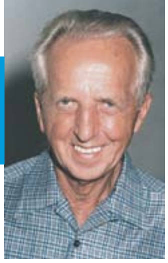

ACADEMIC STUDY

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The Curvature of Peened Almen Strips and Its Applications

INTRODUCTION

Peened Almen strips have a familiar shape—displaying both longitudinal and transverse curving—after release from hold-down. Fig.1 shows how the duplex curving contributes to the measured arc height, h.

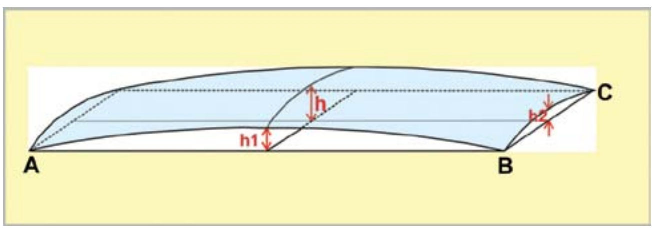

Fig.1. Duplex curving induced in a peened Almen strip.

At a first glance, both curves appear to be arcs of a circle. This is, however, incorrect. The two curves are, in fact, both parabolas. If a parabola is graphed, an optical illusion can be induced. This phenomenon is illustrated by fig.2.

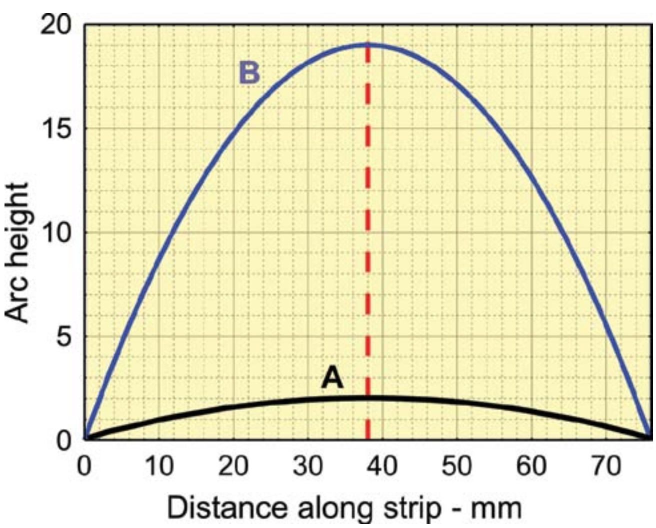

Fig.2. Longitudinal Parabolic shape of a uniformly peened Almen strip.

Curve B has the same equation as curve A but with the vertical scale magnified by a factor of ten. It is difficult to differentiate visually the shape of curve A from that of a circular arc hence the concept of an optical illusion.

This article embraces an analysis of why peened Almen strips have this parabolic shape together with experimental evidence and a consideration of its practical significance. Understanding the cause of strip deflection enables us to identify and control variations in peening procedure. One section is presented in the form of an article. The tutorial includes a demonstration of how calculus can be employed to solve a peening problem!

EXPERIMENTAL EVIDENCE

The results of an experimental program titled "Factors Affecting Almen Strip Curvature Readings" were presented at ICSP7, Warsaw, 1999 by D.Kirk and R. Hollyoak (pp291-300). Measurements were made using a Kemco 600 co-ordinate machine and QCT-3d measuring software operating with digitized points. Figs. 3 and 4 show the longitudinal and transverse deflection variations for a peened N strip.

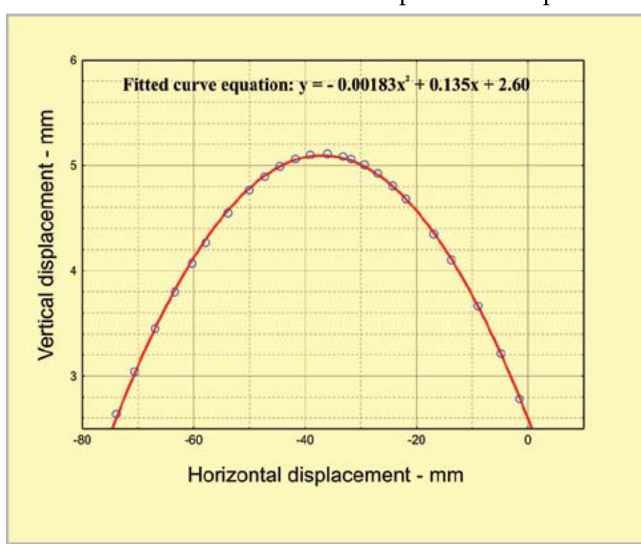

Fig.3. Longitudinal curve shape of a peened Almen N strip.

Figs.3 and 4 are a clear indication of the parabolic nature of the curves induced into a peened Almen strip. The fitted equations were $y = -0.00183x^2\ 0.135x + 2.60$ and $y = -0.00271x^2\ 0.0485x + 6.05$ respectively with goodness of fit, r^2, being 1.00. Both equations correspond to that of a vertical-axis parabola. Other parabolas were noted in the quoted ICSP7 paper.

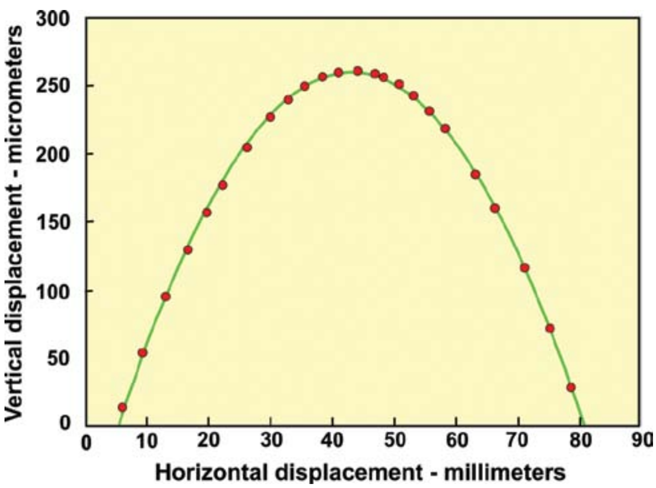

Fig.4. Transverse curve shape of peened Almen N strip.

TUTORIAL ON PARABOLIC BENDING OF STRIPS

"Hi guys. This tutorial is about how bending moments induce parabolic bending of rectangular strips. You will find it useful if you ever aim to be an expert shot peener.

Consider a simple model of an office ruler supported near to each end (as shown in fig.5). Press down on the ruler at a point close to A, using a reasonable force that we will call "**F**". The ruler will hardly bend at all. As we move our finger to a distance **x** we observe that noticeable bending now occurs. If we continue moving the finger we note that maximum bending probably occurs halfway along the ruler at a distance from **A** of **L/2**.

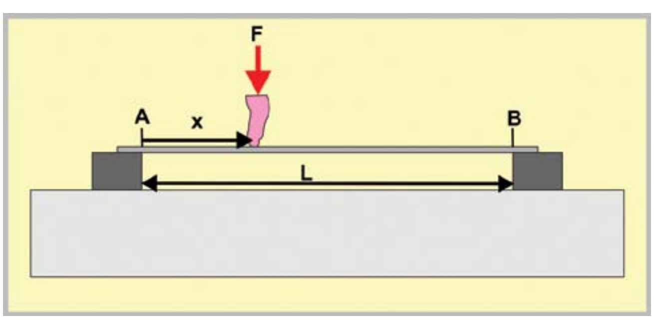

Fig.5. Finger bending of an office ruler.

The combination of **F** and **x** generates what is called a "Bending Moment". Bending moments are so important to engineers that they are given the letter **M**. Next let us think about how we can quantify the variation of **M** with distance **x**. The variation of **M** with a single applied force, **F**, is given by the equation:

$$\mathbf{M = F.x\ (L - x)/L} \text{ so that} \qquad (1)$$
$$\mathbf{M/F = x\ (L - x)/L} \qquad (2)$$

Equations (1) and (2) are vertical axis quadratics.

Keeping **F** and **L** fixed, the bending moment will vary only with **x** and has a magnitude **(L – x)/L**. Time now for some mental arithmetic! Assume that the fixed length **L** is 100. Pressing down with **x** equal to 1 unit **x(L – x)/L** becomes 1(100 – 1)/100 or 99/100 which equals 0.99. Now, for example, if **x** equals 10, **x (L – x)/L** becomes 10(100 -10)/100 which equals 900/100 or 9. The bending moment has therefore been increased by a factor of just over 9. At halfway along the ruler **x** equals 50 so that **x (L – x)/L** becomes 50(100 – 50)/100 or 2500/100 which equals 25.

We can show how the bending moment, **M**, varies continuously when applying a single fixed force **F**, at different distances, **x**, by plotting a graph. Assume that **F** is unity and **L** is 100 so that equation (2) becomes:

$$\mathbf{M_x = x(100 - x)/100} \qquad (3)$$

Use 'Easyplot' (supplied to everyone by University), to graph equation (3). Set the x-axis to go from 0 to 100 and the y-axis to go from 0 to 30 - we already know that the maximum value for **M** will be 25. You should then all get the same graph (fig.6):

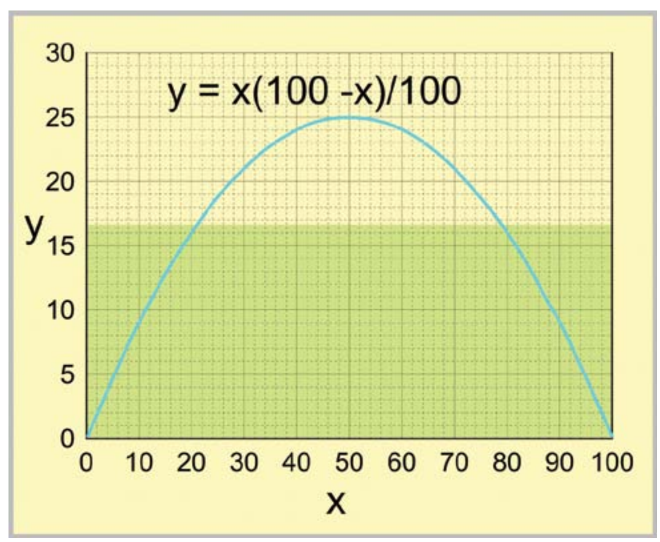

Fig.6. Variation of Maximum Bending Moment for a single applied force.

The total of the bending moments being suffered by the strip is the same as that of the green rectangle included in fig.6.

So far we have only considered how bending moment varies for a single applied force, F. What happens if tiny forces are applied continuously and uniformly along the beam? We have what engineers call "uniformly distributed loading". For example, consider 1N per mm applied uniformly along a beam that is 100 mm long. The total loading will therefore be

100N but what is the magnitude of the bending moment? To tackle this problem we can use a standard textbook equation used by mechanical engineers. The origin of the equation is illustrated in fig.7. **q** is the "uniform loading" equivalent to force per unit length.

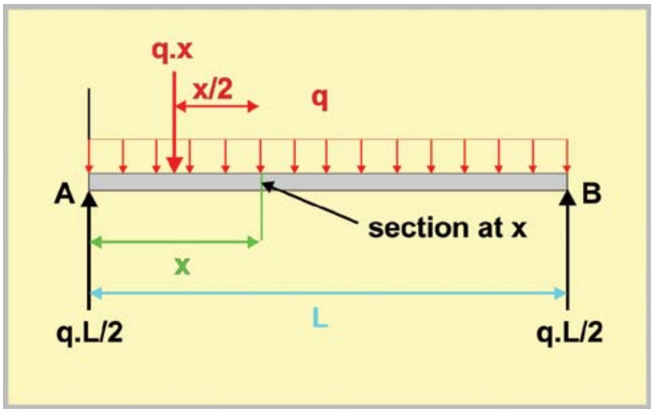

Fig.7. Forces and Bending Moments in a uniformly-loaded beam.

Consider what is happening at the section of the strip that is at a distance **x** from **A**. The upward force at **A** (in Newtons) is **q.L/2** generating a clockwise bending moment of **q.x.L/2**. Between **A** and the section at **x** we have a total downward force of **q.x**. This force acts as if it was at a distance **x/2** from the section at **x**. Hence it generates an anti-clockwise bending moment of **q.x2/2**. The net bending moment at the section **x** is therefore:

$$M_x = q.x.L/2 - q.x^2/2 \text{ or}$$

$$M_x = q.x\,(L - x)/2 \qquad (4)$$

Using 76 mm (the length of an Almen strip) for **L** and unity for **q** gives:

$$M_x = x\,(76 - x)/2 \qquad (5)$$

Plotting equation (5) yields Fig.8. Again we have a parabolic shape.

We could explore uses for the distribution shown in fig.8. However, tutorial time is up—let's go for a beer."

BENDING MOMENT DIAGRAM APPLICATIONS

Applications relating to peened Almen strips depend mainly on the relationships that exist between bending moment, deflection and curvature. The fundamental relationship is given by:

$$M = E.I.1/R \qquad (6)$$

where **M** is bending moment, **I** is the rigidity factor and **1/R** is the curvature. The rigidity factor for a rectangular beam is **width times thickness cubed divided by 12**. The curvature of a strip is therefore related to its arc height, **h**, by:

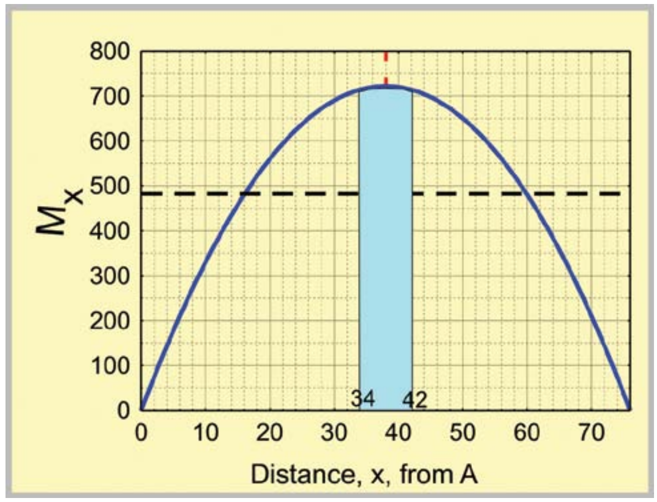

Fig.8. Bending moment distribution for a uniformly loaded Almen strip.

$$1/R = 8h/L^2 \qquad (7)$$

Combining equations (6) and (7), together with the rigidity factor, gives:

$$M = E.I.8h/L^2 \qquad (8)$$

One important inference from equation (8) is that:

Arc height is directly proportional to the induced bending moment

There are several ways in which we can utilize bending moment diagrams of peened Almen strips. Just three of these ways are detailed as follows:

(1) Almen Gage Measurement Sensitivity

The observed parabolic shape of peened Almen strips has an important effect of gage measurement sensitivity. We know intuitively that it is best for the measuring tip of an Almen gage to be on the center line of the strip. This point is illustrated by Fig.9 for which the peened strip is assumed to have a center line deflection of 722 µm. We can quantify the effect of a small deviation from the center line by substituting into the corresponding parabolic equation $h = (76x - x^2)/2$. For example, for x = 36 mm, 38 mm (centerline value) and 40 mm we get h values of 720, 722 and 720 µm respectively. This shows that a deviation of as much as 2 mm from the center line would only result in an error of 2 µm. Modern Almen gages physically restrict such offsetting to a tiny fraction of a millimeter.

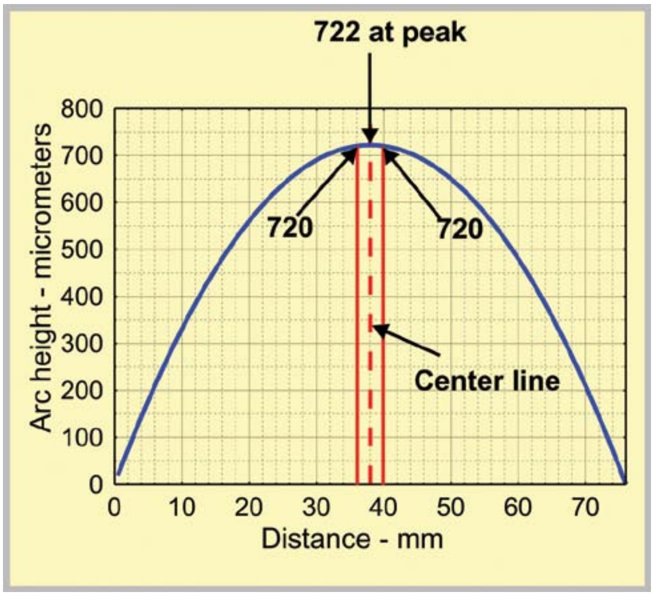

Fig.9. Effect of major point displacement on arc height indication.

(2) Relative Contribution

We see from fig.8 that the bending moment is highest at the center of the strip and disappears at the ends of the strip. The relative contribution to bending of any given portion of the strip is its corresponding area under the curve. One such contribution has been included in fig.8 representing a portion 8 mm wide straddling the center of the strip. The area in blue is some 8 mm x 720 = 5760 (in arbitrary units). Four millimeter-wide portions at the ends of the strip would each contribute (as "half base times perpendicular height") some 2 mm x 135 = 270 giving a total of 540. This is less than a tenth of the contribution by the same width of central portion. These areas can, however, be determined more precisely by using integral calculus.

Every equation has a corresponding derivative that mathematicians describe as its "integral". These are readily obtained nowadays by using web sites. Decades ago integral equations had to be derived using established precepts such as that the integral of $y = x$ is $y^* = x^2/2$ and that the integral of $y = x^2$ is $y^* = x^3/3$. Just those two precepts are all that we need to derive the integral of our Almen strip bending moment equation:

y (or M_x) = x (76 – x)/2 so that its integral equation is

$$y^* \text{ (or } M^*_x) = (76x^2/2 - x^3/3)/2 \qquad (9)$$

Equation (9) is what we need to be able to calculate relative contributions to the bending of a peened Almen strip.

Every integral equation can be used to derive the area under a selected portion of the curve. The method is to substitute the smaller value of x, **a**, into its equation and subtract it from the value obtained by substituting the larger value of x, **b**. These larger and smaller values of x are commonly called "limiting values". For those unfamiliar with calculus consider the following example:

"What is the area under a curve of $y = x$ between limits of 1 and 2?" The integral of $y = x$ is $y^* = x^2/2$. The smaller limiting value for x is 1 which, on substitution, gives us ½. The larger limiting value of x is 2 which, on substitution, gives us 2. Subtracting ½ from 2 gives us 1½ which is the correct area. The general equation for solving areas under **y = x** can be expressed as:

$$\text{Area} = b^2/2 - a^2/2$$

Consider next the area of the blue region shown in fig.8. The "limiting values" are 34 for a and 42 for b. The equation for the integral is more complicated than that for the straight line of the previous example.

$$\text{Area} = (76x^2/2 - x^3/3)/2$$

Substituting the limiting values of 42 and 34 for **b** and **a** into **$(76x^2/2 - x^3/3)/2$** gives us that the required area = $(76x42^2/2 - 42^3/3)/2 - (76x34^2/2 - 34^3/3)/2$. This (with the aid of a calculator) gives us 5,755. This just happens to be very close to the previous manual estimate of 5,760. In order to determine the relative contribution of this strip portion we need a value for the total area under the bending moment curve. This is obtained using 76 mm and 0 mm for the larger and smaller limiting values of x. Hence:

Total bending moment =

$$(76x76^2/2 - 76^3/3)/2 - (76x0^2/2 - 03/3)/2 = 36{,}580$$

The relative contribution of the 8 mm-wide central portion is therefore 5,755/36,580 = 0.157 or 15.7% of the total. We can compare this contribution with that for two, 4 mm-wide, end portions of our peened Almen strip. Using b = 4 and a = 0, we get that the corresponding area under the fig.8 curve is 293 which is the same as when using b = 76 and a = 72. Hence the combined contribution of the two 4 mm-wide end portions is 586. This is only a tenth of the contribution to deflection being made by the central 8 mm wide portion thereby agreeing with the previous manual estimate. As an important practical point we can conclude that:

The contribution to bending made by the central region is far greater than that for equal-area end portions of a peened strip.

This point has previously been established by a study, under the auspices of Electronics Inc., for which the end portions of Almen strips were masked. This masking was found to have only a very small effect on measured deflection when compared with that for unmasked strips.

(3) Shot Stream Intensity Variability

The intensity within every shot stream varies within itself.

Declared peening intensity values are an average of shot stream inherent variability. The origin of this inherent intensity variability is schematically illustrated by fig.10.

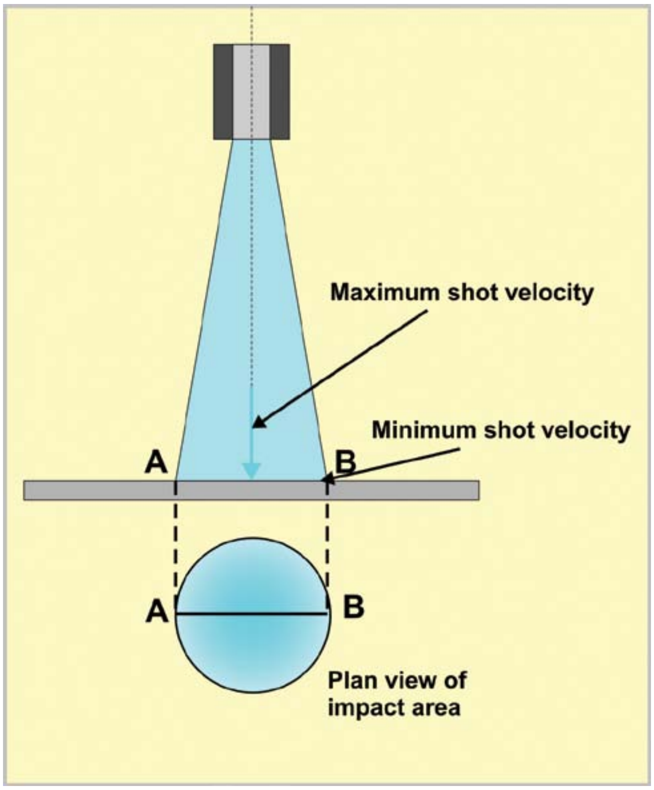

Fig.10. Inherent variability of shot stream velocity.

The maximum shot velocity occurs at the center of the impact area where the impelling air velocity is highest. Minimum shot velocity occurs at the edges, **A** and **B**, of the impact area where the carrying air velocity is lowest. The higher a shot particle's velocity the higher will be its contribution to bending moment. It follows that in order to induce a reasonably uniform peening intensity on actual components there must be substantial overlapping of passes. This effect is very similar to that of trying to achieve reasonably uniform coverage (see "Coverage Variability", The Shot Peener, Winter, 2017).

The effect of shot stream intensity variability on deflection can be studied using a combination of integral equations. One such study is illustrated in fig.11 where it has been assumed that the peening intensity varies by a factor of two (from 10 to 20 units) for a shot stream held constant over a narrow strip. The shape of the bending moment contribution curve is no longer a simple parabola but is now a quartic. With a quartic shape the contributions of end portions become even smaller.

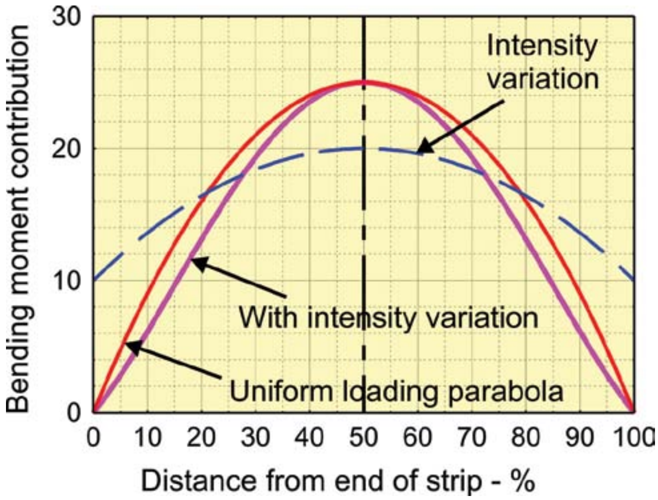

Fig.11. Predicted effect of shot stream intensity variability.

DISCUSSION

This article, being essentially "educational", has sought to show how mathematical techniques can be employed to underpin our understanding of shot peening. Peened Almen strips, being readily available, are particularly useful and their curved shapes can easily be determined. Only longitudinal curvature has been analyzed but the same principles can be applied to crosswise curvature. A very simple (to mathematicians) use of calculus has been included that demonstrates the quantification of curvature contributions. Analysis has been restricted to a limited number of situations largely because of space restrictions for a single article. The relative contributions of residual stress and plastic deformation to measured arc height and studies of the variation of curvature contribution along the length of a peened Almen strip are examples of omissions.

The author showed in 1984 (ICSP2, Kirk D, "Behavior of Peen-formed Steel Strip on Isochronal Annealing") that the two contributions to strip deflection (plastic deformation and residual stress) were roughly equal, giving a ratio of 1:1. This early study involved just one coverage condition. It is disappointing that no institution appears to have carried out more substantial investigations. Theoretical considerations would indicate that the ratio will increase with increase in the amount of coverage. It is probable that the residual stress contribution to curvature peaks at about a nominal 100% coverage and that further bending is due to continued plastic deformation. There is a growing weight of argument that the maximum benefit of shot peening occurs at or below a nominal 100% coverage. A study involving the peening of sets of strips at a range of coverages followed by isochronal annealing would confirm (or otherwise) the theoretical prediction and add weight to the argument about optimum coverage.