quarter spherical FE model as shown in Fig. 4 is utilized. The outer and inner radii of the model are 5 and 0.603 mm. The material under study is aluminum alloy Al2024-T351. The material properties are fitted with power-law hardening model ($\sigma = K\varepsilon^n$) with parameters of elastic modulus 73 GPa, yield stress 343 MPa, strength coefficient (K) 804 MPa and strain-hardening exponent (n) 0.159. The Poisson's ratio of the material is 0.33. In the process of the simulation, symmetry boundary conditions are imposed on the $x=0$, $y=0$ and $z=0$ planes. First, the inner surface of the model is pressed with a pressure of 1114 MPa corresponding to the impacting of an APB1/8 shot with initial velocity 40 m/s. Then, the pressure is unloaded to detect the residual stresses. The parameters of the APB1/8 shot are diameter 3.175 mm and density 7800 kg/m^3.

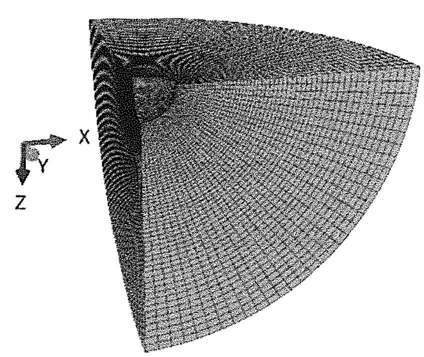


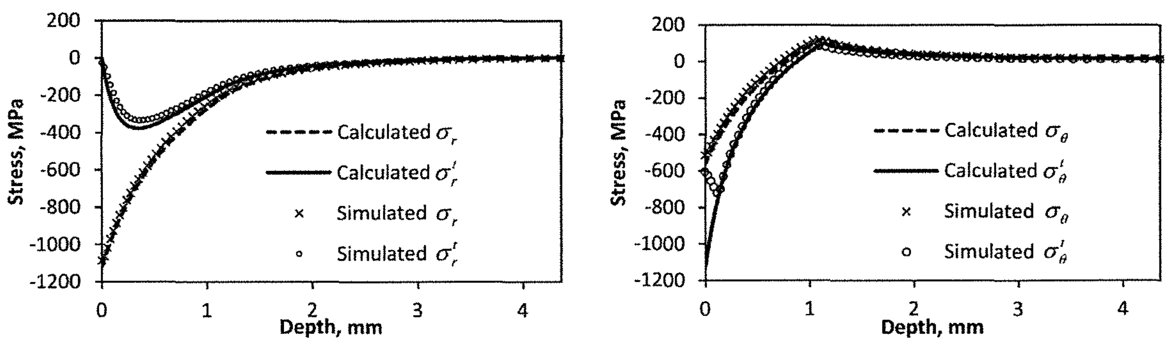
Fig. 4. One-quarter of FE cavity model with outer radius 5 mm and inner radius 0.603 mm.

Experiment

To verify the peening effect of multiple shots impacting predicted with the analytical model, shot peen forming experiments were performed using 2024-T351 aluminum alloy plates with dimensions of 200×200×5 mm. The experiments are performed on the Wheelabrator MP20000 Aircraft Wing Peening System with standard shot of APB1/8. The distance between the peening nozzle and the plate is 200 mm. The rate of shot flow is 10 kg/min. Four plates were peened with air pressure 0.3 MPa and feed speed 2, 4, 6, 8 m/min, respectively. Another four plates were peened with air pressure 0.4 MPa and feed speed 2, 4, 6, 8 m/min, respectively. The shot velocity at the nozzle is measured with the high-speed photographic setup attached to the MP20000. The shot velocities corresponding to air pressure 0.3 and 0.4 MPa are 40 and 48 m/s, respectively. The resulting curvatures of the plates are obtained by measuring the arc heights with an arc height gauge. The peening coverage is optically measured by scanning the peened surface.

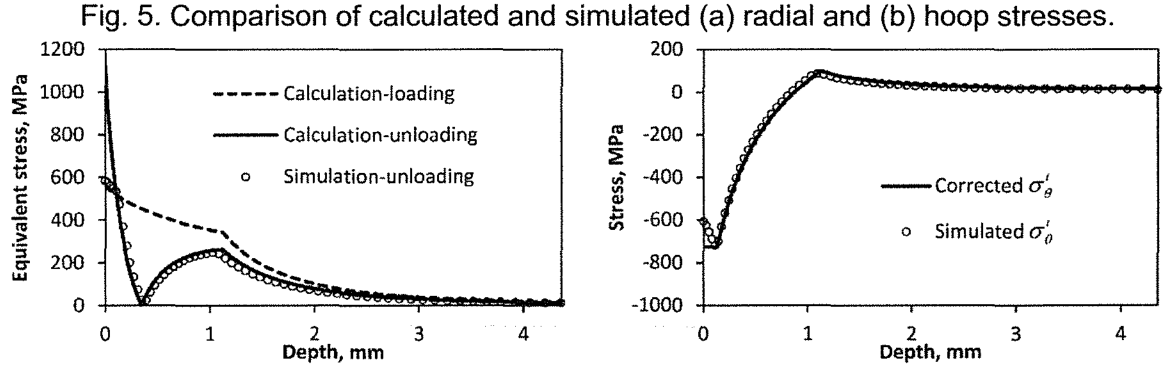
Results

The impacting effects of an APB1/8 shot with initial velocity 40 m/s on an aluminum alloy 2024-T351 plate with thickness 5 mm are predicted with the analytical model. Fig. 5 plots the calculated and simulated radial and hoop stresses for loading and unloading. The loading stresses of calculation and simulation are in good agreement. The calculated unloading radial stress shown in Fig. 5(a) is slightly numerically larger than the simulated values since the analytical model neglecting the elastic volume change when unloading. The unloading hoop stress shown in Fig. 5(b) is numerically larger than the simulated values in the surface layer. The difference is attributed to reverse yielding in the surface layer that is not considered when unloading.



(a) (b)

Fig. 5. Comparison of calculated and simulated (a) radial and (b) hoop stresses.



(a) (b)

Fig. 6. (a) Comparison of calculated and simulated equivalent stresses. (b) Corrected hoop stress according to the equivalent stress and compared with simulated values.

Fig. 6(a) shows the calculated equivalent stresses of loading and unloading and simulated equivalent stresses of unloading. It can be seen that the unloading equivalent stress exceeds the values of loading in the surface layer, where the reverse yielding takes place. To correct the calculated hoop stress of unloading, the hoop stress in the reverse yielding region is set to be equal to the stress of the reverse yield point, as shown in Fig. 6(b).

The peening effect of different peening coverage values and shot velocities on the resulting curvature of the Al 2024-T351 plate with thickness 5 mm are predicted with the analytical model. The predicted results are compared with experiments, as shown in Fig. 7. Three δ values (Fig. 3(b)) of 0.25, 0.35 and 0.45 are taken in calculations. Fig. 7(a) shows the calculated and experimental relationships between resulting curvature and peening coverage for shot velocity of 40 m/s. Fig. 7(b) shows the relationships for shot velocity of 48 m/s. It can be seen that the resulting curvature decreases with the increase of the δ. The experimental values are consistent with the calculated values with the δ of 0.35 mm.

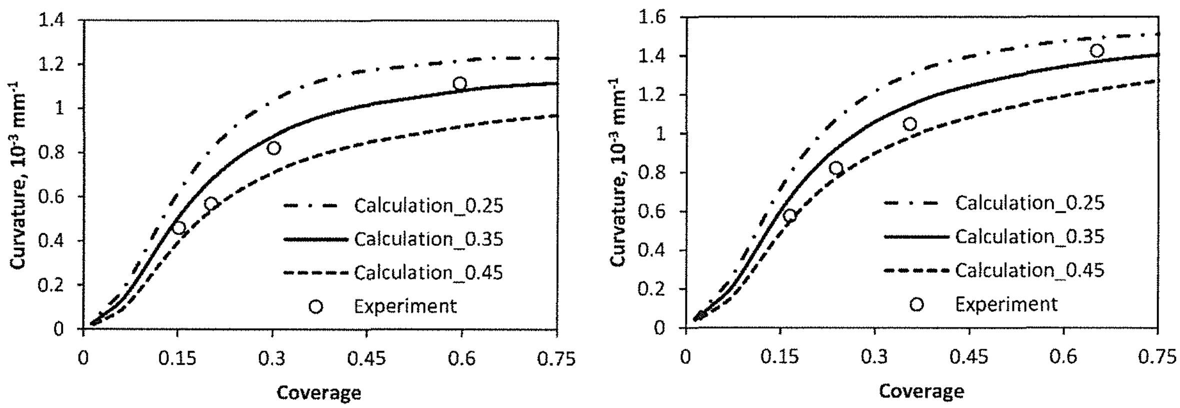


Fig. 7. The calculated and experimental relationships between resulting curvature and peening coverage for shot velocity of 40 and 48 m/s. Three δ values of 0.25, 0.35 and 0.45 are taken in calculations.

Conclusions

To relate the peen forming parameters to the plate deformation, an analytical model was developed to predict the shot-induced residual stress and resulting plate curvature. The calculated results are in good agreement with simulation and experiments. For the application of the model on a plate, the predicted resulting curvatures are partly depended on the thickness of the surface layer for stress release. Associated with experiments, the analytical model can be used to predict the peening effects on the plate deformation.

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