



# ELASTICITY

## The Missing Link

### INTRODUCTION

In spite of its importance, elasticity is rarely even mentioned in peening articles. This rarity justifies it being considered as a “missing link”. This article is divided into two parts. Part A covers the relationship between elasticity and induced indentations. Part B deals with effect of elastic modulus,  $E$ , on the deflection of shot-peened Almen strips.

### PART A

### INTRODUCTION

Elasticity is a very important property for shot peening. Its magnitude is the link that controls the proportion of a flying shot's kinetic energy that is used to produce indentations in components.

When a flying shot particle strikes a component part of its kinetic energy,  $\frac{1}{2}mv^2$ , is absorbed by the component and part is retained as the kinetic energy of the rebounding particle. This important principle is illustrated in fig.1. Imagine a ball bearing being dropped from a height of 1 m onto a steel plate rebounding to the half-height of 0.5 m. This ratio has been shown, by experiment, to be similar to when steel shot strikes steel components.

Two inter-related parameters indicate the degree of elasticity for materials. These are “Coefficient of Restitution” and “Elastic Modulus”. These parameters and their relevance to shot peening form the subject of this article.

### COEFFICIENT OF RESTITUTION

The coefficient of restitution,  $e$ , is defined by:

$$e = \text{rebound velocity} / \text{impact velocity} \quad (1)$$

For dropped bouncing objects  $e$  can be estimated from:

$$e = \sqrt{h/H} \quad (2)$$

where  $h$  is the rebound height and  $H$  is the drop height.

Consider the situation illustrated in fig.1.  $e = \sqrt{(0.5\text{m}/1\text{m})} = \sqrt{0.5} = 0.71$ .

As stated in the Introduction, the bounce height ratio for steel shot striking steel components is also close to 0.5. We can therefore conclude that:

**The coefficient of restitution for shot peening with steel shot is close to 0.71.**

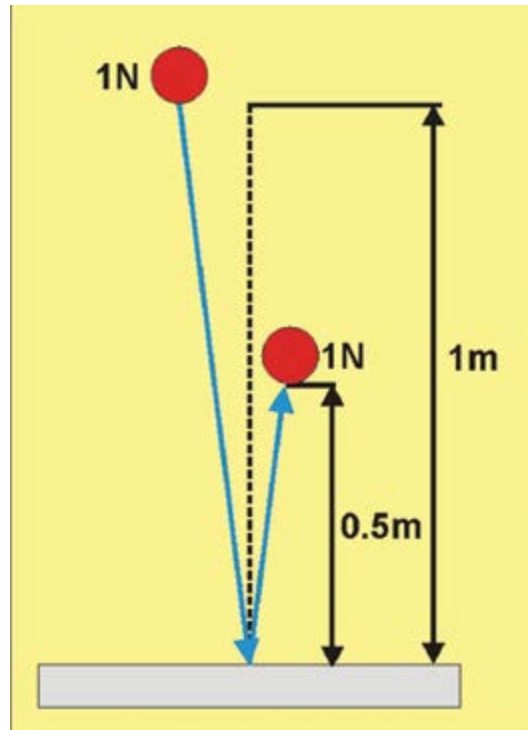


Fig.1. Kinetic energy absorption when impacting a component's surface.

### ENERGY TRANSFER

Experimental results indicate that in the shot peening process, the energy transfer from the shot into the target material is mainly in the form of so-called “Elastic-Plastic deformation energy”. For energy transfer during shot peening, Plastic deformation energy accounts for the major part ( $\geq 72\%$ ) and the rest ( $\leq 28\%$ ) is Elastic deformation energy. The plastic deformation energy is consumed by plastic deformation under the dent. Elastic deformation energy goes into elastic deformation of both the shot particle and the component during impact. Up to 70% of the elastic deformation energy is released during the elastic recovery of the shot particle and the component. The remaining 30% of the elastic deformation energy is stored to provide the driving power for permanent macroscopic deformation of the shot-peened components, e.g., bending.

**ELASTICITY LEVELS**

Shot particles and components are solids and therefore have a certain level of elasticity. Shot particle elasticity is of primary importance. That importance is the tendency of the particle to return to its original shape after impact with a component. During impact, the force the particle exerts on the component increases rapidly. This force causes the particle to increasingly flatten. As the particle rebounds this flattening is removed. It would be disastrous if the particle stayed flattened!

Useful elasticity levels are indicated by two parameters—elastic modulus and yield strength.

We need impacting shot to behave elastically as illustrated by fig.2 (where flattening has been greatly exaggerated).

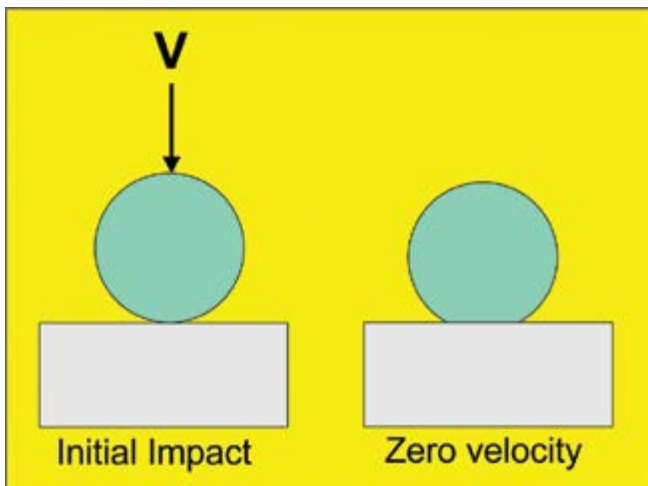


Fig.2. Elastic shot flattening on impact with component.

**Elastic Modulus**

The relationship between applied force and elastic deformation is given by Hooke's Law. This predicts that, for small strains, there is a linear relationship between applied stress and resulting induced strain as illustrated in fig.3. The ratio of applied stress divided by induced strain is known as the elastic modulus, E.

Fig.3 simply indicates that solids can have a wide range of elastic modulus values. Table 1 gives typical numerical values of E and approximate yield strength for some relevant materials.

The great advantages of hard steel as shot material are: low cost, reasonable elastic modulus level, and its ability to be heat treated to very high-yield strength levels. This means that heat-treated steel shot would be incredibly difficult to plastically deform on impact—thank goodness—as illustrated by fig.3. Component properties can, however, require the employment of different shot materials.

**Modulus Measurement**

Elastic modulus can be measured in tension (pull testing), compression, and bend/flex testing. Measurement for indi-

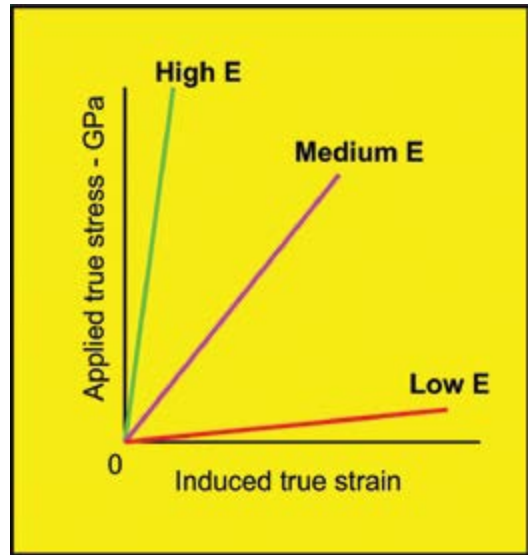


Fig.3. Range of elastic modulus values.

**Table 1 Elastic Modulus and Yield Strength Values**

Material	E, GPa	Yield Strength, MPa
Hard Steel	210	200-2100
Stainless Steel	180	500
Ceramic	345	≈ 150
Aluminum	71	50
Tungsten Carbide	550	350

vidual shot particles would be very tricky! We resort to measurements on rod samples made and heat-treated from the same steel.

**PART B**

**INTRODUCTION**

Shot-peened Almen strips, on release from their fixture, adopt a curved shape, see fig.4.

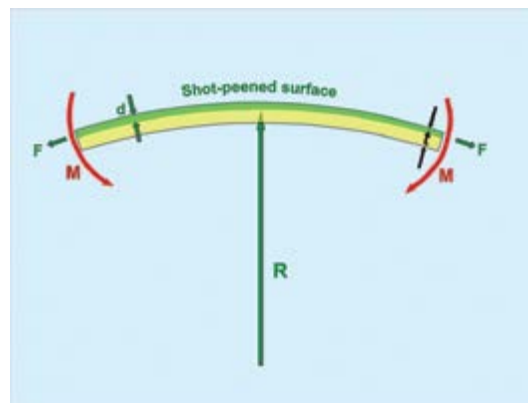


Fig.4. Curved shot-peened Almen Strip.

Curving is due to two factors:

- (1) **Plastic deformation of the peened surface layer and**
- (2) **Compressive residual stress in the peened surface layer.**

The two factors have been shown, experimentally, to be approximately equal in magnitude. This was achieved by measuring the arc height before and after stress-relieving heat treatment. It follows, as an important principle, that:

**Half of measured Almen Arc Height is caused by permanent Plastic Deformation and Half is caused by temporary Residual Stress.**

Plastic deformation is independent of elastic modulus. Residual stress curving is directly dependent on the compressive residual stress in the deformed surface layer.

**CURVATURE VERSUS ALMEN ARC HEIGHT**

The magnitude of curving is classically given the term “Curvature”, which is defined as 1/R. R being the radius of bending, as shown in fig.4. The smaller the value of R the greater will be the magnitude of 1/R. Arc height can be related to curvature using Euclid’s “Intersecting Chord Theorem”. This was presented by him some 2,500 years ago! Fig.5 shows how his theorem can be applied to convert Almen arc height, h, into curvature, 1/R.

**h** is the measured Almen arc height, **x** is half of the strip’s length between ball supports and **R** is the effective radius of curvature (actually (R-h) but h is so tiny relative to R that it can be ignored).

Euclid’s Intersecting Chord Theorem tells us that  $x^2 = h \cdot R$ , hence:

$$\text{Curvature } 1/R = h/x^2 \tag{3}$$

Equation (3) shows that curvature increases directly with the magnitude of arc height.

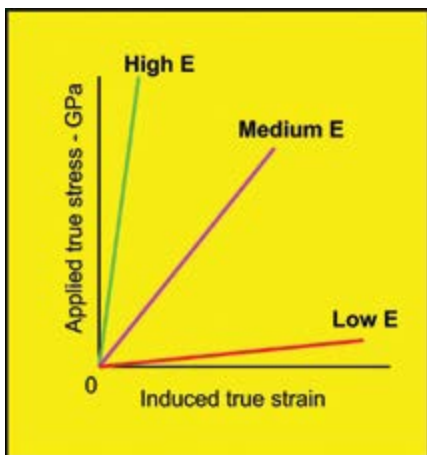


Fig.5. Euclid’s Chord Theorem applied to Almen Strip curvature.

**EFFECT OF ELASTIC MODULUS ON MEASURED ARC HEIGHT**

Compressive residual stress in the peened surface layer causes half of the induced curvature. The amount of curvature depends on the Almen strip’s ability to resist bending. This resistance ability is given by:

$$\text{Bending resistance} = E \cdot I \tag{4}$$

where **E** is elastic modulus and **I** is the “rigidity factor” (technically known as the “second moment of area” of the strip).

Equation (4) tells us that the resistance to elastic bending depends directly on the magnitude of the strip’s elastic modulus. The higher the modulus the greater will be the resistance to bending. It follows that the lower will be the contribution to arc height induced by compressive residual stress in the peened surface layer.

The compressive residual stress in the peened surface layer generates the bending moment, **M**, shown in fig.4. The greater the induced bending moment the greater will be the observed arc height. Basic beam bending theory gives us a simple relationship between the bending moment applied to a beam and its consequent curvature, 1/R :

$$1/R = M/(E \cdot I) \tag{5}$$

where **R** is radius of bending, **E** is elastic modulus, **I** is the “second moment of area” and **M** is applied bending moment.

Equation (5) indicates that curvature (and therefore arc height) increases with increased bending moment but is decreased by increases in either elastic modulus or “second moment of area”.

Bending moment and elastic modulus are familiar parameters. “Second moment of area” is less familiar. It is simply a quantitative measure of the rigidity of a beam. Fortunately Almen strips, because of their rectangular shape, have a simple relationship between “second moment of area”, **I**, and their dimensions:

$$I = w \cdot t^3 / 12 \tag{6}$$

where **w** is strip width and **t** is the strip thickness.

The significance of equation (6) can be appreciated by trying to bend a measuring rule. In one direction the rule bends easily. Turn the rule through 90° and it is virtually impossible to achieve visible bending.

If we substitute the value of **I** given by equation (6) into equation (5) we get:

$$1/R = 12M / (E \cdot w \cdot t^3) \tag{7}$$

Substituting  $h/x^2 = 1/R$  from equation (3) gives:

$$h = 6x^2 \cdot M / (E \cdot w \cdot t^3) \tag{8}$$

where **h** is induced arc height contribution and **x** is half the distance between the support balls of the Almen gage.

Equation (8) is a “definitive equation” that indicates the inter-relationship of all of the significant variation of strip factors.

**ALMEN STRIP ELASTIC MODULUS VARIABILITY MEASUREMENT**

There is no doubt that the Elastic modulus of Almen strips can vary. Variations can occur in both composition and thermomechanical processing. Of these, the incidence of preferred orientation is probably the largest factor. Measurement of strip elastic modulus can be carried out by conventional tensile testing or by a Go-No-Go test.

With conventional tensile testing a “dog bone” shaped specimen is pulled using a tensile testing machine. For small strains loading gives a linear stress/strain plot as shown in fig.5. The ratio of stress/strain, E, is derived from the applied stresses and resulting strains as indicated in fig.3.

A simple Go-No-Go test is illustrated in fig.6. A long length of strip, say 1 metre, is supported on fixed rods placed against drawn “too high” and “too low” background curves. For each thickness of Almen strip, N, A and C, a corresponding appropriate load is hung from the center of the strip. Load hanging could be facilitated by using the type of hook shown in the inset. Loading induces bending into a curve shape. If the strip modulus is deemed to be too high then not enough bending will occur. Too much bending would occur if the modulus was deemed to be too low.

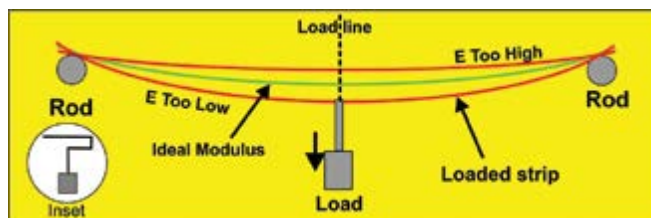


Fig.6. Go-No-Go test facility.

For individual Almen strips a device such as that shown in fig.7 can be used. A force meter is pressed against the center of the peened Almen strip. The Almen gauge then indicates the corresponding strip bending. The Almen gauge readings for different applied forces will yield a straight line graph whose slope is the elastic modulus. This is somewhat tedious but an alternative is to use the device in Go-No-Go mode. For a given thickness of Almen strip, a fixed applied force level will induce a gauge deflection that varies with the magnitude of elastic modulus, E.

**DISCUSSION**

This article has shown that the elastic properties of shot peening components—especially Almen strips—deserve far more attention than they are generally given. As such, Elasticity can rightly be regarded as a “Missing Link” in the shot peening world.



Fig.7. Device for Almen strip elastic modulus determination.

The elasticity of shot particles determines the proportion of their kinetic energy that is used in dent formation. This also applies to the elasticity of the component. More importantly, elastic modulus value affects the amount of bending of Almen strips when subjected to shot peening.

Elastic modulus variability can be determined easily for long Almen strip samples. Individual strips can be tested quickly using the device just described. For both methods it is necessary to check that strip width—and particularly strip thickness—are not intruding variables. ●

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