ACADEMIC STUDY
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Shot Peening Mathematics

SHOT PEENING is, necessarily, quantitative. It therefore relies on the application of a large number of mathematical principles. These principles vary from simple arithmetic procedures to the use of complex equations to predict the effects of peening variables. Examples include estimating particle size variation and curve fitting for peening intensity prediction.

This article aims to show how mathematics is involved in relevant areas of shot peening. The most important factor is the application of established equations. Equations express the interrelationship of variables. As such they can be regarded as being models of behavior.

MODELLING

All models have to be based on a set of assumptions that reflect established knowledge. This requirement is illustrated by the following case study.

Case Study: RESIDUAL STRESS DISTRIBUTION

As is well-known, shot peening induces a thin surface layer of compressed material. This is one of the two major benefits of shot peening—the other being the corresponding work-hardening of the surface. The distributions of residual stress and work-hardening are very similar. The general shape of the residual stress distribution is known to be as shown in fig.1.

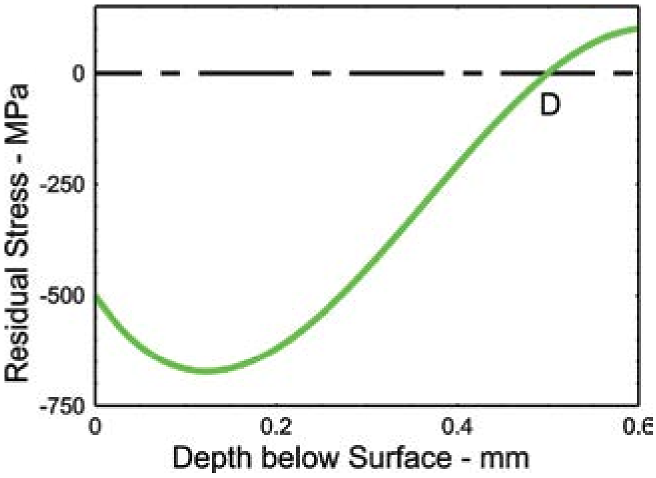

Fig.1. General shape of residual stress distribution in single-peened components.

Fig.1 therefore represents a model of the residual stress distribution. For the illustrated model, the following assumptions were made:

1. The level of surface compressive stress is half of the yield strength of the as-peened material, Y.
2. The maximum level of compressive stress is two-thirds of Y and occurs at 20% of the depth of compressed material.
3. The depth of compressed material, D, is shown, for this illustration, as being 0.5 mm.
4. A balancing tensile stress of 10% of Y is reached at 1.2D (0.6 mm).
5. A cubic polynomial interpolation will be appropriate.
6. The peened material is assumed to have a yield value of 1000 MPa.

The problem to be tackled, as with all models, is to enable predictions to be made. For this case study, it is to enable prediction of residual stress profile curves by varying the assumed parameters.

The model may be extended in several ways to make it more generally applicable:

1. Because we rarely know the yield strength of the as-peened material we can use some other measure. It is suggested that the ultimate tensile strength (U.T.S.) of the unpeened component material is a good indication of the yield strength of as-peened material. That is because the U.T.S., as measured in a tensile test, indicates the strength of material deformed to the point of plastic instability. After the U.T.S. is reached, further strengthening (true strength) occurs up to the point of fracture. During peening the material is subjected to multiple impacts that strengthen the material to about the U.T.S. level without any chance of plastic instability occurring.
2. Because we cannot know the depth of the compressed layer in advance we can make an assumption that it is equal to the dimple diameter. Dimple diameter can either be measured for a given peening situation or can be predicted.

This case study is an example of developing a model that will predict a given type of curve without having to produce any actual experimental data. The reliability of the predicted curve is only as good as the assumptions that have been made. Hence extreme care has to be taken before any reliance can be put on the predictions. Conversely it can be a very good guide as to the peening parameters that may lead to a required residual stress depth profile. Measured residual stress profiles can be used to confirm the applicability of the model.

ARC HEIGHT VARIATION WITH PEENING TIME

The variation of arc height with peening time forms the basis for estimating peening intensity. In the early days of intensity, estimation hand-drawn graphs were produced. These enabled a subjective point to be selected as representing intensity, using as a guide "10% or less". Nowadays mathematical models have taken over, allowing objective estimates to be made of peening intensity. "Objective" means that everyone gets precisely the same value when using the same set of peening data and model equation.

Again we require that intensity prediction models should to be based on a set of assumptions that reflect established knowledge. Numerous intensity curves indicate that their general shape is similar to that of a coverage/peening time curve tending to approach a maximum. The most useful data is probably that published by Wieland (R. C. Wieland, "A Statistical Analysis of the Shot Peening Intensity Measurement, ICSP5, 1993, pp 27-38). A total of 388 Almen strips were shot peened using the same closely controlled conditions but with varying exposure times. The averages of about thirty deflections, measured at each different exposure time, have been plotted as fig.2. As would be expected, such averages smooth out measurement variability.

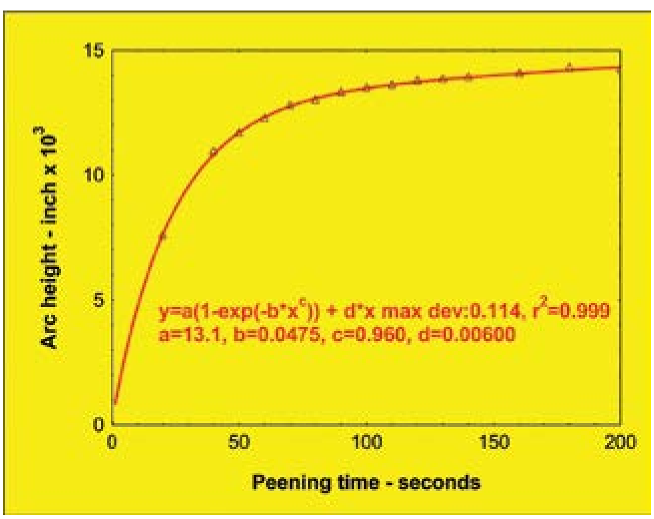

Fig.2. Variation of Arc Height with peening Time.

The mathematical equation included in fig.2. is based on the assumptions that:

1. The general shape is that of a three-exponent exponential function, together with
2. A very small linear element (d only equalling 0.00600x).

The goodness-of-fit is indicated by the value of r^2 being virtually unity. It should be noted that some sixteen different peening times have been involved—far more than would be available with routine intensity curve testing. The four-parameter equation used to establish the precise shape, but involving sixteen points, is therefore inappropriate for routine testing. The author's Solver Suite of appropriate fitting equations cuts out the very small linear component. Two choices are offered: One with just two parameters and the other with three. The three-parameter equation gives a closer fit but is best used with more than the bare minimum of four data points in a set. Corrections can be applied for pre-bow of Almen strips.

VARIABILITY OF INDIVIDUAL ALMEN ARC HEIGHT MEASUREMENTS

All arc height measurements have some degree of variability. Consider the two hypothetical sets of Almen arc height data, A and B, given in Table 1. These are for sets of twelve identical strips peened using the same conditions but by different operators. The objective in both cases was to impose an arc height of 0.0063".

Table 1. Variability indicated by two sets of Almen arc height data.

Strip Number	Arc heights (inch x 1000) SET A	Arc heights (inch x 1000) SET B
1	6.2	6.3
2	6.3	6.5
3	6.3	5.9
4	6.2	6.7
5	6.5	6.0
6	6.3	5.9
7	6.3	6.4
8	6.4	6.3
9	6.2	6.2
10	6.3	6.5
11	6.3	6.7
12	6.1	5.9
AVERAGES	**6.3**	**6.3**
STANDARD DEVIATION	**0.1**	**0.3**

It can be seen that both operators were successful on average. The variability of arc heights for operator A was, however, much less than that for operator B. This difference is quantified by the respective standard deviations of 0.0001" and 0.0003". (Standard deviations are easily calculated using Excel. We highlight a cell and insert, for example, "=STDEV(A1:A12)" where A1:A12 contains our twelve arc height values.) We do not need to understand the mathematical basis of "standard deviation" in order to use it effectively. (We can drive within speed limits without knowing how a speedometer works.) The term standard deviation refers to the "spread" to be expected from a set of values that are "normally distributed".

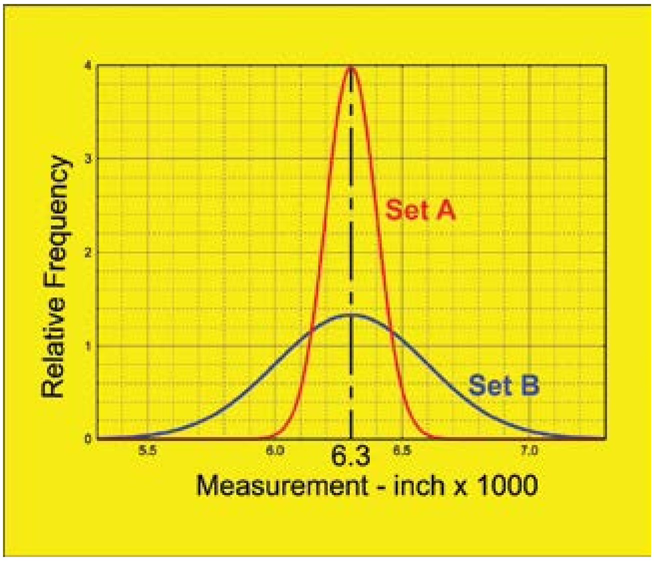

Fig.3. Normal distributions for Sets A and B.

Fig.3 shows the normal distributions representing Sets A and B. The wider spread of measurements for Set B becomes very apparent. It was assumed that the two sets of data were normally distributed. Close examination reveals that this is not necessarily correct.

SHOT PARTICLE KINETIC ENERGY

Shot particles, when accelerated by air or wheel, gain kinetic energy, $\frac{1}{2}mv^2$, where m is mass and v is velocity. Mass is volume multiplied by density with volume equal to πd3/6. One way of visualizing the enormous range of available kinetic energies is to use information such as that contained in Table 2. For cast steel particles there is a range of 6,700 to 1 in the mass of the particles. Hence, for a given shot velocity the kinetic energy will vary by 6,700 to 1.

As an example of kinetic energy calculation, consider S170 accelerated to 50 ms^{-1}. The kinetic energy, K_{S170}, is given by:

$$K = \frac{1}{2}*0.33134*10^{-3}g*(50ms^{-1})^2 \text{ or}$$
$$K = 0.414m^2s^{-2}g$$

COVERAGE

Coverage of a component by shot-induced dents is a vital feature of peening. The development of coverage, C, with peening time, t, closely follows the curve:

$$C = 100\,(1 - \exp(-A.R.t)) \qquad (1)$$

Where **A** is the area of each dent and **R** is the rate of creation of dents.

We can use equation (1) to plot how coverage increases with peening time. As an example, if A = 1 mm^2, R = 0.2 dent per second per square millimeter and t is 1 second (see fig.4 on page 32).

Table 2. Variation of Size, Mass and Particles per 100 g of Cast Steel Particles.

SHOT	DIAMETER		MASS	PARTICLES
	- inch	- mm	- mg	PER 100 g
S70	0.0070	0.1778	0.02313	4322983
S110	0.0110	0.2794	0.08976	1114037
S170	0.0170	0.4318	0.33134	301808
S230	0.0230	0.5842	0.82055	121869
S280	0.0280	0.7112	1.48046	67547
S330	0.0330	0.8382	2.42362	41261
S390	0.0390	0.9906	4.00052	24997
S460	0.0460	1.1684	6.56441	15234
S550	0.0550	1.3970	11.22045	8912
S660	0.0660	1.6764	19.38894	5158
S780	0.0780	1.9812	32.00414	3125
S930	0.0930	2.3622	54.24643	1843
S1110	0.1110	2.8194	92.23404	1084
S1320	0.1320	3.3528	155.11154	645
Ratios				
highest/ lowest	19:1	19:1	6700:1	6700:1

More examples can be accessed from Proceedings of ICSP5, "Theoretical basis of shot peening coverage control", pp183-190.

In order to use equation (1), the area, **A**, of each dent can be estimated by direct measurement of the dents in a lightly peened component. Getting a good estimate of **R** is more involved. **R** = **M/m** where **M** is the feed rate per unit area of the peening contact zone and **m** is the average mass per particle.

SHOT VELOCITY

Shot velocity is obviously of prime importance because it governs shot's ability to create dents. The control factors for shot velocity are completely different for air-blast and wheel-blast peening.

Air-Blast Shot Velocity

The outlet from an air compressor goes into a ballast tank and thence to an air supply pipe, preferably via a drying unit. The compressed air flows as a stream through the pipe. This can then be connected to a shot feed and nozzle system. Ballast tanks even out pressure fluctuations from the compressor and provide a reservoir of compressed air. One or more pressure control valves, **PCV**, will be present in the air supply line. The compressed air, at a pressure, $\mathbf{p_1}$, is fed into a blast hose of length **L**, at the other end of which is a nozzle where the pressure will then be $\mathbf{p_2}$, see fig.5.

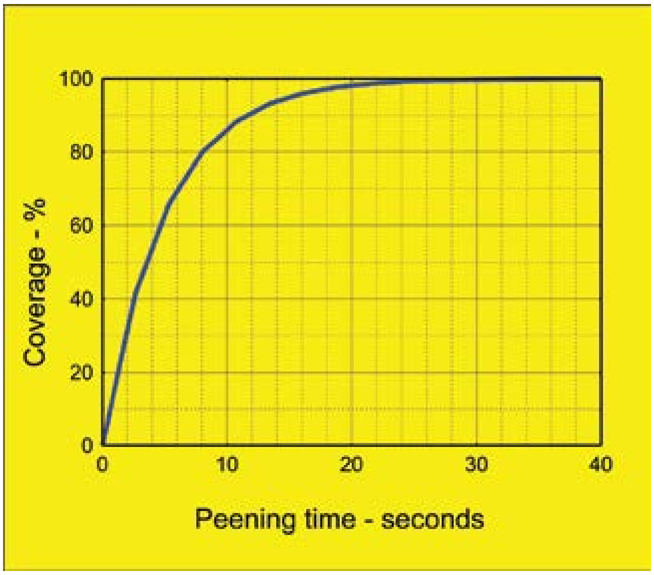

Fig.4. Variation of coverage with peening time.

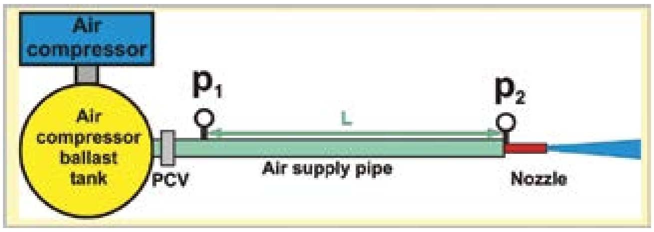

Fig.5. Schematic representation of air stream component elements, not to scale.

Fig.6 is a simplified schematic representation of how the nozzle air velocity changes with increase of nozzle air pressure (assuming that the nozzle vents to 1 atm pressure in a peening unit). A "sonic barrier" exists at the narrowest part of the nozzle, caused by the difference in pressure in the nozzle as compared with that in the peening unit. This barrier occurs when the air pressure difference is about 1.9 atm. Because all practical peening involves a pressure difference of more than 2 atm (29.4 psi), we have a fixed limited air velocity in the nozzle regardless of nozzle pressure and nozzle diameter.

The constancy of air velocity exiting the nozzle begs the question: "What effect does air pressure have if it does not affect air velocity?" The answer is that at higher pressures the air is more compressed so that it has a greater density but has the same velocity. Increasing the nozzle pressure increases the "mass flow" of air. Alternatively we could say: **"As we increase nozzle air pressure we are firing heavier air but at a constant velocity."**

A previous article (TSP, Winter, 2007) described the derivation of a formula for estimating air-blast shot velocity, **vs**:

$$\mathbf{v_S = (1{\cdot}5.C_D.\rho_A.s/\pi d.\rho_S)^{0.5}\,(v_a - v_S)} \qquad (2)$$

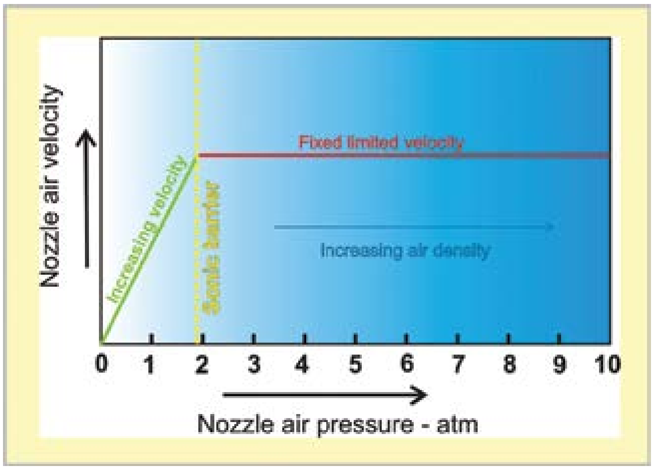

Fig.6. Schematic representation of air velocity variation with applied air pressure.

where **C_D** is the "drag coefficient" (a dimensionless number that depends upon the shape of the object and for a smooth sphere **C_D** ≈ 0.5), **ρ_A** is the density of the **compressed** air (1.2 kgm^{-3} times the compression ratio), **s** is nozzle length, **d** is nozzle diameter, **ρ_S** is shot density, **va** is the velocity of the air stream and **vs** is the velocity of the shot particle. **(v_a – v_s)** is termed the "relative velocity" of the particle compared with that of the air stream.

Equation (2) represents a model of air-blast shot velocity generation. Its best use is for estimating the effect on shot velocity of changing the value of the variables.

Wheel-Blast Shot Velocity

The physical components of a wheel-blast machine are quite different from those for air-blasting as illustrated by fig.7 on page 34.

Wheel-blast velocity is generated by employing two components—tangential velocity, **V_T**, and radial velocity, **V_R**. Again, the relevant formulae were described in a previous article (TSP, Spring, 2007). Fig.8 shows how the two components combine to give **V_S**.

Tangential velocity, **V_T**, is given by:

$$\mathbf{V_T = 2\pi.R.N} \qquad (3)$$

Where **R** is the blade length and **N** is the number of revolutions per second. As an example, if **R** = 0.250 m and **N** = 50 r.p.s. then **V_T** = 78.5 m.s^{-1}.

Radial velocity, **V_R**, is given by:

$$\mathbf{V_R = 2.\pi.N(2.R.L - L^2)^{0.5}} \qquad (4)$$

The combined wheel-blast shot velocity, **V_S**, is given by:

$$\mathbf{V_S = 2.\pi.N(R^2 + 2.R.L - L^2)^{0.5}} \qquad (5)$$

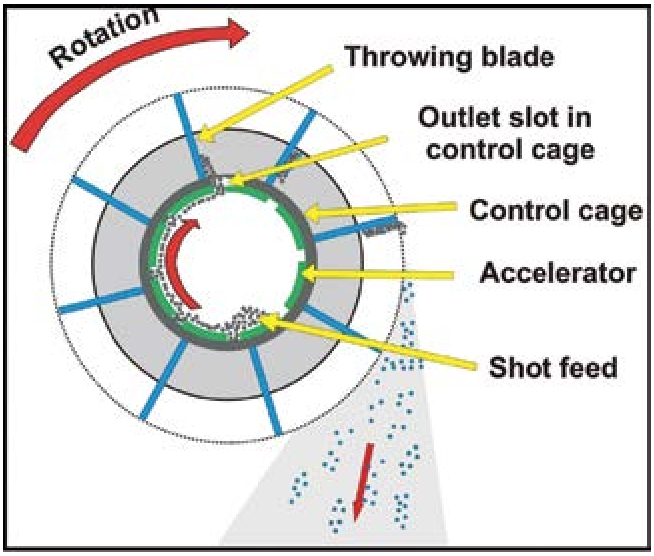

Fig.7. Wheel-blast system with "open" throwing blades.

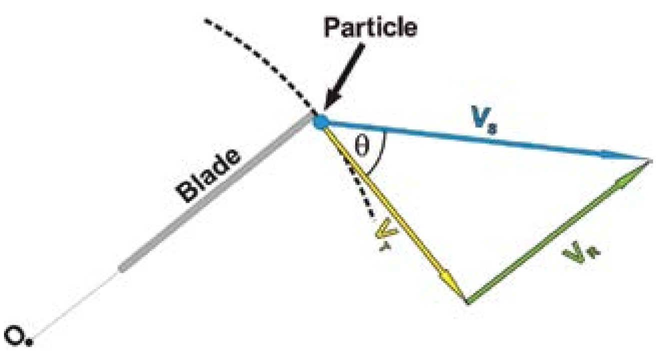

Fig.8. Individual particle leaving blade tip with vector-combined velocity, **Vs**.

DENT SIZE

Dent size can, of course, be measured directly. However, a previous article (TSP, Spring 2004) presented an equation that shows how different parameters influence dent size:

$$\mathbf{d = 0.02284*D*(1 - e^2)^{0.25} *\rho^{0.25}*v^{0.5}/B^{0.25}} \qquad (6)$$

Where **d** = dent diameter, $\mathbf{(1 - e^2)}$ = proportion of absorbed impact energy, **D** = shot diameter, $\boldsymbol{\rho}$ = shot density, **v** = shot velocity and **B** = Brinell hardness of component. All of the parameters in equation (6) are known except for the proportion of absorbed impact energy. This can be assumed to be close to 0.5. Dent diameter is directly proportional to shot diameter with other parameters remaining constant. Shot velocity is the next most influential parameter followed by the others.

SURFACE HEATING CAUSED BY PEENING

Energy cannot be destroyed—it can only be transferred. For example, some 90% of the energy absorbed by impacting shot particles is transferred into heat. Fig.9 shows experimental results obtained when air-blasting Almen N strips using different combinations of air pressure and flow rate with S170 shot.

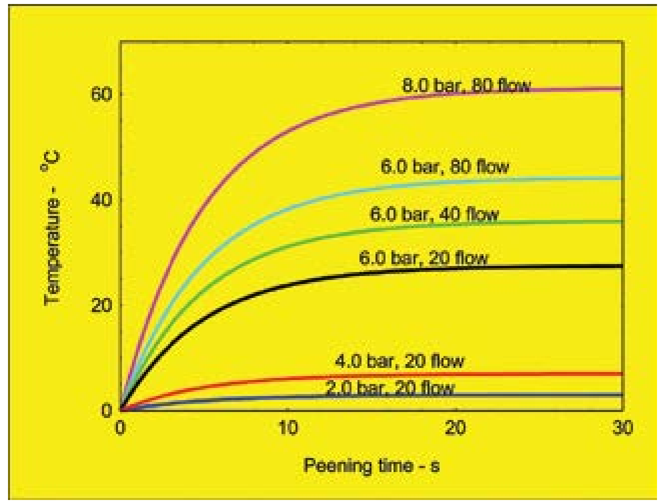

Fig.9. Surface heating curves for Almen N strips and S170 shot.

Surface heating was described in a previous TSP article (Summer, 2003). The measurements indicate that significant surface heating can be expected when shot peening.

DISCUSSION

An attempt has been made to show how mathematics pervades all aspects of shot peening. Mathematical techniques have allowed shot peening to graduate into a technologically advanced process. Most of the techniques are models that approximate, more or less closely, to real-life situations.

A fundamental advantage of mathematical techniques is that they are objective. Unless we make a mistake, we should all get the same answer.