Crack shape effects on the fatigue behaviour of HFMI treated welds under variable amplitude loading conditions
Rakesh Ranjan, Scott Walbridge
University of Waterloo, Canada, rranjan@uwaterloo.ca, swalbrid@uwaterloo.ca

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Introduction
High frequency mechanical impact (HFMI) treatments have received increasing attention in recent years by researchers and engineers as an effective means for improving the fatigue performance of the welds in cyclically loaded components and structures [1,2]. It is generally recognized that the primary mechanism whereby this improvement is achieved is through the compressive residual stresses introduced by the treatment. Having the ability to accurately predict the fatigue behaviour and performance of HFMI treated welds is of vital importance for establishing design rules and quality control guidelines, as well as for enabling accurate assessment of the economic implications of employing these treatments in new designs and fatigue retrofitting projects. Previous research by the author group has shown that nonlinear fracture mechanics can serve as a valuable tool for predicting the effects of HFMI treatment for various materials (e.g. aluminium, mild steel, and high strength steel) under both constant and variable amplitude loading conditions [3-5].

Objectives
The objectives of the research presented in the current paper are to: 1) briefly describe a previously-developed 1D nonlinear fracture mechanics model and show how it tends to result in systematically conservative predictions of the fatigue performance of HFMI treated mild steel welds tested under two variable amplitude loading histories, and 2) explain how this 1D model can be implemented in a 2D crack framework in order to predict the crack shape under fatigue loading, and 3) investigate and assess how well the 2D model is able to predict the test results and whether these predictions represent an improvement over those made by the simpler 1D model.

Methodology
CSA G40.21 350W mild steel was used to fabricate cruciform welded joint specimens out of 300 mm wide, 9.5(3/8”) mm thick plate. Welding of the transverse stiffeners was performed using the flux core Arc Welding (FCAW) process. The welded plates were HFMI treated and then cut into 50 mm wide strips. They were then “dog boned” using a computer numerical control (CNC) cutting machine in the middle region as shown in Figure 1. The specimens were tested under three types of uniaxial loading namely constant amplitude (CA) loading with load ratio 0.1 and two variable amplitude loading histories, VA1 and VA2, as shown in Figure 2. Material tests were performed to get the input parameters for the fracture mechanics model, which are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Magnitude</th>
<th>Units</th>
<th>Parameter</th>
<th>Magnitude</th>
<th>Units</th>
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<td>MPa</td>
<td>ΔKth</td>
<td>80</td>
<td>MPa·√mm</td>
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<td>396.3</td>
<td>MPa</td>
<td>K’</td>
<td>947.6</td>
<td>MPa</td>
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<tr>
<td>σu</td>
<td>574.3</td>
<td>MPa</td>
<td>n’</td>
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<td>-</td>
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<td>MPa, mm</td>
<td>a₀</td>
<td>0.15</td>
<td>mm</td>
</tr>
<tr>
<td>m</td>
<td>3</td>
<td>MPa, mm</td>
<td>µ</td>
<td>0.002</td>
<td>-</td>
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</tbody>
</table>
The basis for the nonlinear fracture mechanics model employed in the analysis presented in this paper is the Paris-Erdogan crack growth law, commonly used in linear elastic fracture mechanics (LEFM) analysis, modified to consider crack closure effects and a threshold stress intensity factor (SIF) range, $\Delta K_{th}$, and integrated over a crack depth range, $a_i$ to $a_c$:

$$N = \int_{a_i}^{a_c} \frac{da}{C \cdot \text{MAX} \left( \Delta K_{eff}^m - \Delta K_{th}^m, 0 \right)}$$

(1)

where, $C$ and $m$ are material constants. The effective SIF range, $\Delta K_{eff}$, considering crack closure (or opening) stress effects, is determined by the following expression:

$$\Delta K_{eff} = K_{max} - \text{MAX} \left( K_{op}, K_{min} \right)$$

(2)

where $K_{max}$ and $K_{min}$ are the SIFs due to the maximum and minimum local strain levels ($\varepsilon$) for each load cycle and $K_{op}$ is the SIF corresponding with the crack opening strain level for a given load cycle.
The following expression is used to calculate each SIF:

$$K = Y \cdot E \cdot \varepsilon \cdot \sqrt{\pi \cdot (a + a_0)}$$  \hspace{1cm} (3)$$

where, \(a_0\) is a material constant to account for small crack behaviour and \(Y\) is a correction factor to account for the crack shape, the free surface on one side of the crack, and the finite thickness of the cracked plate. The constant \(a_0\) can be taken as:

$$a_0 = \left( \frac{\Delta K_{th}}{\Delta \sigma_e} \right)^2 \cdot \frac{1}{\pi}$$  \hspace{1cm} (4)$$

where, \(\Delta \sigma_e\) is the fatigue limit for \(R = -1\) (\(\approx 0.5 \cdot \sigma_u\)). To calculate the local stresses and strains, \(\sigma\) and \(\varepsilon\), for each load cycle, a Ramberg-Osgood material model is used, which requires the cyclic material parameters: \(K'\) and \(n'\). Strain histories are determined using Neuber’s rule. Crack closure is modelled using formulas by Newman. These require as input: the maximum stress, \(\sigma_{\text{max}}\), the stress ratio, \(R\), the flow stress, \(\sigma_0\) (i.e. the average of the yield and ultimate strength, \(\sigma_y\) and \(\sigma_u\)), and a plastic constraint factor, \(\alpha\). In order to consider the non-uniform stress distribution, the stress concentration factor (SCF), which relates the elastic local stress at crack depth, \(a\) to the nominal stress (and is calculated by a linear elastic finite element analysis), is multiplied by a correction factor \(k_p\):

$$k_p = \frac{\int_a^0 \sigma(x) \cdot m(x,a) \cdot dx}{Y \cdot E \cdot \varepsilon \cdot \sqrt{\pi \cdot a}}$$  \hspace{1cm} (5)$$

In Equation (5), the numerator is the SIF for the nonuniform stress distribution associated with the weld toe notch, calculated by the weight function method. The denominator is the SIF for a uniform stress equal to the local elastic stress at crack depth, \(a\), and is calculated using a readily available correction factor, \(Y\), to account for the crack shape, finite plate thickness, etc. Other aspects of the model – in particular concerning the manner in which crack closure effects under variable amplitude (VA) loading conditions are treated – are elaborated upon further in [3-5]. The model is thought to be particularly well-suited for analysing impact treated welds, under VA loading conditions including periodic large cycles or overload events, which may result in relaxation of the treatment-induced residual stresses due to the nonlinear material behaviour.

In previous studies by this research group, this fracture mechanics model was applied in a 1D form, with the crack shape needed to determine the SIFs at each crack depth forced to evolve according to a predefined empirical function, based on crack shape data either measured or reported by others. A semi-elliptical surface crack was assumed with a depth, \(a\), and width, \(2 \cdot c\).

For the current paper, a 2D version of the model was developed, where the crack shape was allowed to evolve based on calculated crack growth rates in the depth (\(a\)) and width (\(c\)) directions. To do this, Equation (1) was integrated numerically in increments of crack depth, \(a\). For each crack depth increment, a rate of crack growth in the width direction, \(dc/dN\), was calculated, using equations similar to Equations (1)-(5), expressed in terms of crack width, \(c\), rather than depth, \(a\). Weight functions, and correction factor, \(Y\), for crack growth rate at the surface point of a semi-elliptic surface crack were used. The new crack width dimension, \(c\), was then calculated, based on the crack growth rate, \(dc/dN\), and the number of cycles required for the crack depth increment.
In addition to extending the fracture mechanics model to 2D, weight functions and correction factors were also integrated to allow the analysis of quarter-elliptical corner cracks, reflecting the observation from inspection of the fracture surfaces, that some treated specimens appeared to fail due to cracks propagating from the edges rather than the middle of the specimen.

The resulting model allows trends to be modelled that have been observed experimentally. For example, since impact treatment slows crack growth rates near the treated surface, as the crack gets deeper, it stands to reason that it will grow more rapidly in the depth direction, as opposed to the width direction. The model allows the crack closure stresses in depth and width directions to differ. However, it simplistically ignores any effect crack closure at one location may have on the crack growth rate at the other. Another limitation of such 2D crack growth models, is that they do not consider the effects of coalescence of cracks initiating at multiple sites. Given these advantages and disadvantages of the 2D model, a comparison with experimental data is of interest, in order to assess whether or not it provides an improved degree of prediction accuracy.

**Results and analysis**

In Figure 3 to 6, experimental data and estimated S-N curves are presented for as-welded and HFMI treated specimens under the VA1 and VA2 loading histories. In these figures, SE and QE refer to semi-elliptical and quarter-elliptical cracks. 1D analyses results are plotted for two extremes: \( \frac{a}{c} = 1.0 \) (a semi-circular or quarter-circular crack), and 0.001 (essentially a through crack). Looking at these figures, it can be observed that experimental results are inside the envelopes of estimated S-N curves corresponding to different crack shape assumptions. For the as-welded specimens, a through crack shape was typically observed at failure, as seen in Figures 3 (right) and 4 (right). On the other hand, the 2D results in these figures tend to lie closer to the \( \frac{a}{c} = 1.0 \) results, suggesting that crack coalescence may have played a significant role for the as-welded specimens.

![Figure 3: Results for as-welded specimen under VA1 loading (left) and final crack shape (right).](image-url)
In Figures 5 and 6, it can be seen that the crack shape at fracture for the treated specimens tended to be quarter-elliptical with a higher aspect ratio \((a/c)\) than was observed in the as-welded specimens. In some cases, cracks were seen to grow from the sides of the specimens, indicating that the treatment was effective enough to shift the crack initiation site entirely. Looking at the fracture mechanics results, it can be seen that the 2D model predicts a high aspect ratio at higher stress levels for the treated specimens. At the lower stress levels, the 2D S-N curve approaches the 1D curve for a through crack, suggesting flatter crack shape at the lower stress ranges.
Conclusions

Based on the results presented in this paper, it is concluded that HFMI treatment affects the crack shape evolution under the fatigue loading and proper consideration should therefore be given to crack shape evolution modeling for better estimation of test results. It was observed that for the as-welded specimens, the crack shape at fracture resembles a through crack while for HFMI treated specimens, the crack shape was found to be closer to circular and often a corner crack. At the higher stress levels, the 2D model appears to be predicting this trend. It is possible, however, that the lower aspect ratio observed in the as-welded samples is due in part to the coalescence of cracks from multiple initiation sites—a phenomenon not captured by the 2D model. Further investigation may be needed to fully understand the impact of the various model parameters on crack shape to answer this question. In continuing this research, efforts will be made to further validate the 2D model by comparison with larger databases of test results for as-welded and HFMI treated specimens under CA and VA loading conditions. The long term goal of this research will be to establish an accurate fracture mechanics model to aid in the development of improved design code provisions and tools for predicting the effectiveness of HFMI treatment in weld retrofitting applications.

References